HANDBOOK OF ACOUSTICS

By T. F. HARRIS

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HANDBOOK

OF

ACOUSTICS

(CURWEN'S EDITION, 5005.)

FOR THE USE OF MUSICAL STUDENTS.

BY

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PREFACE.

Although many works on the subject of Acoustics have been written for the use of musical students, the author of this book has not met with one which gives, in an elementary form, more than a partial view of the science. Thus, there are several admirable treatises on the purely physical and experimental part, but most if not all of them stop short just when the subject begins to be of especial interest to the student of music. On the other hand, there are many excellent works, which treat of the bearings of purely acoustical phenomena on the science and art of music, but which presuppose a knowledge of such phenomena and their causes on the part of the reader. Thus the ordinary musical student, who can probably give but a limited amount of time to this part of his studies, is at the disadvantage of having to master several works, each probably written in a totally different style, and possibly not all agreeing perfectly with one another as to details. This disadvantage has been felt by the author, in his classes for some years past, and the present work has been written with the object of furnishing to the student, as far as is possible in an elementary work, a complete view of Acoustical science and its bearings on the art of music.

In the arrangement of the subject, the reader should observe that up to and including the 7th Chapter, the sounds treated of are supposed to be *simple;* the next four chapters treat of sounds—both simple and compound—*singly*, that is to say, only one tone is supposed to be produced at a time; the phenomena accompanying the simultaneous production of two or more sounds are reserved for the remaining chapters.

The movable Sol-fa names for the notes of the scale have been used throughout, as they are so much better adapted to scientific treatment than the fixed Staff Notation symbols. It may be useful to readers not acquainted with the Tonic Sol-fa Notation to mention that in this system, the symbol **d** is taken to represent a sound of any assumed pitch, and the letters, **r, m, f, s, l, t,** represent the other tones of the diatonic scale in ascending order. The sharp of any one of these tones is denoted by placing the letter **e** after its symbol: thus, the sharp of **s** is **se**; of **r**, **re**; and so on. The flat of any tone is denoted by placing the letter **a** after its symbol: thus the flat of **t** is **ta**; of **m**, **ma**; and so on. The upper or lower octaves of these notes are expressed by marks above or below their symbols: thus $\mathbf{d}^1$ is one octave, $\mathbf{d}^2$ two octaves above **d**; $\mathbf{s}_1$ is one octave, $\mathbf{s}_2$ two octaves below **s**. Absolute pitch has been denoted throughout by the ordinary symbols, C representing the note on the ledger line below the treble staff. Its successive higher octaves are denoted by placing the figures 1, 2, 3, &c., above it, and its lower octaves by writing the same figures beneath it; thus, C^1, C^2, C^3, &c.; C_1, C_2, C_3, &c.

It is perhaps as well to observe, that although Helmholtz's theory as to the origin of Combination Tones given in Chap. XII is at present the received one, it is possible that in the future it may require modification, in view of the recent researches of Preyer, Kœnig, and Bosanquet.

Although this book has not been written expressly for the use of students preparing for any particular examination, it will be found that a mastery of its contents will enable a candidate to successfully work any papers set in Acoustics at the ordinary musical examinations, including those of the Tonic Sol-fa College, Trinity College, and the examinations for the degree of Bachelor of Music at Cambridge and London. The papers set at these examinations during the last two years, together with the answers to the questions, will be found at the end of the book.

The text will be found to be fully illustrated by figures, of which Nos. 1, 2, 20, 21, 23, 36, 37, 50, 51, 64 are taken from Deschanel's Treatise on Natural Philosophy, and Nos. 4, 5, 58, 72, 78 from Lees' Acoustics, Light, and Heat, by permission of Messrs. Blackie & Sons and Collins & Son respectively. All the other figures have been cut expressly for this work.

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HAND-BOOK OF ACOUSTICS.

CHAPTER I.

Introductory: The Origin of a Musical Sound.

It must be evident to every one, that the cause of the sensation we term "sound," is something external to us. It is almost equally obvious that this external cause is motion. To be convinced of this

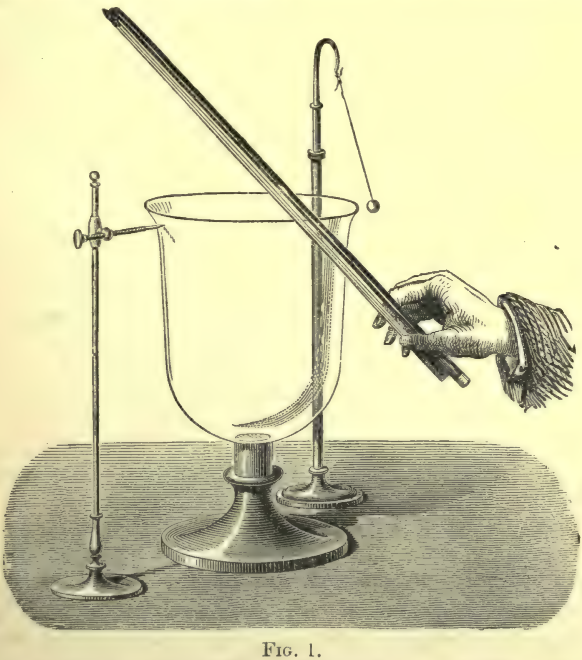


Fig. 1.

fact, it is only necessary to trace any sound to its origin; the sound from a piano, for example, to its vibrating string, or that from a harmonium to its oscillating tongue. If the glass bell (fig. 1) be bowed, it will emit a sound, and the little suspended weight will be violently dashed away; the rattle of the moving glass against the projecting point will also be plainly heard. Even where the movement cannot be seen, as in most wind instruments, it may easily be felt.

Although all sounds are thus produced by motion, movements do not always give rise to the sensation, sound. We have therefore to ascertain, what particular kind of motion is capable of producing the sensation, and the conditions necessary for its production. Sounds may be roughly classified as musical or unmusical. As we are only concerned here with the former, it will be as well first to distinguish as far as possible between the two classes. For acoustical purposes, we may define a musical sound to be that, which, whether it lasts for a long or short period of time, does not vary in pitch. In other words, a musical sound is a steady sound. In an ordinary way, we say that a sound is musical or unmusical, according as it is pleasant or otherwise, and on examination, this will be found to agree fairly well with the more rigid definition above, especially if we bear in mind the fact, that most sounds consist of musical and unmusical elements, and that the resulting sound is agreeable or the reverse, according as the former or the latter predominate. For example, the sound produced by an organ pipe consists of the steady sound proper to the pipe, and of the unsteady fluttering or hissing sound, caused by the current of air striking the thin edge of the embouchure; but, as the former predominates greatly over the latter, the resulting sound is termed musical. Again, in the roar of a waterfall we have the same two elements, but in this case, the unsteady predominates over the steady, and an unmusical sound, or noise, is the result.

We have just seen that the external cause of a musical sound is motion; we shall further find on examination, that this motion is a periodic one. A periodic motion is one that repeats itself at equal intervals of time; as, for example, the motion of a common pendulum. In order to satisfy ourselves that a musical sound is caused by a periodic motion, we will examine into the origin of the sounds produced by strings, reeds, and flue pipes.

A very simple experiment will suffice in the case of the first named. Stretch a yard of common elastic somewhat loosely between two pegs. On plucking it in the middle, it begins vibrating, and although its motion is somewhat rapid, yet we have no difficulty in counting the vibrations; or at any rate, we can see that they follow one another regularly at equal intervals of time. Further, we may notice, that this is the case, whether we pluck the string gently or violently, that is, whether the vibrations are of large or small extent. The motion of the string is therefore periodic, — its vibrations are all executed in equal times. If now the elastic be stretched a little more, the vibrations become too rapid for the eye to follow. We see only a hazy spindle, yet we cannot doubt but that the kind of motion is the same as before. Stretch the string still more, and now a musical sound is heard, which is thus caused by the rapid periodic motion of the string.

A similar experiment proves the same fact with regard to reed instruments. Fasten one end of a long thin strip of metal in a vice (fig. 2). Displace the other end (D) of the strip, and let it go.

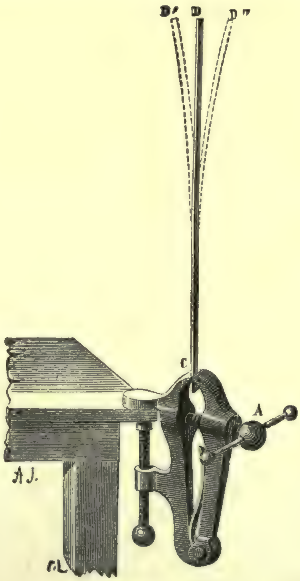


FIG. 2.

The strip vibrates slowly enough for us to count its vibrations, and these we find to recur regularly; that is, the motion is periodic. Gradually shorten the strip, and the vibrations will follow one another faster and faster, till at length a musical sound is heard. Although we cannot now follow the rapid motion of the strip, yet, as in the case of the string above, we may fairly conclude that its character remains unaltered; that is, the motion is still periodic.

In such an instrument as a flue pipe, the vibrating body is the air. Although this itself is invisible, it is not difficult to render its motion visible. Fixed vertically in the stand (fig. 3) is a glass

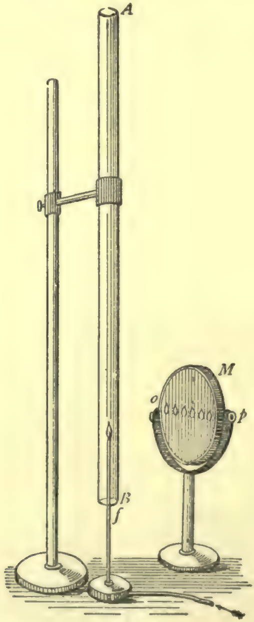


FIG. 3.

tube A B, about 2ft. long and an inch in diameter. Passing into the lower end of the tube is a pin-hole gas jet, (f), joined to the ordinary gas supply by india-rubber tubing. Before the jet is introduced into the tube, it is ignited, and the gas turned down, until the flame is about an inch or less in height. On inserting this into the glass tube, after a little adjustment, a musical sound is heard coming from the tube. It is, in fact, the well-known singing flame. The particles of air in the tube are in rapid vibration, moving towards the centre and from it, alternately. The air particles at that part of the tube, where the flame is situated, will therefore be alternately, crowded together and scattered wider apart; that is, the pressure of the air upon the flame will be alternately greater and less than the ordinary atmospheric pressure. The effect of the greater pressure upon the flame will be to force it down, or even extinguish it altogether; the effect of the lesser pressure will be to enlarge it. Thus the flame will rise and fall at every vibration of the air in the tube. These movements of the flame are too rapid, however, to be followed by the eye, and the flame itself will still appear to be at rest. In order to observe them, recourse must be had to a common optical device. First, reduce the tube to silence, by lowering the position of the jet. Having then darkened the room, rotate a mirror (M) on a vertical axis behind the flame. The latter now appears in the mirror as a continuous yellow band of light, for precisely the same reason, that a lighted stick, on being whirled round, presents the appearance of a luminous circle. Now restore the jet to its former position in the tube. The latter begins to sing, and on rotating the mirror we no longer see a continuous band of light, but a series of distinct flames (*o p*) joined together below by a very thin band of light. This clearly shows, that the flame is alternately large and very small; that is, alternately rising and falling, as described above. Now while the mirror is being rotated at an even rate, notice that the intervals between the flames are all equal, and also that the flames themselves are all of the same size. From what is stated above, it will be seen that this proves our point, namely, that the sound in this case is produced by the periodic or vibratory motion of the particles of air.

By examining in this way into the origin of other sounds, it will be found that all musical tones are caused by the periodic motion of some body. Further, a periodic or vibratory motion will always produce a musical sound, provided, (1), that the vibrations recur with sufficient rapidity; (2), that they do not recur too rapidly; (3), that they are sufficiently extensive, and the moving body large enough. The following experiments will illustrate this.

Fig. 4 represents an ordinary cogwheel (B), having some 80 or 90 teeth, which can be rapidly rotated by means of the multiplying wheel (A). Holding a card (E) so as just to touch the cogs, we slowly turn the handle of the multiplying wheel. The card is

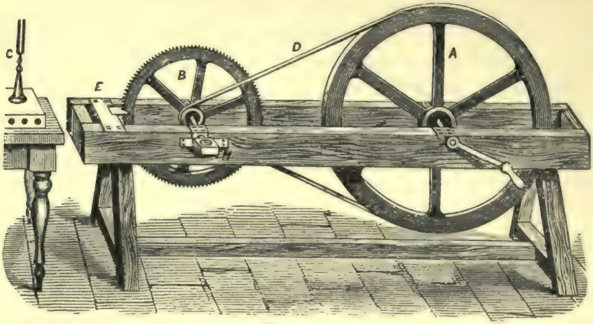


FIG. 4.

lifted slightly by each cog as it passes, but is almost immediately released, and falls back against the succeeding one; that is, it vibrates once for every cog that passes it. As long as the wheel is revolving slowly, the card may thus be heard striking against each cog separately. If, however, the speed be increased, the taps will succeed one another so rapidly as to coalesce, and then a continuous sound will be heard.

The well-known Trevelyan's rocker is intended to illustrate the same thing. It consists of a rectangular-shaped piece of copper about 6 inches long, $2\frac{1}{2}$ inches broad, and 1 inch thick. The lower side is bevelled, and has a longitudinal groove running down the middle, as shown in fig. 5. Attached to one end is a somewhat

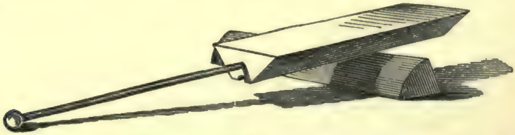


FIG. 5.

slender steel rod, terminating in a brass ball. If we place the rocker, with its bevelled face resting against a block of lead, and with the ball at the other end resting on the smooth surface of a table, it will rock from side to side on being slightly displaced, but not quickly enough to produce a musical sound. If, however, we hold the rocker in the flame of a Bunsen burner, or heat it over a fire, for a few minutes, and then place it as before against the leaden block, we shall find it giving forth a clear and continuous sound. This phenomenon may be explained thus:—When any point of the heated rocker on one side of the groove touches the lead, it imparts its heat to the latter at that spot. This causes the lead at that particular point to expand. A little pimple, as it were, darts up and pushes the rocker back (fig. 6), so that it falls over on

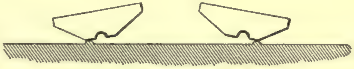


FIG. 6.

to the other side of the groove. A second heat pimple is raised here, and the rocker is tilted in the contrary direction, and thus the motion is continued, as long as the rocker is sufficiently hot.

SUMMARY.

The cause of the sensation *sound*, is *motion*.

A musical sound is one which remains steadily at a definite pitch.

A musical sound originates in the periodic motion of some body.

A periodic or regular vibratory motion will always give rise to a musical sound, if the vibrations recur with sufficient, but not too great rapidity, and provided they are extensive enough.

CHAPTER II.

The Transmission of Sound.

We have seen in the preceding chapter, that all sounds originate in the vibratory movements of bodies. This vibratory motion is capable of being communicated to, and transmitted by, almost all substances, to a greater or less extent. Wood, glass, water, brass, iron, and metals in general may be taken as examples of good conductors of sound. But the substance, which in the vast majority of cases transmits to our ears the vibratory motion, which gives rise to the sensation, sound, is the *air*.

A very little reflection is sufficient to show us that some medium is absolutely necessary for the transmission of sound; for inasmuch as sound is caused by vibratory motion, it is plain that this motion cannot pass through a vacuum. This fact can, however, be easily proved experimentally. Under the receiver of an air-pump a bell is suspended. We shake the pump, and the bell may be heard ringing; for its vibrations are communicated by the air to the glass of the receiver, and by the latter to the outer air, and so to our ears. We now pump as much air as possible out of the receiver, and again ring the bell. It can still be heard faintly, for we cannot remove all the air. We now allow dry hydrogen to pass into the receiver, and on again ringing the bell, there is very little increase of sound, this gas being so very light—only about $\frac{1}{14}$ as heavy as air. If we now exhaust the receiver, we shall be able still further to attenuate the atmosphere within it, and then, although we may violently shake the apparatus, no sound will be heard.

As the air is generally the medium, by means of which the vibratory motion reaches our ears, we shall now have to carefully study the manner in which this transmission takes place. When we stand on the sea shore, or better still, on a cliff near it, and watch the waves rolling in from afar, our first idea is, that the water is actually moving towards us, and it is difficult at the time to get rid of this notion. If, however, we examine a little more closely, we see that the boats and other objects floating about, do not travel forward with the wave, but simply rise and fall as it passes them. Hence we conclude that the water of which the wave is composed, is not moving towards us. What is it, then, that is being transmitted? What is it that is moving forwards? The up and down movement—*the wave motion.* Now the vibratory movements which give rise to sound are, as we shall presently see, transmitted through the air much in the same way; and therefore, although a sound wave is not exactly analogous to a water wave, yet a brief study of the latter will help us more easily to understand the former.

Let ADECB (fig. 7) represent the section of a water wave, and the dotted line AEB the surface of still water. That part of the wave ADE above the dotted line, is termed the crest, and the part ECB below, the trough. Through D and C, the highest and lowest points, draw DH and FC, parallel to AB, and from the same points draw DF and CH perpendicular to it. Then the distance AB is termed the *length* of the wave, DF or CH is its *amplitude*, and the outline ADECB is its *form*.

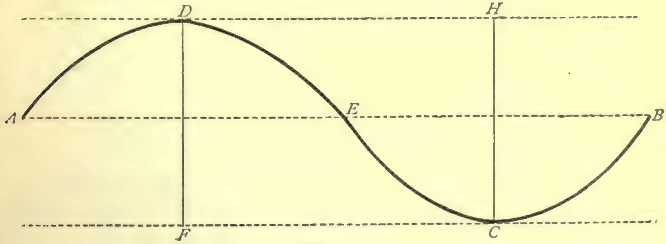


FIG. 7.

These three elements completely determine a wave, in the same way as the length, breadth and thickness of a rectangular block of wood, determine its size; and further, as any one of these three dimensions may vary independently of the other two, so any one of the three elements—length, amplitude, and form—may vary, the other two remaining constant. Thus in fig. 8 (1), we have three waves of the same amplitude and form, but varying in length; in fig. 8 (2), we have three waves of same length and form, but of

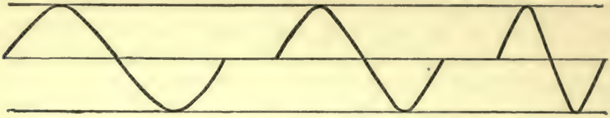


Fig. 8 (1).

different amplitudes; while in fig. 8 (3), we have three waves of same length and amplitude, but of varying forms.

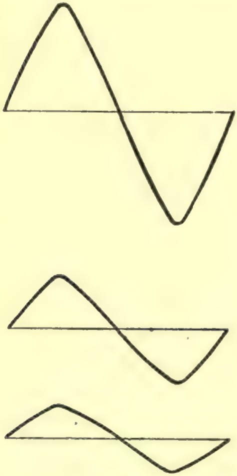


Fig. 8 (2).

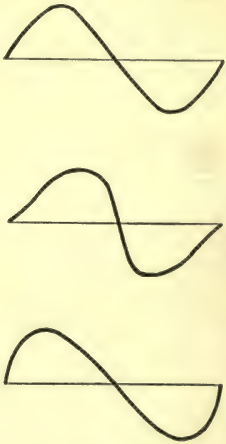


Fig. 8 (3).

We now have to inquire into the nature of the movements of the particles of water which form the wave. The following apparatus will assist the student in this inquiry. Fig. 9 represents a box

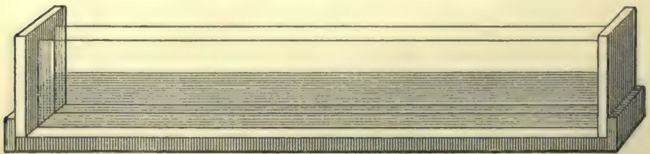


Fig. 9.

about four feet long, four inches broad, and six inches deep, with either one or both the long sides of glass, and caulked with marine glue, so as to be water-tight. This is more than half filled with water, in which are immersed at different depths, balls of wax mixed with just so much iron filings, as will make them of the same specific gravity as water. Now alternately raise and depress one end of the box, so as to give rise to waves. The balls will describe closed curves in a vertical plane, the horizontal diameter of which will much exceed the vertical. If a deeper trough be taken, the difference between the horizontal and vertical diameters of these curves will become less; and in fact, in very deep water the two diameters become of the same length; that is, the closed curves become circles. It will be noticed also, that each particle of water describes its curve and returns to its original position, in the same time as the wave takes to move through its own length. Bearing in mind these facts, we may plot out a water wave in the manner shown in fig. 10. In the top row are 17 dots, representing

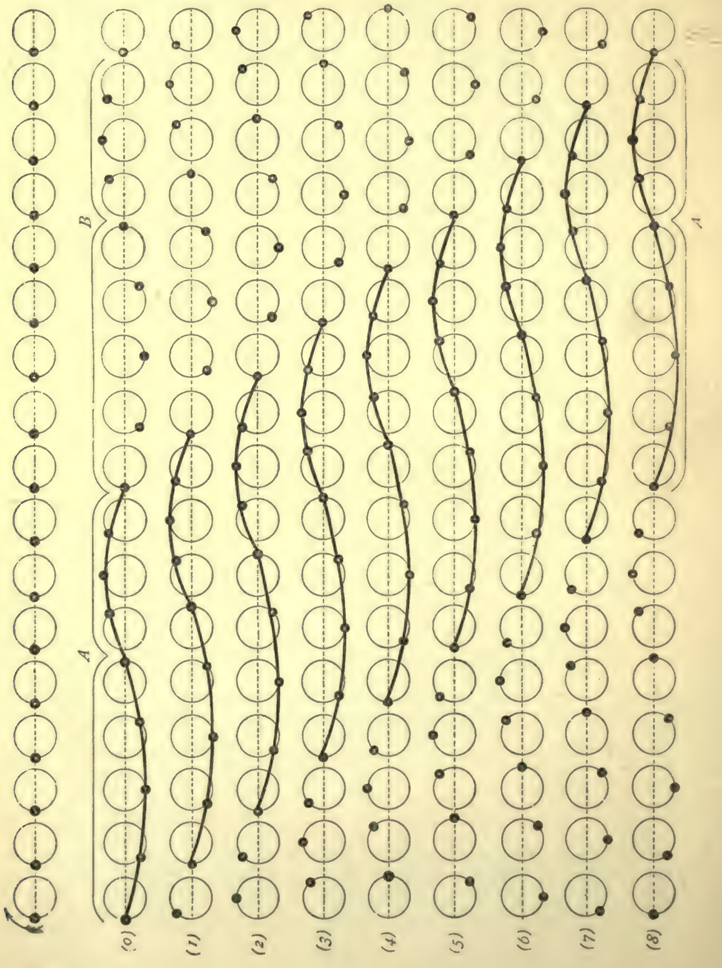


FIG. 10.

17 equidistant particles of water at rest, and the circles in which they are about to move, in the same direction as the hands of a clock. In (0), the position of each of these particles is given at the moment when they form parts of the two complete waves A and B, which are supposed to be passing from left to right; the 8th particle from the left has just passed through $\frac{1}{8}$th of its journey, that is $\frac{1}{8}$th of the whole circumference; the 7th particle has passed through $\frac{2}{8} = \frac{1}{4}$; the 6th through $\frac{3}{8}$; the 5th has just performed $\frac{1}{2}$ its journey; the 4th, $\frac{5}{8}$; the 3rd, $\frac{3}{4}$; the 2nd, $\frac{7}{8}$; and the first has just completed its course and regained its original position. In (1) each particle has moved through $\frac{1}{8}$ more of its curve, and the wave has passed through a space equal to $\frac{1}{8}$ of its length. In (2) each particle has moved through $\frac{1}{8}$ more, and the wave has again advanced as before; and so on, in (3), (4), (5), (6), (7), and (8). Each particle in (8) has the same position as in (0), having made one complete journey; and the wave has advanced one wave length. Thus we see, as in the experiment with the trough, that each particle makes one complete journey in the same time as the wave takes to travel its own length.

Another variety of wave motion may be studied thus: Fill an india-rubber tube, about 12 feet long and $\frac{1}{4}$ inch in diameter, with sand, and fasten one end to the ceiling of a room. Hold the other end in the hand, and jerk it sharply on one side. Notice the wave motion thus communicated to the tube, the wave consisting of two protuberances, one on each side of the position of the tube at rest; the one corresponding to the crest of the water wave, and the other to its trough. Let the uppermost row of dots in fig. 11, represent

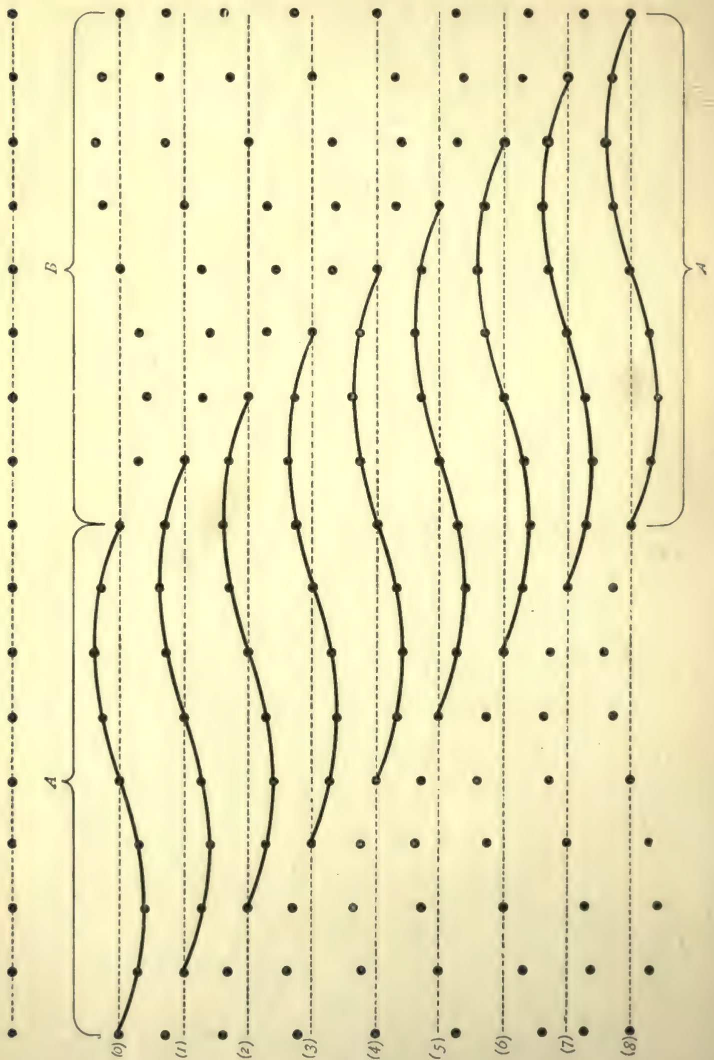


FIG. 11.

17 equidistant particles of the tube at rest; (0) gives the positions of the particles when they form parts of two complete waves A and B; and (1) to (8) show their positions at successive intervals of time, each equal to $\frac{1}{8}$ of the time that it takes a particle to perform one complete vibration. By a careful inspection of this diagram, it will be seen that while each particle performs one complete vibration, the wave travels through a space equal to its own length. Taking the 1st particle in A, for example :—in (1), it has risen half way to its highest point; in (2), it has reached its highest point; in (3), it is descending again; in (4), it is passing downwards through its original position; in (5), it is still sinking; in (6), it has reached its lowest position; in (7), it is ascending again; in (8), it has reached its original position, having made one complete vibration. Now it is obvious from the figure, that during this time the wave A has passed through a space equal to its own length.

Let the curve in fig. 12 represent the outline section of the wave A in fig. 11. Through the deepest and highest points (d) and (c) of the protuberances, draw the dotted straight lines parallel to AB, the position of the tube when at rest. Through (d) draw (df) at right angles to AB; then as before (df) is the amplitude, of the wave. Now on comparing this with fig. 11 it will be seen that this amplitude is the same thing as extent of vibration of each particle.

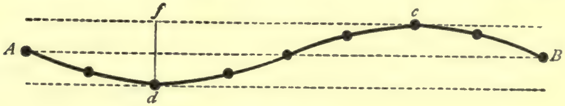


FIG. 12.

If we suppose, that a particle takes a certain definite time, say one second, to perform its vibration, it is evident that the number of different modes in which it may get over its ground in this time is infinite. Thus it may move slowly at first, then quickly and again slowly; or it may start quickly, then slacken, again quicken, slacken again, and finish up quickly; and so on. In fig. 13, two waves of equal length and amplitude are represented after the manner of fig. 12. The extent and time of particle vibration being the same, they only differ in the mode of vibration. In (b) the particle at first moves more quickly than in (a); it then moves more slowly; and so on. Thus for example: supposing the time of a complete vibration to be one second, the particle in (a) reaches its highest position in one quarter, and its lowest, in three quarters of a second after the start; while in (*b*) the particle reaches the same points in $\frac{1}{8}$, and $\frac{7}{8}$ of a second, respectively. Now it is plain from the figure, that this difference in mode of particle vibration is accompanied by a difference in the form of the wave, and a little reflection will show that this must always be the case. We have thus seen how the length, amplitude, and form of a wave are respectively connected with the time, extent, and mode of particle vibration.

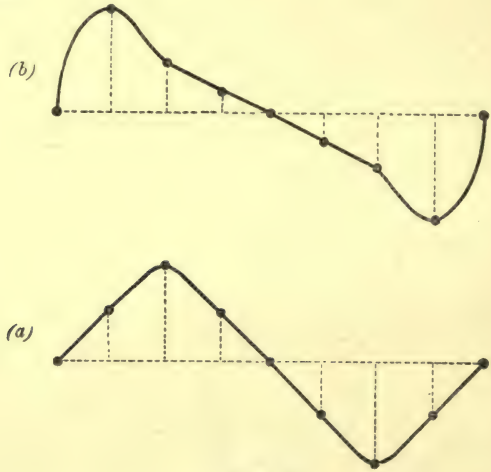


FIG. 13.

Having now made some acquaintance with wave motion in general, we may pass on to consider the particular case of the sound wave. The first point to which we must turn our attention, is the nature of the medium. Air, like every other gas, is supposed to be made up of almost infinitely small particles. These particles cannot be very closely packed together; in fact, they must be at considerable distances apart in proportion to their size, for several hundred volumes of air can be compressed into one volume. When air is thus compressed, the particles of which it consists, press against one another, and against the sides of the containing vessel in all directions; and that with a force which is the greater, the closer the particles are pressed together; in other words, the air is elastic. An ordinary popgun will illustrate this: a certain volume of air having been confined between the cork and the rammer of the gun, the latter is gradually pushed in; the air particles being thus crowded closer and closer together, exert an ever increasing pressure, until at length the weakest point, the cork, gives way.

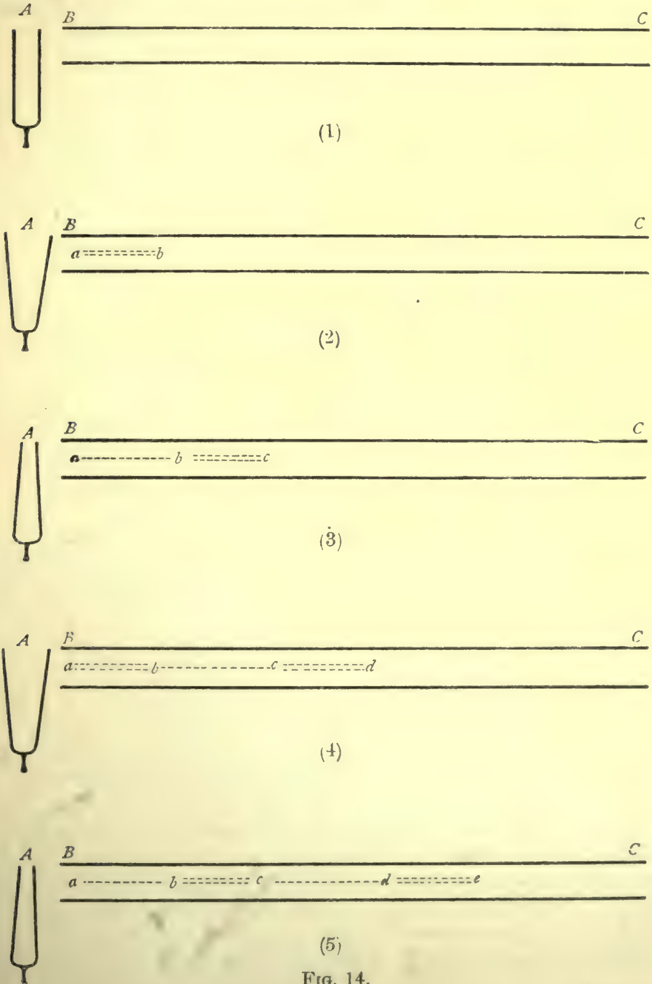


Fig. 14.

Let A, fig. 14 (1), represent a tuning-fork, and BC a long tube open at both ends. Suppose now, that the fork begins vibrating, its prongs first performing an outward journey. The air in BC will be condensed, but in consequence of the swiftness of the fork's motion, and the great elasticity of air, this condensation will be confined during the outward journey of the prongs, to a comparatively small portion of the tube BC: say to the portion (*ab*) fig. 14 (2). The air in (*ab*), being now denser than that in advance of it, will expand, acting on the air in (*b*C) as the tuning-fork acted upon it, thus causing a condensation in (*bc*) fig. 14 (3), while the air in (*ab*) itself, overshooting the mark, as it were, becomes rarefied, an effect which is increased by the simultaneous retreat of the prong A. Again, the air in (*bc*) fig. 14 (3), being denser than that before and behind, expands both ways, forming condensations (*cd*) and (*ab*) fig. 14 (4), in both directions; while in its place is formed the rarefaction (*bc*), the formation of the condensation (*ab*) being assisted by the outward journey of A. Next the air in (*ab*) and (*cd*) fig. 14 (4), being denser than that on either side, expands in both directions, forming the condensations (*bc*) and (*de*) fig. 14 (5); rarefactions being formed in (*ab*) and (*cd*), the former assisted by the retreat of the prong A. By further repetitions of this process the sound waves (*ac*) and (*ce*) will be propagated along the tube.

For the sake of simplicity, the motion has been supposed to take place in a tube. This restriction may now be removed. The movement of the air passing outward in every direction from the sounding body, the successive condensations and rarefactions form spherical concentric shells round it.

In fig. 14 (5), (*ac*) and (*ce*) form two complete sound waves, (*bc*) and (*de*) being the condensed, and (*ab*) and (*cd*) the rarefied parts. In studying the motion of the particles of air forming these sound waves, it will be simplest to consider those adjoining the prong A, for their motion will necessarily be similar to that of the prong itself. The first point to notice is, that the direction of particle vibration in this case is the same as that of the wave motion, and not transversely to it as in the case of the sand tube. In the next place, it will be observed, that the tuning-fork makes one complete vibration while the wave passes through a space equal to its own length; that is, each particle executes one complete vibration in exactly the same time as the wave takes to pass through a distance equal to its own length. Again, the amplitude of the wave is evidently the distance through which the individual particles of air vibrate; that is, the extent of particle vibration. Further, it is plain that, the greater the swing of the prong A, the greater will be the degree of condensation and rarefaction in (ab); that is, the greater the extent of particle vibration, the greater will be the degree of rarefaction and condensation. Lastly, just as the particles in the case of the sand tube could perform their vibrations in an infinite number of modes, so the fork A, and therefore the air particles, may perform their vibrations in an endless variety of modes, giving rise to as many wave forms.

It is often convenient to represent a sound wave after the same manner as a water wave. Let fig. 15 represent the section of a

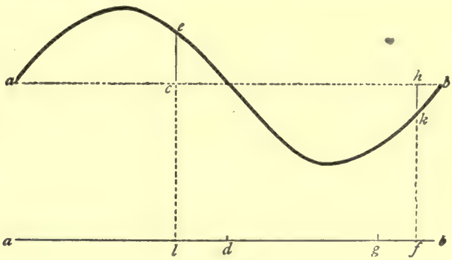


FIG. 15.

water wave, the dotted line being the surface of the water at rest. This may also represent a sound wave the length of which is (ab) distance above the dotted line representing extent of forward, and distance below extent of backward longitudinal vibration. For example, the air particles which are at the points l and f when at rest, would be at d and g, ($ld = ce$ and $hk = gf$) in the state of things represented in the figure. The curve in the figure, related in this way to the wave of condensation and rarefaction, is termed its associated wave.

The velocity with which these sound waves travel is very great, compared with that of water waves. The method of determining it is very simple in principle. Two stations are chosen within sight of each other, the distance between them being accurately known. A gun is fired at one station, and an observer at the other counts the number of seconds that elapse between seeing the flash and hearing the sound. As light passes almost instantaneously over any terrestrial distance, the time that sound takes to travel over a given distance is thus known. Dividing this distance by the number of seconds, we determine the space through which sound travels per second. This is found to vary considerably with the temperature. At 0° (Centigrade) or 32° (Fahrenheit) the velocity of sound is 1,090 feet per second. M. Wertheim gives the following results at other temperatures :—

Temperature.		Velocity
Centigrade.	Fahrenheit.	of Sound.
—·5°	31·1°	1,089
2·1°	35·8°	1,091
8·5°	47·3°	1,109
12°	53·6°	1,113
26·6°	79·9°	1,140

The velocity at any temperature may be approximately found by adding on 2 feet for a rise of one degree Centigrade, and 1 foot for a degree Fahrenheit.

The velocity of sound in any medium varies directly as the square root of the elasticity, and inversely as the square root of the density of that medium. Therefore, as all gases have the same elasticity, the velocity of sound in gases varies inversely as the square root of their densities. Thus oxygen is 16 times as heavy as hydrogen, and the velocity of sound in the latter gas is 4 times the velocity in the former.

Sound travels more quickly through water than through gases. Colladon and Sturm proved that the velocity of sound in water is 4,708 feet per second, at 8° Centigrade. Their experiments were conducted on the Lake of Geneva, on the opposite sides of which the observers were stationed in two boats. The sound was emitted under water by striking a bell with a hammer, and after travelling through a known distance was received by a long speaking tube, the larger orifice of which, covered with a vibrating membrane, was sunk beneath the surface. The same movement which gave rise to the sound, also at the same instant ignited some gunpowder, and the number of seconds elapsing between the flash and the sound was determined by a chronometer.

In solids, the velocity of sound is usually greater than in liquids. It may be calculated from the formula

$$V = \sqrt{\frac{E}{D}}$$

when the elasticity E and density D of the solid are known.

The various experimental methods that have been employed for determining it, will be given later on. The following are some of the principal determinations made by Wertheim.

Lead	-	-	4,030 feet per second at 20^{0} C.
Gold	-	-	5,717 ,, ,, ,, ,,
Silver	-	-	8,553 ,, ,, ,, ,,
Copper	-	-	11,666 ,, ,, ,, ,,
Platinum	-	-	8,815 ,, ,, ,, ,,
Iron	-	-	16,822 ,, ,, ,, ,,
Steel	-	-	16,357 ,, ,, ,, ,,
Wood (along fibre)		from	10,000 to 15,000 ,, ,,
,, (across ,,)		,,	3,000 to 5,000 ,, ,,

The superior conducting power of elastic solids may be illustrated by a variety of experiments. Thus, strike a tuning-fork and place the stem against the end of a rod of wood 12 or 15 feet long. So perfect is the transmission, that if a person applies his ear to the other end, the sound will appear to come from that part of the rod. A still better result is obtained by placing one end of the rod against a door panel, and applying the vibrating tuning fork to the other. Again, place a watch well between the teeth, without however touching them, and note the loudness of its tick. Now gently bite the watch, and observe how much more plainly it can be heard. In the first case, the vibrations pass through the air to the ears, in the second case, through the solid bones of the skull. This is the principle of the audiphone.

Summary.

Sound cannot be transmitted through a vacuum.

The transmission of sound is a particular case of wave motion, of which, water waves and rope waves are other examples. The peculiar characteristic of a wave motion is, that the material particles through which the wave is passing, do not move onwards with the wave, but simply oscillate about their position of rest. In the rope wave, for example, the particles of the rope oscillate at *right angles* to the direction in which the wave is advancing; while, on the other hand, in the sound wave, the air particles oscillate in the *same* direction as the wave is moving.

Just as a water or rope wave consists of two parts, a crest and a trough, so a sound wave is made up of two portions, viz., a *condensation* and a *rarefaction.*

A sound wave, like a water or rope wave, is determined by three elements, viz., its *length*, *amplitude*, and *form*. The *length* of a wave is the distance from any point in one wave to the corresponding point in the succeeding one; the *amplitude* is measured by the extent of vibration of its air particles; while the *form* is determined by the varying velocities of these particles as they perform their excursions.

The greater the amplitude (that is, the extent of particle vibration), the greater will be the degree of condensation and rarefaction.

In an *associated sound wave*, the direction of particle vibration is represented, for the sake of convenience, as being at right angles to its true direction.

The velocity of sound at the freezing point is 1,090 feet per second, and increases as the temperature rises.

The velocity of sound in water is 4,708 feet per second at 8° Centigrade.

The velocity of sound in elastic solids, such as iron and wood, is much greater than the above.

CHAPTER III.

On the Ear.

The Human Ear is separable into three distinct parts—the External, the Middle, and the Internal.

The External ear consists of the cartilaginous lobe or Auricle, and the External Meatus (E. M. fig. 16). The latter is a tube about an inch and a quarter long, directed inwards and slightly forwards, and closed at its inner extremity by the Tympanum (Ty., fig. 16) or drum of the ear, which is stretched obliquely across it.

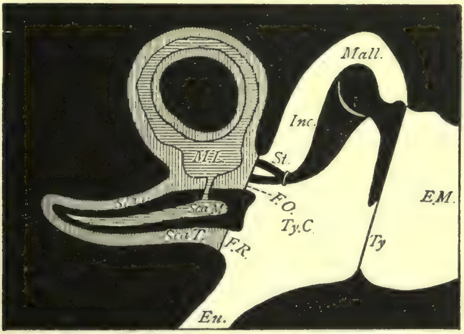


Fig 16. Diagrammatic Section of the Human Ear.

The Middle ear, which is separated from the External by the Tympanum, is a cavity in the bony wall of the skull, called the Tympanic Cavity (Ty. C., fig. 16). From this, a tube, about an inch and a half long, termed the Eustachian Tube (Eu., fig. 16) leads to the upper part of the throat, and thus places the air in the Tympanic Cavity in communication with the external air. On the side opposite to the tympanum, there are two small apertures in the bony wall of the cavity, both of which, however, are closed with membrane. The lower of these two apertures, which is about the size of a large pin's head, is called from its shape the Fenestra Rotunda, or round window (F.R., fig. 16); the upper and rather larger one, the Fenestra Ovalis, or oval window (F. O., fig. 16).

A chain of three small bones, stretches between the Tympanum and the Fenestra Ovalis. One of these, the Malleus or Hammer Bone (Mall., fig. 16) is firmly fastened by one of its processes or arms to the Tympanum, while the other process projects upwards into the cavity, and is articulated to the second bone—the Incus or Anvil (Inc., fig. 16). The lower part of this last again, is articulated to the third bone—the Stapes or Stirrup Bone (St., fig. 16), which in its turn is firmly fastened by the flat end of the Stirrup to the membrane which closes the Fenestra Ovalis. These three bones are suspended in the tympanic cavity in such a way, that they are capable of turning as a whole upon a horizontal axis, which is formed by processes of the Incus and Malleus, which processes fit into depressions in the side walls of the cavity. This axis in fig. 16 is perpendicular to the plane of the paper, and would pass through the head or upper part of the Malleus.

There are also two small muscles in the tympanic cavity, by the contractions and relaxations of which, the membrane of the Fenestra Ovalis and the tympanic membrane can be rendered more or less tense. One, called the Stapedius, passes from the floor of the tympanic cavity to the Stapes, and the other—the Tensor Tympani—from the wall of the Eustachian tube to the tympanum.

Before going further, let us consider the functions of these various parts. The sound waves which enter the External Meatus, pass down it, assisted in their passage by its configuration, and strike against the Tympanum. Now, as the air in the cavity of the tympanum is in communication with the outer air, as long as the latter is at rest, they will be of the same density, and hence the Tympanum will sustain an exactly equal pressure from both sides. When, however, the condensed part of a sound wave comes into contact with it, this equilibrium will no longer exist; the air on the outer side will exert a greater pressure than that on the inner, and the Tympanum will bulge inwards in consequence. The condensed part of the wave will be immediately followed by the rarefied part, and now the state of things is reversed; there will be a greater pressure from the inside, and consequently the Tympanum will move outward, a movement assisted by its own elasticity. We see therefore, that the Tympanum will execute one complete vibration for every wave that reaches it; that is, as we know from the preceding chapter, it will perform one vibration for every vibration of the sound-producing body.

Further, it follows from the arrangement of the Malleus, Incus, and Stapes, and the attachment of the first and last to the Tympanum and the Fenestra Ovalis respectively, that every movement of the Tympanum must cause a similar movement of the Fenestra Ovalis. We see, therefore, that this latter membrane faithfully repeats every vibration of the Tympanum, that is, every movement of the sound-originating body.

The Internal ear is of a much more complicated nature than the parts already described. It consists essentially of a closed membranous bag of a very irregular and intricate shape, which contains a liquid termed Endolymph, and which floats in another liquid called Perilymph. Both the Endolymph and the Perilymph are little else than water. This membranous bag may be divided, for the purpose of explanation, into two parts—the Membranous Labyrinth (M. L., fig. 16) and the Scala Media of the Cochlea (Sca. M., fig. 16).

The Membranous Labyrinth comprises the Utriculus (Ut., fig 17), the Sacculus, and the three Semi-circular Canals. All these parts

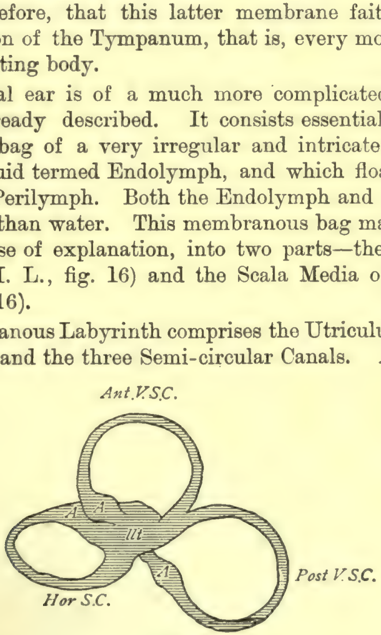


FIG. 17.

communicate with one another, forming one vessel; the two first lying one behind the other, and the three canals springing from the Utriculus. One end of each of the Semi-circular Canals is dilated at the point where it joins the Utriculus into a swelling called an Ampulla (A, fig. 17). Of the three canals two are placed vertically and the other horizontally; hence their names—the Anterior Vertical Semi-circular Canal (Ant. V. S. C., fig. 17), the Posterior Vertical (Post. V. S. C., fig. 17) and the Horizontal (Hor. S. C., fig. 17). All these parts are contained in a bony casing, which follows their outline pretty closely, so that there is a bony labyrinth, bony semi-circular canals, and so on. There is however a space between the bony casing and the membranous parts, which is filled by the Perilymph referred to above.

Every part of the Membranous Labyrinth is lined by an exceedingly delicate coat—the Epithelium—the cells of which, at certain parts, are prolonged into very minute hairs, which thus project into the Endolymph. In close communication with these cells are the ultimate fibres of one branch of the Auditory Nerve, which, ramifying in the wall of the Membranous Labyrinth, pierces its bony casing and proceeds to the brain. Floating in the Endolymph, there are also minute hard solid particles called Otoliths.

The remaining part of the Internal ear is more difficult to describe. It consists essentially of a long tube of bone closed at one end, of which fig. 18 is a diagrammatic section. A thin bony partition—

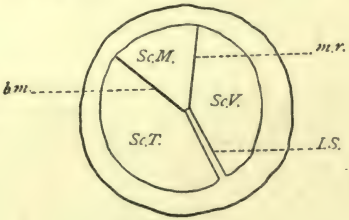


Fig. 18.

the Lamina Spiralis, L. S.—projecting more than half way into the interior, runs along the tube from the bottom, nearly but not quite to the top, so that the two chambers—Sc. V. and Sc. T—communicate at the closed end of the tube. These two chambers—the Scala Vestibuli and the Scala Tympani—are filled with Perilymph. Diverging from the interior edge of the Lamina Spiralis and terminating in the bony wall of the tube, are two membranous partitions, the Membrane of Reissner — *m.r.* — and the Basilar Membrane—*b. m.* The chamber between these two membranes which is termed the Scala Media—Sc. M.—is filled with Endolymph. So far the tube has been described for simplicity as if it were straight. Now, we must imagine it forming a close coil of two and a half turns round a central axis of bone—the Modiolus; the Scala Media being outwards and the Lamina Spiralis springing from the axis. The whole arrangement is not unlike a small snail's shell, and is termed the Cochlea. It is so placed with reference to the other parts of the ear, that the Scala Tympani is closed at the lower end by the Fenestra Rotunda (F. R., fig. 16) while the Perilymph of the Scala Vestibuli is continuous with that of the Labyrinth. On the other hand, the Endolymph which fills the Scala Media is in communication with that of the Sacculus.

Resting on the upper side of the elastic Basilar membrane are the arches or rods of Corti. Each of these rods consists of two filaments, joined at an angle like the rafters of a house. Altogether there are some three or four thousand of them, lying side by side, stretching along the whole length of the Scala Media like the keys of a pianoforte. A branch of the auditory nerve, entering the Modiolus, gives off fibres which pass through the Lamina Spiralis, their ultimate ramifications probably coming into close connection with Corti's rods.

Not much is known for certain of the functions of the various parts of the Internal ear. We have seen how the vibrations of the sounding body are transmitted to, and imitated by, the membrane of the Fenestra Ovalis. These vibrations are necessarily taken up by the Perilymph, which bathes its inner surface, and hence communicated to the Endolymph of the Membranous Labyrinth and the Scala Media. It was formerly thought, that both of these organs were necessary to the perception of sound, but the researches of Goltz have shown that the special function of the Labyrinth, is to enable us to perceive the turning of the head. Thus, the only part of the Internal Ear which is engaged in transmitting sound vibrations to the auditory nerves, is the Cochlea.

The Basilar Membrane of the Cochlea consists of a series of radial fibres, lying side by side, united by a delicate membrane, and it is believed by Helmholtz, that the faculty of discriminating between sounds of different pitch is due to these radial fibres. The Basilar Membrane gradually increases in width, that is, its radial fibres gradually increase in length, as we pass from the Fenestra Ovalis to the apex of the Cochlea, being ten times as long at the latter, as at the former. Helmholtz likens them to a series of stretched strings of gradually increasing lengths, the membranous connection between them simply serving to give a fulcrum to the pressure of the fluid against the strings. He further assumes that each of these fibres is tuned to a note of definite pitch, and capable of taking up its vibratory motion. He considers that the arches of Corti which rest on these fibres, serve the purpose of transmitting this vibratory motion to the terminal appendages of the nerve, each arch being connected with its own nerve ending.

As already mentioned, there are about 3,000 of these arches in the human ear, which would give about 400 to the octave. When a simple tone is sounded in the neighbourhood of the ear, the radial fibre of the Basilar Membrane in unison with it, is supposed to take up its vibrations, which are then transmitted by the arch of Corti in connection with it, to the particular nerve termination with which it is in communication.

For a complete discussion of this theory, the reader is referred to Helmholtz's "Sensations of Tone," Part I, Chap. vi.

Summary.

The Human Ear may be divided, for descriptive purposes, into three parts: the *External*, *Middle*, and *Internal* ears.

The *External* ear consists of the *Lobe*, and the tube or *Meatus* leading inwards, which is closed by the *Tympanum*.

The *Middle* ear contains a chain of three small bones, the *Malleus*, *Incus*, and *Stapes*, which serves to connect the *Tympanum* with the *Fenestra Ovalis*. The air in the cavity of the Middle ear is in communication with the external air, by means of the *Eustachian Tube*.

The *Internal* ear consists essentially of a membranous bag of exceedingly complicated form, filled with a liquid—*Endolymph*. This bag floats in another liquid—*Perilymph*—contained in a bony cavity, which is separated from the cavity of the Middle ear by the membranes of the *Fenestra Ovalis* and *Fenestra Rotunda*.

The nerves of hearing ramify on the walls of this membranous bag, and their ultimate fibres project into the *Endolymph* therein contained.

The alternate condensations and rarefactions of the sound waves which enter the External ear, strike against the Tympanum and set it vibrating. These vibrations are then transmitted by means of the small bones of the Middle ear to the Fenestra Ovalis, and from this again through the Perilymph and Endolymph to the minute terminations of the auditory nerve, which lie in the latter liquid.

One particular part of the membrane of the Internal ear, in which lie the *Fibres of Corti*, is specially modified, and is supposed to be the region of the ear which serves to discriminate the pitch and quality of musical sounds.

CHAPTER IV.

On the Pitch of Musical Sounds.

In order to fully describe a Musical Sound, it is necessary, besides specifying its duration, to particularize three things about it, viz., its Pitch, its Intensity, and its Quality. By pitch is meant its height or depth in the musical scale, by intensity its degree of loudness, and by its quality, that character which distinguishes it from another sound of the same pitch and intensity.

In the present chapter we have only to do with Pitch. Inasmuch as we have seen that the Sensation of Sound is due to the vibratory motion of some body, the first question that arises is,—What change takes place in the vibratory movement, when the pitch of the sound changes?

If we stretch a violin string loosely, and pluck it, we get a low sound; stretch it more tightly, and again pluck it, a higher sound is obtained, and at the same time, we can see that it is vibrating *more rapidly*. Again, take in the same way a short and a long tongue of metal, and the former will be found to give a higher sound and to vibrate *more quickly* than the latter.

But the instrument that best shows the fact, that the more rapid the rate of vibration, the higher is the pitch of the sound produced, is perhaps the Wheel Syren (fig. 19). This consists of a disc of zinc or other metal, about 18 inches or more in diameter, mounted on a horizontal axis, and capable of being rapidly revolved by means of a multiplying wheel. The disc is perforated by a number of holes arranged in concentric circles, all the holes in each circle being the same distance apart. In order to work the instrument, a current of air is blown through an India-rubber tube and jet, against one of the circles of holes while the disc is slowly revolving. Whenever a perforation comes in front of the jet, a puff of air passes through,

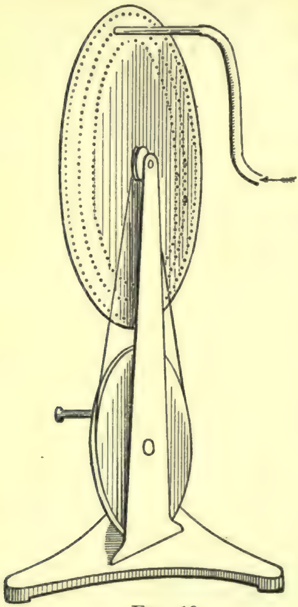


FIG. 19.

causing a condensation on the other side. The current of air is immediately cut off by the revolution of the disc, and consequently a rarefaction follows the previous condensation—in short, a complete sound wave is formed. Another hole appears opposite the jet, and another wave follows the first, and so on, one wave for each hole. While the disc is being slowly revolved, the waves do not follow one another quickly enough to give rise to a musical sound,—each puff is heard separately; but on gradually increasing the speed, they succeed each other more and more quickly, till at last a low musical sound is heard. If we now still further increase the speed, the pitch of the sound will gradually rise in proportion.

Savart's toothed wheel (fig. 4, p. 6) is another instrument which proves the same fact. While the wheel is being slowly turned, each tap on the card is heard separately, but on increasing the speed, we get a musical sound, which rises in pitch as the rate of revolution increases, that is, as the rate of vibration of the card increases.

We see, therefore, that the pitch of a sound depends upon the vibration rate of the body, which gives rise to it. By the vibration rate, is meant the number of complete vibrations—journeys to and fro—which it makes in a given time. In expressing the vibration rate, it is most convenient to take a second as the unit of time. The pitch of any sound may therefore be defined, by stating the number of vibrations per second required to produce it; and to avoid circumlocution, this number may be termed the "vibration number" of that sound. We shall now proceed to describe the principal methods which have been devised for ascertaining the vibration number of any given sound.

One of the earliest instruments constructed for this purpose was Savart's toothed wheel (fig. 4) which has just been referred to. A registering apparatus, H, which can be instantly connected or disconnected with the toothed wheel by touching a spring, records the number of revolutions made by it. In order to discover the vibration number of any given sound, the velocity of the wheel must be gradually increased until the sound it produces is in unison with the given sound. The registering apparatus is now thrown into action, while the same speed of rotation is maintained for a certain time, say a minute. The number of revolutions the wheel makes during this time, multiplied by the number of teeth on the wheel, gives the number of vibrations performed in one minute. This number divided by 60 gives the number of vibrations per second required to produce the given sound, that is, its vibration number.

Cagniard de Latour's Syren is an instrument constructed on the same principle as the Wheel Syren, but different in detail. Two

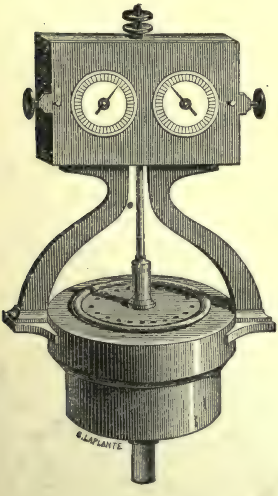


FIG. 20.

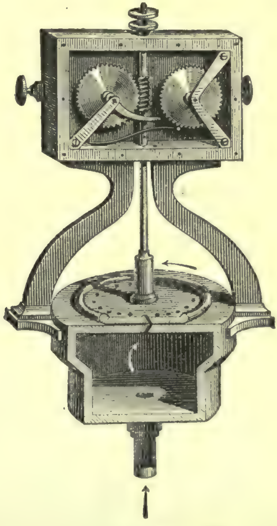


FIG. 21.

brass discs, rather more than an inch in diameter, are each similarly pierced with the same number of holes arranged in a circle (figs. 20 & 21). The upper disc revolves with the least possible amount of friction on the lower one, which is stationary, and forms the upper side of a wind chest. When air is forced into this chest by means of the tube below, and the disc is made to revolve, a puff will pass through each hole, every time the holes coincide. Thus, supposing there are 15 holes, they will coincide 15 times for every revolution, and therefore one revolution will give rise to 15 sound waves. By an ingenious device, which, however, in the long run is a disadvantage, the current of air itself moves the upper disc. This is brought about by boring the holes in the upper disc in a slanting direction, while those in the lower one are bored in the opposite direction, as shown in fig. 21. On the upper part of the axis of the revolving disc, a screw is cut, in the threads of which work the teeth of the wheel shown on the left in fig. 21. Further, the right hand wheel in fig. 21 is so connected with the left hand one, that while the latter makes one revolution the former moves only one tooth. Each of these wheels has 100 teeth. On the front of the instrument (fig. 20) are two dials, each divided into 100 parts corresponding to the 100 teeth on each of the wheels, the axles of which passing through the centre of the dials, carry the hands seen in the figure. Thus the right hand dial records the number of single revolutions up to 100, and the left hand one the number of hundreds. Finally, this registering apparatus can be instantly thrown in and out of gear, by pushing the nuts seen on both sides.

Suppose now we wish to obtain, by means of this instrument, the vibration number of a certain sound on a harmonium or organ. We first see that the registering apparatus is out of gear, and then placing the syren in connection with an acoustical bellows, gradually blow air into the instrument. This causes the disc to revolve, and at the same time a sound, at first low, but gradually rising in pitch, is heard. As we continue to increase the wind pressure, the pitch gradually rises until at last it is in unison with the sound we are testing. Having now got the two into exact unison, we throw the registering apparatus into gear, and maintain the same rate of rotation, that is, keep the two sounds at the same pitch for a measured interval of time—say one minute. At the expiration of this time the recording apparatus is thrown out of gear again. We now read off the number of revolutions: suppose the dial on the left stands at 21, and that on the right 36, then we know the disc has made 2,136 revolutions. Multiply this by 15 (for we have seen that each revolution gives rise to 15 waves), and we get 32,040 as the number of waves given off in one minute. Divide this by

60, and the quotient, 534. is the vibration number of the given sound.

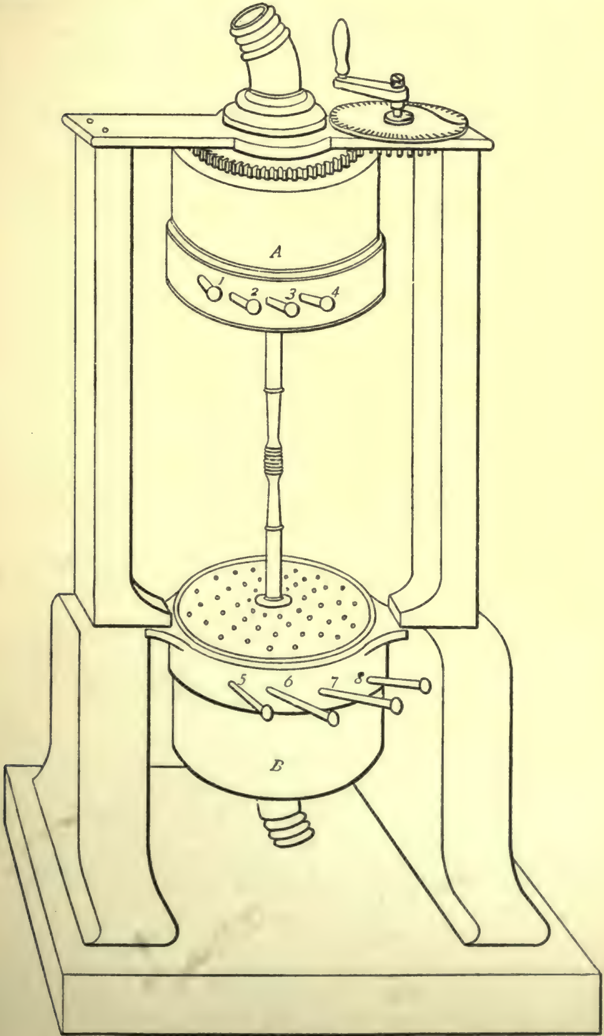


Fig. 22.

Although this instrument does very well for purposes of lecture illustration, it has several practical defects. When, for instance, the registering apparatus is thrown into gear, the increased work which the wind has to perform in turning the cogwheels, slackens the speed, and consequently lowers the pitch. Then, again, it is very difficult to keep the blast at a constant pressure—that is, to keep the sound steady. Further, there is an opening for error in noting the exact time of opening and closing the recording apparatus. From these causes, this form of Syren cannot be depended upon, according to Mr. Ellis, within ten vibrations per second.

Helmholtz has devised a form of Syren, in which these sources of error are avoided. It consists really of two Syrens, A and B, fig. 22, facing one another, the discs of which are both mounted on the same axis, and driven with uniform velocity, not by the pressure of the wind, but by an electro-motor. Each disc has 4 circles of equidistant holes, the number of holes in the circles being, in the lower disc 8, 10, 12, 18, and in the upper 9, 12, 15, 16 respectively. By means of the handles 1, 2, 3, &c., projecting from the wind chests, any or all of these circles may be closed or opened, so that two or any number of sounds up to eight, may be heard simultaneously.

Knowing the length of a stretched string, the stretching weight, and the weight of the string itself, it is possible to ascertain the vibration number of the sound it gives by calculation. The instrument used in applying this method, is termed a Sonometer

Fig 23.

or Monochord. It consists (fig. 23) of a sound box of wood, about 5 feet long, 7 inches broad, and 5 inches deep. A steel wire is fastened to a peg at one end, passes over bridges as shown in the figure, and is stretched by a weight at the other. The sound is evolved by drawing a violin bow over the wire. In order to find the vibrational number of any given sound, the wire is first tuned in unison with it, by varying the length of the wire (by means of the movable bridge shown in fig. 23) or the stretching weight. This weight is then noted, and the vibrating part of the wire measured and weighed. If the number of grains in the stretching weight (including the weight of the adjacent non-vibrating part of the wire) be denoted by W; the number of inches in the vibrating wire by L; and the number of grains in the same by w; then the vibrational number is

$$\frac{\pi}{2}\sqrt{\frac{W \times P}{w \times L}}$$

P being the length of the seconds pendulum at the place of observation, (= 39·14 at Greenwich), and π the constant 3·14159. Although theoretically perfect, the practical difficulties of determining the unison, measuring the length, ascertaining the weights, obtaining perfect uniformity in the wire, keeping the temperature constant, together with those arising from the thickness of the wire, are so great, as to render this method of no avail where accuracy is required.

A far more accurate method of counting vibrations is that known as the Graphic method, which is, however, only applicable in general to tuning-forks. The principle of this method will be readily understood on reference to fig. 24. A light style is attached to one of the prongs of a tuning fork. A piece of paper or glass, which

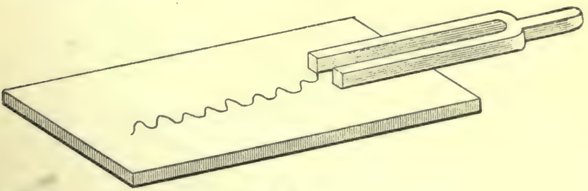


FIG. 24.

has been coated with lamp-black by holding it in the smoke of burning camphor or turpentine, is placed below the tuning fork so that the style just touches it. If now, while the glass remains at rest, the fork be set in vibration, the style will trace a straight line on the glass or paper, by removing the lamp black in its path as it moves to and fro. But if the glass be moved rapidly and steadily

Fig. 25.

in the direction of the fork's length, a complete record of the motion of the fork will be left on the lamp-blacked surface in the form of a wavy line, each double sinuosity in which will correspond to a complete vibration of the fork. A more convenient arrangement is to fasten the lamp-blacked paper round a rotating cylinder, which is also made to travel slowly from right to left by means of a screw on the axis, so as to prevent the tracing from overlapping itself. If this cylinder be kept rotating for a certain number of seconds, the number of sinuosities traced in that time can be counted; we have then only to divide this number by the number of seconds, to obtain the vibration number of the fork.

This method has been brought to a very high degree of perfection by Professor Mayer, of New Jersey. He uses lamp-blacked paper, wrapped round a rotating metallic cylinder or drum, as above described. The wave curve is traced on this by an aluminium style, attached to the end of one of the prongs of the tuning fork under examination. A pendulum (*a*, fig. 25), beating seconds, has a platinum wedge fastened to it below, which, at every swing, makes contact with a small basin containing mercury. This basin is in communication with one pole of a battery (*b*), a wire from the other pole being attached to the primary coil (*p*) of an inductorium, from which again a wire proceeds to the top of the pendulum. The wire from one end of the secondary coil (*s*) is attached to the stem of the tuning fork (*t*), and that from the other end to the revolving cylinder (*d*). When the apparatus is at work, it is obvious that a spark will pierce the smoked paper at every contact of the pendulum with the mercury, leaving a minute perforation. The number of complete sinuosities between two consecutive perforations, will consequently be the vibration number of the fork.

A new instrument for counting rapid vibrations, called by its inventors, the *Cycloscope*, was devised a few years ago by McLeod and Clarke. A diagrammatic representation of the essential parts of the apparatus is shown in fig. 26. A is a drum, rotating on a horizontal axis, and capable of being revolved at any definite rate.

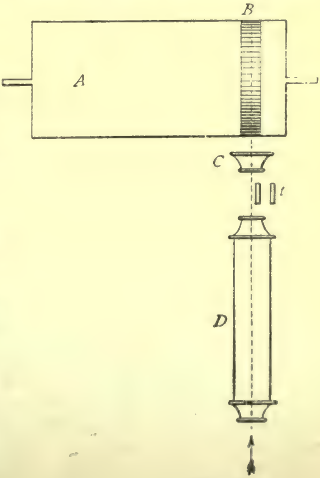


FIG. 26.

Round this is fastened a strip of dark paper B, ruled with equidistant white lines. C is a 2″ objective, giving an image of the white lines, which is viewed through the microscope, D. The tuning-fork (t) to be tested is fastened vertically in a vice, so that one of the prongs is situated in the common focus in such a way as to obscure about one-fourth of the field of view. Thus, on looking through the microscope when everything is at rest, an image of the white lines is seen, and the part of the prong (*a*, fig. 27 A), of the tuning-fork

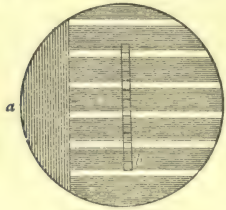


Fig. 27 (A).

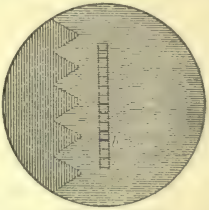


Fig. 27 (B).

partially obscuring them; (b) is a scale fixed in the eye-piece. When the fork is set vibrating and the drum is rapidly rotated. the lines can no longer be separately distinguished; but, just as, in the Graphic method, we found a wave to result from two movements at right angles, so by the composition of the fork's motion with that of the white lines, a wave makes its appearance, (fig. 27, B). If the white lines pass through a space equal to the distance between two of them, while the fork makes one vibration, then the length of the wave will be the distance between two white lines as seen through the microscope; furthermore, if this is the case exactly, the waves will appear stationary. If, however, the drum rotates faster or slower than this, the wave will have a slow apparent motion, either upwards or downwards. As the velocity of rotation of the drum is under the control of the observer, it is easy to keep it at such a speed, that the wave appears stationary, that is, at such a speed that the white lines pass through a space equal to the distance between two of them while the fork makes one vibration. It follows, therefore, that the number of white lines that pass over the field of view during the time of the experiment, is equal to the number of vibrations executed by the fork in that time. An electric pendulum, beating seconds, gives the time, while an electric counter records the number of revolutions made by the drum. A fine pointed tube, filled with magenta, automatically marks the paper on the drum at the beginning and end of the experiment, so as to give the fractional part of a revolution. The number of white lines that pass the field of view is thus easily obtained, by multiplying the number of white lines round the drum by the number of revolutions and fractional part of a revolution made by it. This, divided by the number of seconds the experiment lasted, gives the vibration number of the fork. In the hands of the inventors, this instrument has given extremely accurate results.

Another very accurate counting instrument is the Tonometer, an account of which is deferred, until the principles on which it is constructed have been explained (see Chap. xiii).

The exactness with which pitch can now be determined, is shown by the following abridged table, taken from Mr. Ellis's "History of Musical Pitch," p. 402. In the first column are the names of five particular forks, the vibrational numbers of which are given in the second, third, and fourth columns, as determined independently by McLeod with the Cycloscope, Ellis with the Tonometer, and Mayer with his modification of the Graphic method, respectively.

Name of fork.	Mc Leod.	Ellis.	Mayer.
Conservatoire	439·55	439·54	439·51
Tuileries	434·33	434·25	434·33
Feydeau	423·02	423·01	422·98
Versailles	395·83	395·79	395·78
Marloye	255·98	255·96	256·02

Knowing the velocity of sound in air, and having ascertained the vibrational rate of any sounding body by one of the preceding methods, it is easy to deduce the length of the sound waves emitted by it. For, taking the velocity of sound as 1,100 feet per second, suppose a tuning-fork, the vibration number of which is say 100, to vibrate for exactly one second. During this time it will have given rise to exactly 100 waves, the first of which at the end of the second will be 1,100 feet distant from the fork; the second one will be immediately behind the first, and the third behind the second, an so on: the last one emitted being close to the fork. Thus the combined lengths of the 100 waves will evidently be 1,100 feet, and as they are all equal in length, the length of one wave will be

$$\frac{1100}{100} = 11 \text{ ft.}$$

To find the wave length of any sound, therefore, it is only necessary to divide the velocity of sound by the vibration number of the sound in question. It will be noticed that as the velocity of sound varies with the temperature of the air, so the wave length of any particular sound must vary with the temperature.

We have seen that the pitch of any sound depends ultimately upon the rapidity with which the sound waves strike the tympanum of the ear, and usually this corresponds with the rate of vibration of the sounding body. If, however, from any cause, more sound waves from the sounding body, strike the ear, in a given time, than are emitted from it during that time, the apparent pitch of the note will be correspondingly raised, and *vice versa.* Such a case occurs when the sounding body is moving towards or from us, or when we are advancing or receding from the sounding body. When, for example, a locomotive, with the whistle sounding, is advancing rapidly towards the observer, the pitch will appear perceptibly sharper than after it has passed. This fact may be easily illustrated by fastening a whistle in one end of a piece of india-rubber tubing 4 or 5 feet long, and blowing through the other; at the same time whirling the tube in a horizontal circle above the head. A person at a distance will perceive a rise in pitch as the whistle is advancing towards him and a fall as it recedes.

The lowest sound used in music, is found in the lowest note of the largest modern organs, and is produced by 16½ vibrations per second; but so little of musical character does it possess, that it is never used except with its higher octaves. The musical character continues to be very imperfect for some distance above this limit; in fact, until we get to above twice this number of vibrations per second. The highest limit of musical pitch at the present time is about C^4, a sound corresponding to about 4,000 vibrations per second. Very much higher sounds than this can be heard, but they are too shrill to be of any use in music. Fig. 28, which explains itself, shows the limits of pitch of the chief musical instruments.

The note C in the treble staff is the sound that musicians usually take as a basis of pitch; this, once fixed, all the other sounds of the musical scale are readily determined. But, unfortunately, at the present day, there are a very large number of vibration numbers, corresponding to this note; in other words, there is no universally recognised standard. It appears from Mr. Ellis's paper on "The History of Musical Pitch" that the vibration number

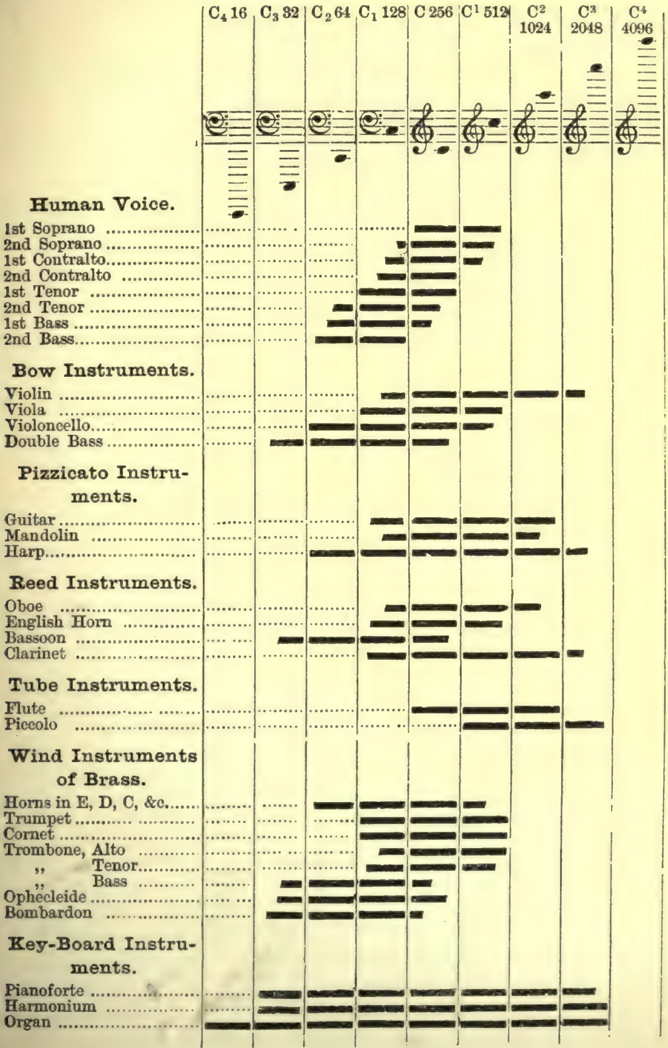

C_4 16
C_3 32
C_2 64
C_1 128
C 256
C^1 512
C^2 1024
C^3 2048
C^4 4096
Human Voice.
1st Soprano
2nd Soprano
1st Contralto
2nd Contralto
1st Tenor
2nd Tenor
1st Bass
2nd Bass
Bow Instruments.
Violin
Viola
Violoncello
Double Bass
Pizzicato Instruments.
Guitar
Mandolin
Harp
Reed Instruments.
Oboe
English Horn
Bassoon
Clarinet
Tube Instruments.
Flute
Piccolo
Wind Instruments of Brass.
Horns in E, D, C, &c.
Trumpet
Cornet
Trombone, Alto
" Tenor
" Bass
Ophecleide
Bombardon
Key-Board Instruments.
Pianoforte
Harmonium
Organ

FIG. 28.

corresponding to C has been gradually rising for many years. The following are a few of the chief standards.

Standard.	A	C'
Diapason Normal, 1859	435	517·3
Society of Arts, 1886	—	517·3
Tonic Sol-fa	—	517·3
Pianoforte Manufacturers' Association, New York, 1891	—	517·3
Philharmonic, 1897	439	522
,, before 1897	—	538
Kneller Hall, 1890	—	538
Crystal Palace	—	538

Summary.

A musical sound has three elements: *Pitch*, *Intensity*, and *Quality*.

The *Pitch* of a musical sound depends solely upon the vibration rate of the body that gives rise to it.

The *Vibration Number* of a given musical sound is the number of vibrations per second necessary to produce a sound of that particular pitch.

The principal instruments which have been used from time to time in determining the vibration numbers of musical sounds are:

Savart's Toothed Wheel.
The Syren.
The Sonometer or Monochord.
Mayer's Graphic Method.
The Cycloscope.
The Tonometer.

The last three are by far the most accurate.

To ascertain the wave length of any given sound—Divide the velocity of sound by its vibration number.

The wave length of any given sound, increases with the temperature.

The temperature remaining constant, the length of the sound wave determines the pitch of the sound produced.

The range of musical pitch is from about 40 to 4,000 vibrations per second.

CHAPTER V.

The Melodic Relations of the Sounds of the Common Scale.

In describing the form of Syren devised by Helmholtz, it was mentioned, that the lower revolving plate was pierced with four circles of 8, 10, 12, and 18 holes, and the upper with four circles of 9, 12, 15, and 16. If only the "8-hole circle" on the lower and the "16-hole" circle on the upper be opened, while the Syren is working, two sounds are produced, the interval between which, the musician at once recognises as the Octave. When the speed of rotation is increased, both sounds rise in pitch, but they always remain an Octave apart. The same interval is heard, if the circles of 9 and 18 holes be opened together. It follows from these experiments, that when two sounds are at the interval of an Octave, the vibrational number of the higher one is exactly twice that of the lower. An Octave, therefore, may be acoustically defined as the interval between two sounds, the vibration number of the higher of which is twice that of the lower. Musically, it may be distinguished from all other intervals by the fact, that, if any particular sound be taken, another sound an octave above this, another an octave above this last, and so on, and all these be simultaneously produced, there is nothing in the resulting sound unpleasant to the ear.

Since the ratio of the vibration numbers of two sounds at the interval of an octave is as 2 : 1, it is easy to divide the whole range of musical sound into octaves. Taking the lowest sound to be produced by 16 vibrations per second, we have

1st Octave, from		16	to	32	vibrations per second.	
2nd	,,	32	to	64	,,	,,
3rd	,,	64	to	128	,,	,,
4th	,,	128	to	256	,,	,,
5th	,,	256	to	512	,,	,,
6th	,,	512	to	1,024	,,	,,
7th	,,	1,024	to	2,048	,,	,,
8th	,,	2,048	to	4,096	,,	,,

Thus all the sounds used in music are comprised within the compass of about eight octaves.

Returning to the Syren: if the 8 and 12 "hole circles" be opened together, we hear two sounds at an interval of a Fifth, and as in the case of the octave, this is the fact, whatever the velocity of rotation. The same result is obtained on opening the 10 and 15, or the 12 and 18 circles. When, therefore, two sounds are at an interval of a Fifth, for every 8 vibrations of the lower sound, there are 12 of the upper, or for every 10 of the lower there are 15 of the upper, or for every 12 of the lower there are 18 of the upper. But

$$8 : 12 :: 2 : 3.$$
$$10 : 15 :: 2 : 3.$$
$$12 : 18 :: 2 : 3.$$

Therefore two sounds are at the interval of a Fifth when their vibration numbers are as 2 to 3; that is when 2 vibrations of the one are performed in exactly the same time as 3 vibrations of the other. This may be conveniently expressed by saying that the *vibration ratio* or *vibration fraction* of a Fifth is $3 : 2$ or $\frac{3}{2}$. Similarly the vibration ratio of an Octave is $2 : 1$ or $\frac{2}{1}$.

Again, on opening the circles of 8 and 10 holes, two sounds are heard at the interval of a Major Third. The same interval is obtained with the 12 and 15 circles. Now $8 : 10 :: 4 : 5$ and $12 : 15 :: 4 : 5$. Therefore two sounds are at the interval of a Major Third, when their vibration numbers are as $4 : 5$; or more concisely, the vibration ratio of a Major Third is $\frac{5}{4}$.

With the results, thus experimentally obtained, it is easy to calculate the vibration numbers of all the other sounds of the musical scale, when the vibration number of one is given. For example, let the vibration number of **d** be 288, or shortly, let $\mathbf{d} = 288$; then the higher Octave $\mathbf{d}^{\mathsf{I}} = 288 \times 2 = 576$. Also the vibration ratio of a Fifth $= \frac{3}{2}$; therefore the vibration number of **s** is to that of **d**, as $3 : 2$; that is, $\mathbf{s} = \frac{3}{2} \times 288 = 432$. Similarly the interval $\left\{\begin{smallmatrix}\mathbf{m}\\ \mathbf{d}\end{smallmatrix}\right.$ is a Major Third; but the vibration ratio of a Major Third we have found to be, $\frac{5}{4}$; therefore $\mathbf{m} : \mathbf{d} :: 5 : 4$, that is $\mathbf{m} = \frac{5}{4} \times 288 = 360$. Again, $\left\{\begin{smallmatrix}\mathbf{t}\\ \mathbf{s}\end{smallmatrix}\right.$ is a Major Third; therefore $\mathbf{t} = \frac{5}{4} \times 432 = 540$. Further, $\left\{\begin{smallmatrix}\mathbf{r}^{\mathsf{I}}\\ \mathbf{s}\end{smallmatrix}\right.$ is a Fifth; therefore $\mathbf{r}^{\mathsf{I}} = \frac{3}{2} \times 432 = 648$, and its lower octave $\mathbf{r} = \frac{648}{2} = 324$. It only remains to obtain the vibration numbers of **f** and **l**. Now $\left\{\begin{smallmatrix}\mathbf{d}^{\mathsf{I}}\\ \mathbf{f}\end{smallmatrix}\right.$ is a Fifth, thus the vibrational number of **f** is to that of $\mathbf{d}^{\mathsf{I}}$ as $2 : 3$; therefore $\mathbf{f} = \frac{2}{3} \times 576 = 384$; and $\left\{\begin{smallmatrix}\mathbf{l}\\ \mathbf{f}\end{smallmatrix}\right.$ is a Major Third, consequently $\mathbf{l} = 384 \times \frac{5}{4} = 480$. Tabulating these results we have

$$
\begin{aligned}
d^{I} &= 576. \\
t &= 540. \\
l &= 480. \\
s &= 432. \\
f &= 384. \\
m &= 360. \\
r &= 324. \\
d &= 288.
\end{aligned}
$$

The vibration numbers of the upper or lower octaves of these notes, are of course at once obtained by doubling or halving them.

It will be noticed that a scale may be constructed on any vibration number as a foundation. The only reason for selecting 288 was, to avoid fractions of a vibration and so simplify the calculations. As another example let us take d = 200. Proceeding in the same way as before, but tabulating at once, for the sake of brevity, we get

$$
\begin{array}{lllr}
d^{I} = 200 \times 2 & = 400. & & (2). \\
t = \frac{300}{1} \times \frac{5}{4} & = 375. & & (5). \\
l = 266\frac{2}{3} \times \frac{5}{4} & = 333\frac{1}{3}. & & (8). \\
s = \frac{200}{1} \times \frac{3}{2} & = 300. & & (3). \\
f = \frac{400}{1} \times \frac{2}{3} & = 266\frac{2}{3}. & & (7). \\
m = \frac{200}{1} \times \frac{5}{4} & = 250. & & (4). \\
r = \frac{300}{1} \times \frac{3}{2} \times \frac{1}{2} & = 225. & & (6). \\
d = & = 200. & & (1).
\end{array}
$$

We may now adopt the reverse process, that is, from the vibration numbers, obtain the vibration ratios. For example, using the first scale, we find that the vibration number of t is to that of m as 540 : 360, that is (dividing each by 180, for the purpose of simplifying) as 3 : 2; or more concisely

$$\Big\}\frac{t}{m} = \frac{540}{360} = \frac{3}{2}$$

The interval $\Big\}\frac{t}{m}$ is therefore a perfect Fifth. Again,

$$\Big\}\frac{l}{r} = \frac{480}{324}$$

Now the vibration fraction of a perfect Fifth $= \frac{3}{2} = \frac{480}{320}$, therefore $\Big\}\frac{l}{r}$ is not a perfect Fifth. We shall return to this matter further on, at present it will be sufficient to notice the fact. The student must take particular care *not to subtract or add* vibration numbers, in order to find the interval between them; thus the difference between the vibration numbers of **t** and **m** in the second scale is $375 - 250 = 125$, but this does not express the interval between them, viz., a Fifth, but merely the difference between the vibration numbers of these particular sounds. To make this clearer, take the difference between the vibration numbers of **d** and **s** in the second table $= 300 - 200 = 100$, and between **d** and **s** in the first $= 432 - 288 = 144$. Here we have different results, although the interval is the same. Take the ratio, however, and we shall get the same in each case for

$$\frac{300}{200} = \frac{3}{2} \text{ and } \frac{432}{288} = \frac{3}{2}.$$

We shall now proceed to ascertain the vibration ratios of the intervals between the successive sounds of the scale, using the first of the two scales given on the preceding page :—

$\left\{ \begin{matrix} d' \\ t \end{matrix} \right.$	=	$\frac{576}{540}$	=	$\frac{96}{90}$	=	$\frac{16}{15}$
$\left\{ \begin{matrix} t \\ l \end{matrix} \right.$	=	$\frac{540}{480}$	=	$\frac{54}{48}$	=	$\frac{9}{8}$
$\left\{ \begin{matrix} l \\ s \end{matrix} \right.$	=	$\frac{480}{432}$	=	$\frac{120}{108}$	=	$\frac{10}{9}$
$\left\{ \begin{matrix} s \\ f \end{matrix} \right.$	=	$\frac{432}{384}$	=	$\frac{54}{48}$	=	$\frac{9}{8}$
$\left\{ \begin{matrix} f \\ m \end{matrix} \right.$	=	$\frac{384}{360}$	=	$\frac{96}{90}$	=	$\frac{16}{15}$
$\left\{ \begin{matrix} m \\ r \end{matrix} \right.$	=	$\frac{360}{324}$	=	$\frac{90}{81}$	=	$\frac{10}{9}$
$\left\{ \begin{matrix} r \\ d \end{matrix} \right.$	=	$\frac{324}{288}$	=	$\frac{81}{72}$	=	$\frac{9}{8}$

There are, therefore, three kinds of intervals between the consecutive sounds of the scale, the vibration ratios of which are $\frac{9}{8}$, $\frac{10}{9}$, and $\frac{16}{15}$. The first of these intervals, which has been termed the Greater Step or Major Tone, occurs three times in the diatonic scale, viz.,

$$\left\{ \begin{matrix} t \\ l \end{matrix} \right. \quad \left\{ \begin{matrix} s \\ f \end{matrix} \right. \quad \left\{ \begin{matrix} r \\ d \end{matrix} \right.$$

The next is the Smaller Step or Minor Tone, and is found twice, viz.,

$$\left\{ \begin{matrix} l \\ s \end{matrix} \right. \quad \left\{ \begin{matrix} m \\ r \end{matrix} \right.$$

The last is the Diatonic Semitone, and also occurs twice, viz.,

$$\begin{cases} d' \\ t \end{cases} \quad \begin{cases} f \\ m \end{cases}$$

We may now calculate the vibration ratios of the remaining intervals of the scale. $\begin{cases} f \\ d \end{cases}$ may be selected as the type of the Fourth. Taking again the vibration numbers of the first scale, the vibration ratio of this interval is

$$\frac{384}{288} = \frac{96}{72} = \frac{8}{6} = \frac{4}{3}$$

This result may be verified on the Syren by opening the 12 and 9 or 16 and 12 circles.

Taking $\begin{cases} s \\ m \end{cases}$ as an example of a Minor Third, its vibration ratio is

$$\frac{432}{360} = \frac{48}{40} = \frac{6}{5}.$$

This can also be verified by the Syren with the 12 and 10 circles.

Again, the vibration ratio of $\begin{cases} d' \\ m \end{cases}$ a Minor Sixth, is

$$\frac{576}{360} = \frac{72}{45} = \frac{8}{5}.$$

and this, too, may be confirmed on the Syren, with the 16 and 10 circle.

The vibration ratio of $\begin{cases} l \\ d \end{cases}$, a Major Sixth, is

$$\frac{480}{288} = \frac{60}{36} = \frac{5}{3};$$

which may be confirmed with the 15 and 9 circles.

The vibration ratio of the Major Seventh $\begin{cases} t \\ d \end{cases}$ is

$$\frac{540}{288} = \frac{135}{72} = \frac{15}{8};$$

and this can be verified with the 15 and 8 circles.

The vibration ratio of the Minor Seventh $\begin{cases} f \\ s_{1} \end{cases}$ is

$$\frac{384}{216} = \frac{96}{54} = \frac{16}{9};$$

capable of verification with the 16 and 9 circles.

The vibration fraction of the Diminished Fifth $\begin{cases} f \\ t_{1} \end{cases}$ is

$$\frac{384}{270} = \frac{64}{45};$$

and that of the Tritone, or Pluperfect Fourth $\begin{cases} t \\ f \end{cases}$ is

$$\frac{540}{384} = \frac{90}{64} = \frac{45}{32}.$$

In order to find the vibration ratio of the sum of two intervals, the vibration ratios of which are given, it is only necessary to multiply them together as if they were vulgar fractions, thus, given $\left\{{\mathbf{s} \atop \mathbf{m}}\right. = \frac{6}{5}$, and $\left\{{\mathbf{m} \atop \mathbf{d}}\right. = \frac{5}{4}$; to find $\left\{{\mathbf{s} \atop \mathbf{d}}\right.$:—

$$\left\{{\mathbf{s} \atop \mathbf{d}}\right. = \frac{6}{5} \times \frac{5}{4} = \frac{6}{4} = \frac{3}{2};$$

which we already know to be the case. The reason of the process may be seen from the following considerations. From $\left\{{\mathbf{s} \atop \mathbf{m}}\right. = \frac{6}{5}$, and $\left\{{\mathbf{m} \atop \mathbf{d}}\right. = \frac{5}{4}$ we know that,

	for every	6	vibrations of	**s**,	there are	5	of	**m**;
and	,,	,,	5	,,	,, **m**,	,,	,,	4	,,	**d**;
Therefore	,,	,,	6	,,	,, **s**,	,,	,,	4	,,	**d**;
that is	,,	,,	3	,,	,, **s**,	,,	,,	2	,,	**d**.

Again, in order to find the vibration ratio of the difference of two intervals, the vibration ratios of which are given, the greater of these must be divided by the less, just as if they were vulgar fractions. For example, given $\left\{{\mathbf{d}' \atop \mathbf{d}}\right. = \frac{2}{1}$, and $\left\{{\mathbf{m} \atop \mathbf{d}}\right. = \frac{5}{4}$, to find $\left\{{\mathbf{d}' \atop \mathbf{m}}\right.$:

$$\left\{{\mathbf{d}' \atop \mathbf{m}}\right. = \frac{2}{1} \div \frac{5}{4} = \frac{2}{1} \times \frac{4}{5} = \frac{8}{5}.$$

The reason for the rule will be seen from the following considerations. From the given vibration ratios we know that,

	for every	2	vibrations of	**d'**,	there is	1	of	**d**;
that is	,,	,,	8	,,	,, **d'**,	,,	are	4	,,	**d**;
and	,,	,,	4	,,	,, **d**,	,,	,,	5	,,	**m**;
therefore	,,	,,	8	,,	,, **d'**,	,,	,,	5	,,	**m**.

We shall apply this rule, to find the vibration ratios of a few other intervals. The Greater Chromatic Semitone is the difference between the Greater Step and the Diatonic Semitone. $\left\{{\mathbf{fe} \atop \mathbf{f}}\right.$ is an example of the Greater Chromatic Semitone, being the difference between $\left\{{\mathbf{s} \atop \mathbf{f}}\right.$ a Greater Step, and $\left\{{\mathbf{s} \atop \mathbf{fe}}\right.$ a Diatonic Semitone. Now $\left\{{\mathbf{s} \atop \mathbf{f}}\right. = \frac{9}{8}$, and $\left\{{\mathbf{s} \atop \mathbf{fe}}\right. = \frac{16}{15}$ (for it is the same interval as $\left\{{\mathbf{d}' \atop \mathbf{t}}\right.$); therefore

$$\left\{{\mathbf{fe} \atop \mathbf{f}}\right. = \frac{9}{8} \div \frac{16}{15} = \frac{9}{8} \times \frac{15}{16} = \frac{135}{128}.$$

The Lesser Chromatic Semitone is the difference between the Smaller Step and the Diatonic Semitone; $\left\{{\mathbf{se} \atop \mathbf{s}}\right.$, for example, which is the difference between $\left\{{\mathbf{l} \atop \mathbf{s}}\right.$ and $\left\{{\mathbf{l} \atop \mathbf{se}}\right.$. Now $\left\{{\mathbf{l} \atop \mathbf{s}}\right. = \frac{10}{9}$ and $\left\{{\mathbf{l} \atop \mathbf{se}}\right. = \left\{{\mathbf{d}' \atop \mathbf{t}}\right. = \frac{16}{15}$; therefore

$$\left\{{\mathbf{se} \atop \mathbf{s}}\right. = \frac{10}{9} \div \frac{16}{15} = \frac{10}{9} \times \frac{15}{16} = \frac{25}{24}.$$

This is also the difference between a Major and a Minor Third, for

$$\frac{5}{4} \div \frac{6}{5} = \frac{5}{4} \times \frac{5}{6} = \frac{25}{24}$$

The interval between the Greater and Lesser Chromatic Semitones will be

$$\frac{135}{128} \div \frac{25}{24} = \frac{135}{128} \times \frac{24}{25} = \frac{81}{80};$$

which is usually termed the Comma or Komma.

Referring to the first table of vibration numbers on page 46, we have $\mathbf{l} = 480$, and $\mathbf{r} = 324$; therefore

$$\Big\{\frac{\mathbf{l}}{\mathbf{r}} = \frac{480}{324} = \frac{40}{27};$$

and thus, as noticed above, it is not a Perfect Fifth. To form a Perfect Fifth with $\mathbf{l}$, a note $\mathbf{r}^{\backprime}$ would be required, such that

$$\Big\{\frac{\mathbf{l}}{\mathbf{r}^{\backprime}} = \frac{3}{2}.$$

It is easy to find the vibration number of this note if that of $\mathbf{l}$ be given, thus:—

$$\Big\{\frac{\mathbf{l}}{\mathbf{r}^{\backprime}} = \frac{3}{2},$$

$$\text{that is, } \frac{480}{\mathbf{r}^{\backprime}} = \frac{3}{2};$$

$$\text{therefore } \frac{\mathbf{r}^{\backprime}}{480} = \frac{2}{3},$$

$$\mathbf{r}^{\backprime} = \frac{2}{3} \times \frac{480}{1} = 320.$$

This note has been termed *rah* or grave $\mathbf{r}$, and may be conveniently written, $\mathbf{r}^{\backprime}$. Similarly $\Big\{\frac{\mathbf{f}}{\mathbf{r}}$ is not a true Minor Third, for its vibration ratio is

$$\frac{384}{324} = \frac{96}{81} = \frac{32}{27};$$

but $\Big\{\frac{\mathbf{f}}{\mathbf{r}^{\backprime}}$ is a true Minor Third, for its vibration ratio is

$$\frac{384}{320} = \frac{48}{40} = \frac{6}{5}.$$

The interval between $\mathbf{r}$ and $\mathbf{r}^{\backprime}$ is the comma, its vibration ratio being evidently

$$\frac{324}{320} = \frac{81}{80}.$$

SUMMARY.

The sounds used in Music lie within the compass of about eight Octaves.

The *vibration ratio* or *vibration fraction* of an interval, is the ratio of the vibration numbers of the two sounds forming that interval.

The vibration ratios of the principal musical intervals have been exactly verified by Helmholtz's modification of the Double Syren.

It may be shown, by means of this instrument, that the vibration numbers of the three tones of a Major Triad, in its normal position —$\left\{\begin{matrix}\text{G}\\ \text{E},\\ \text{C}\end{matrix}\right.$ or $\left\{\begin{matrix}\text{s}\\ \text{m},\\ \text{d}\end{matrix}\right.$ for example,—are as

$$4 : 5 : 6.$$

Starting from this experimental foundation, the vibration numbers of all the tones of the modern scale can readily be calculated on any basis; and from these results, the vibration ratio of any interval used in modern music may be obtained.

Vibration ratios must never be *added* or *subtracted.*

To find the vibration ratio of the sum of two or more intervals, multiply their vibration ratios together.

To find the vibration ratio of the difference of two intervals, divide the vibration ratio of the greater interval by that of the smaller.

The vibration ratios of the principal intervals of the modern musical scale are as follows:—

Komma	$\frac{81}{80}$
Lesser Chromatic Semitone	$\frac{25}{24}$
Greater ,, ,,	$\frac{135}{128}$
Diatonic Semitone	$\frac{16}{15}$
Smaller Step or Minor Tone	$\frac{10}{9}$
Greater Step or Major Tone	$\frac{9}{8}$
Minor Third	$\frac{6}{5}$
Major Third	$\frac{5}{4}$
Fourth	$\frac{4}{3}$
Tritone	$\frac{45}{32}$
Diminished Fifth	$\frac{64}{45}$
Fifth	$\frac{3}{2}$
Minor Sixth	$\frac{8}{5}$
Major Sixth	$\frac{5}{3}$
Minor Seventh	$\frac{16}{9}$
Major Seventh	$\frac{15}{8}$
Octave	$\frac{2}{1}$

To find the vibration ratio of any of the above intervals increased by an Octave, multiply by $\frac{2}{1}$; thus the vibration ratio of a Major Tenth is

$$\tfrac{5}{4} \times \tfrac{2}{1} = \tfrac{10}{4} = \tfrac{5}{2}.$$

CHAPTER VI.

On the Intensity or Loudness of Musical Sounds.

We have seen that the pitch of a sound depends solely upon the rapidity with which the vibrations succeed one another. We have next to study the question: "Upon what does the Loudness or Intensity of a sound depend?"

Gently pluck a violin string. Notice the intensity of the resulting sound, and also observe the extent or amplitude of the string's vibration. Pluck it harder; a louder sound is heard, and the string is seen to vibrate through a greater space. Pluck it harder still; a yet louder sound is produced, and the amplitude of the vibrations is still greater. We may conclude, from this experiment, that as long as we keep to the same sounding body, the intensity of the sound it produces, depends upon the amplitude of its vibrations; the greater the amplitude, the louder the sound. This fact may be strikingly illustrated by the following experiment. Fasten a style of paper, or better still, parchment, to one prong of a large tuning-fork. Coat a slip of glass on one side with lamp-black, and lay it, with the coated side upwards, on a smooth board,

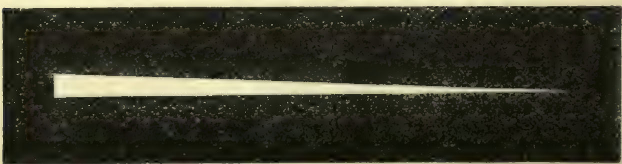


Fig. 29.

having previously nailed on the latter a straight strip of wood, to serve as a guide in subsequently moving the glass slip. Now strike the fork sharply, and immediately hold it parallel to the glass, in such a way, that the vibrating style just touches the lamp-black. Move the glass slip slowly along under the tuning-fork. The latter, as it vibrates, will remove the lamp-black, and leave a clean wedge-shaped trace on the glass, as seen in fig. 29. As the width of the trace at any point is evidently the amplitude of the vibration of the fork, at the time that point was below it, we see that the amplitude of the vibrations of the fork gradually decreases till the fork comes to rest; and as the sound decreases gradually till the fork becomes silent, we see that the intensity of its sound depends upon the amplitude of its vibrations.

It is obvious, that the greater the amplitude of the vibrations of a sounding body, the greater will be the amplitude of the vibrations of the air particles in its neighbourhood; thus we may conclude, that the intensity of a sound depends upon the amplitude of vibration of the air particles in the sound wave. But it is a matter of common experience, that a sound becomes fainter and fainter, the farther we depart from its origin; therefore, we must limit the above statement thus: the intensity of a given sound, as perceived by our ears, depends upon the amplitude of those air particles of its sound wave, which are in the immediate neighbourhood of our ears. This leads us to the question: "At what rate does the intensity of a sound diminish, as we recede from its origin?" We may ascertain the answer to this question, by proceeding as in the analogous case of heat or light. Thus, let A, fig. 30 be the origin of a given sound. At centre A, and with radii of say 1 yd., 2 yds., 3 yds., describe three imaginary spheres, B, C, D. Now, looking on sound, for the moment, as a quantity, it is evident that the quantity of sound which passes through the surface of the sphere B is identical with the quantity that passes through the surface of the spheres C and D. But the surfaces of spheres vary as the squares of their radii; therefore, as the radii of the spheres B, C, and D are 1, 2, and 3 yds. respectively, their surfaces are as $1^2 : 2^2 : 3^2$; that is, the spherical surface C is four times as great, and D 9 times as great, as the spherical surface B. We see, therefore, that the quantity of sound, which passes through the surface of B, is, as it were, spread out fourfold as it passes through C, and ninefold as it passes through D. It follows, therefore, that one square inch of C will only receive $\frac{1}{4}$ as much sound as a square inch of B, and one square inch of D only $\frac{1}{9}$ as much. Thus, at distances of 1, 2, 3, from a sounding body, the intensities are as 1, $\frac{1}{4}$, and $\frac{1}{9}$; that is, as we recede from a sounding body, the intensity diminishes in proportion to the square of our distance from the body, or more concisely, "The intensity of a sound varies inversely as the square of the distance from its origin.

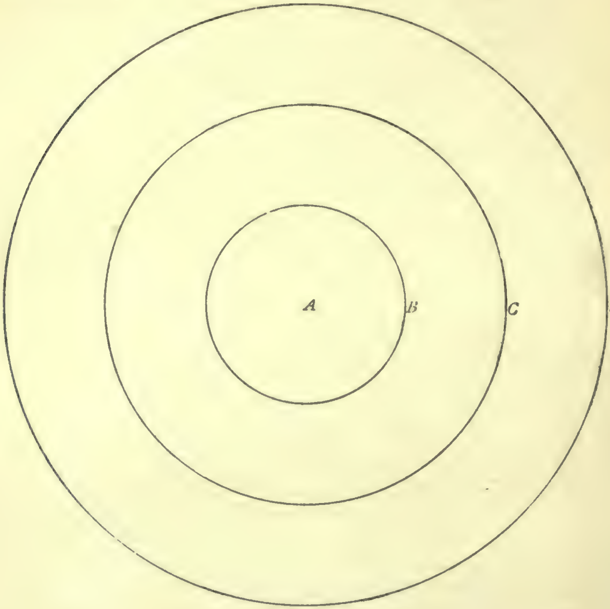


Fig. 30.

It should be clearly noted, however, that the conditions under which the above "law of inverse squares," as it is called, is true, rarely or never obtain. The chief disturbing elements in the application of this law are echoes. When a ray of light strikes any reflecting surface at right angles, it is reflected back in the direction whence it came. If a ray of light, A C, fig. 31, does not fall at right angles upon a reflecting surface P Q, it is reflected along a line C B, which is so situated, that the angle B C H is equal to the angle A C H; H C being at right angles to P Q. Just so with sound. A person standing at B would hear a sound from A, first as it reaches him in the direction A B, and directly after, along the line C B. If the distances A B and A C B were each only a few yards, the two sounds would be indistinguishable, but if there were any considerable difference between these two

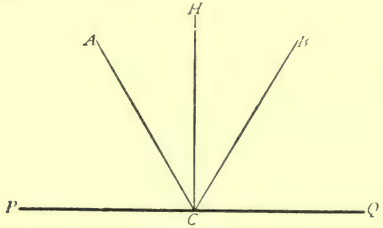


FIG. 31.

distances, the two sounds would be *separately heard*, the latter being termed the echo of the former. Therefore, when a vibrating body emits a sound in any room or hall, the waves which proceed from it in all directions, strike the walls, floor, ceiling, and also the reflecting surfaces of the various objects in the place, and are reflected again and again from them. Thus the direct and reflected sounds coalesce, and interfere with one another, in the most complicated manner, and the simple law of inverse squares is no longer applicable. This is still the case, even in the open air, away from all surrounding objects, for the ground will here present a reflecting surface, and other invisible reflectors are found, as Professor Tyndall has shown, in the surfaces which separate bodies of air of different hygrometric states and of different temperatures.

This may be put in another way. It is a condition of the truth of the law of inverse squares, as above shown, that the sound shall be able to spread outwards in all directions; if this is not the case, the law no longer holds good. Now, in a building, this is not the case; the sound is prevented from spreading by the roof, floor, and walls. If the sound can be entirely prevented from spreading, its intensity will not diminish at all. This is the principle of the speaking tube. In this instrument, the vibrations of the air particles are transmitted undiminished, except by friction against the side of the tube, and by that part of the motion which is given up to the substance of the tube itself; thus sound can be transmitted to great distances in such tubes. Regnault, experimenting with the sewer conduits of Paris, found that the report of a pistol was audible through them, for a distance of 6 miles.

The bad acoustical properties of a building are generally due to echoes. A sound from the lips of a speaker, in a building, reaches the ear of the listener directly, and also after one or more reflections from the ceiling, walls, floor, and so on. If the building be of considerable dimensions, these echoes may reach the listener's ear at an appreciable interval of time after the direct sound, or after one another, and will then so combine with the succeeding direct sound from the speaker as to make his words quite indistinguishable. The roof is often the chief culprit in this matter, especially when lofty, and constructed of wood, this latter affording an excellent reflecting surface. An obvious remedy is to cover such a surface with some badly-reflecting substance, such as a textile fabric. A sound-board over the speaker's head, will also prevent the sound from passing directly to the roof. The bodies and clothes of the persons forming an audience, are also valuable in preventing echoes. Professor Tyndall, to whose work on sound the student is referred for further information on this subject, says that, having to deliver a lecture in a certain hall, he tried its acoustical properties beforehand, and was startled to find that when he spoke from the platform, a friend he had with him, seated in the body of the empty hall, could not distinguish a word, in consequence of the echoes. Subsequently, when the hall was filled with people, the Professor had no difficulty in making himself distinctly heard in every part. Again, everyone must have noticed the difference between speaking in an empty and uncarpeted room, in which the echoes reinforce the direct sound, and speaking in the same room carpeted, and furnished, the echoes in this case being deadened by the carpets, curtains, &c.

Summary.

The *Intensity* of the sound produced by a vibratory body, depends upon the *amplitude* of its vibrations.

The Intensity of a sound varies inversely as the square of the distance from its origin, only when the sound waves can radiate freely in all directions without interruption.

Sound is reflected from elastic surfaces in the same way as light, thereby producing echoes.

Sound is well reflected from such surfaces as wood, iron, stone, &c., while cloths, carpets, curtains, and textile fabrics in general, scarcely reflect at all.

CHAPTER VII.

Resonance, Co-vibration, or Sympathy of Tones.

Select two tuning-forks which are exactly in unison. Having taken one in each hand, strike that in the right hand pretty sharply, and immediately hold it with its prongs parallel, and close to the prongs of the other, but without touching it. After the lapse of not less than one second, on damping the fork in the right hand, that in the left will be found to be giving out a feeble tone. To this phenomenon, the names of Resonance, Co-vibration, and Sympathy of tones have been given, the first being the one most commonly used in English works. The explanation of this effect will be better understood after a consideration of the following analogous experiment.

Let a heavy weight be suspended at the end of a long cord, and to it attach a fibre of silk or cotton. The weight being at rest, pull the fibre gently so as not to break it. The weight will thus be pulled forwards through an exceedingly small, perhaps imperceptible distance. Now relax the pull on the fibre, till the weight has swung through its original position, and reached the limit of its backward movement. If another gentle pull be then given, the weight will swing forward a trifle further than at first. The weight then swings backwards as before, and again a properly-timed pull will still further extend its excursion. By proceeding in this way, after a time, the total effect of these accumulated impulses will have been sufficient to impart to the weight a considerable oscillation. On examination it will be found that this experiment is analogous to the last one. The regularly timed impulses in the second experiment, correspond to the regularly vibrating fork in the right hand; the weight to the fork in the left hand, and the fibre to the air between the forks. And here it must be observed, that just as the forks execute their vibrations in equal times, whether their amplitudes be great or small, so the weight performs its swings (as long as they are not too violent) in equal times, whether their range be small or great. Again, the conditions of success are the same in both cases; for, in the first place, the impulses in the second experiment must be exactly timed, that is, they must be repeated at an interval of time which is identical with the time taken by the weight to perform a complete swing. In other words, the hand which pulls the fibre must move in perfect unison with the weight. If this were not so, the impulses would destroy one another's effects. Just so with the forks; they must be in the most rigorous unison, in order that the effects of the impulses may accumulate. Again, in each experiment a certain lapse of time is necessary to allow the effects of the successive impulses to accumulate.

The complete explanation of the experiment with the two tuning-forks is as follows. Let the prong A of the fork in the right hand be supposed to be advancing in the direction of the non-vibrating fork B; the air between A and B will be compressed, and thus the pressure on this side of the prong B will be greater than that on the other; the latter prong will therefore move through an infinitesimal space away from A. Now suppose the prong A has reached its extreme position and is returning; then, as both forks execute their vibrations in exactly equal times, whether these be of large or small extent, it follows that B must be returning also; but as A moves through a greater space than B, the air between the two will become rarefied, and thus the pressure on this side of B will be less than that on the other; B will therefore receive another impulse, which will slightly increase its amplitude. On its return, it will receive another slight impulse, and thus, by these minute successive additions, the amplitude is soon sufficiently increased to produce an audible sound.

In the preceding experiment, the exciting fork communicates a small portion of its motion to the air between the forks, and then this latter gives up part of its motion to the other fork. Now, as the density or weight of air is so exceedingly small in comparison with that of the steel fork, the amplitude of the vibrations thus set up in the latter must necessarily be always very small, that is, its sound will be very faint. By using a medium of greater elasticity, the sound may be obtained of sufficient intensity to be heard by several persons at once. Thus, let one fork be struck sharply, and the end be immediately applied to a sounding board, on which the end of the non-vibrating fork is already resting; after the lapse of a second or so, the latter will be heard giving forth a sound of considerable intensity, the motion in this case having been transmitted through the board.

The following experiments illustrate the phenomenon of resonance or co-vibration in the case of stretched strings. Press down the loud pedal of a pianoforte, so as to raise the dampers from the strings. Each sound on the pianoforte is generally produced by the vibration of two or three wires tuned in unison. Set one of these vibrating, by plucking it with the finger. After the lapse of a second or so, damp it, and the other wire will be heard vibrating. Again, having raised the dampers of a pianoforte, sing loudly any note of the piano, near and towards the sound-board. On ceasing, the piano will be heard sending back the sound sung into it. The full meaning of this experiment will be explained hereafter.

The resonance of strings may be visibly demonstrated to an audience in the following manner. Tune two strings on the monochord or any sound-board, in perfect unison, and upon one of them place a rider of thin cardboard or paper. On bowing the other string very gently, the rider will be violently agitated, and on increasing the force of the bowing, will be thrown off.

In these experiments the sound-board plays an important, or rather an essential part. Thus, in the above experiment with the piano, the sound waves from the larynx of the singer strike the sound-board of the piano, setting up vibrations in it, which are communicated through the bridges to the wires. It will be found that in these experiments with stretched strings, such perfection of unison as was necessary with the forks is not absolutely essential, and the reason is obvious; for in the first place, the medium by which the vibrations are communicated in the former, viz., the wood, is much more elastic than in the latter; and secondly, the light string or wire is much more easily set in vibration than the heavy steel of the fork. A smaller number of impulses is therefore sufficient to excite the string, and consequently such a rigorous unison is not absolutely essential. As we might expect, however, the more exact the unison, the louder is the sound produced.

In consequence of their small density, masses of enclosed air are very readily thrown into powerful co-vibration. Strike a C^{I} tuning-fork, and hold the vibrating prongs over the end of an open tube, about 13 inches long and about an inch in diameter. The sound of the fork, which before was very faint, swells out with considerable intensity. The material of the tube is without influence on the result. A sheet of paper rolled up so as to form a tube answers very well. On experimenting with tubes of different lengths, it will be found that the sound of the C^{I} fork is most powerfully reinforced by a tube of a certain length, viz., about 13 inches. A slightly shorter or longer tube will resound to a smaller extent, but little resonance will be obtained, if it differs much from the length given. Before reading the following explanation of this phenomenon, the student should read over again the account given in Chapter II of the propagation of sound.

Let A B, fig. 32, represent the tube, with the vibrating tuning-fork above it. As the lower prong of the latter descends, it will press upon the air particles beneath it, giving rise to a condensation, A C.

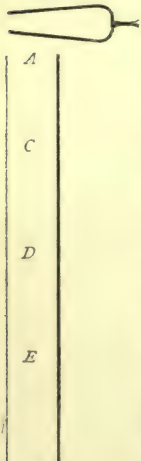


Fig. 32.

The particles in A C being thus crowded together, press upon those below, giving rise to a condensation C D; the particles in which, in their turn, press upon those beneath, thus transmitting the wave of condensation to D E. In this way, the condensation passes through the tube, and at length reaches the end, E B. The crowded particles in E B will now press outwards in all directions, and overshooting the mark, will leave the remainder farther apart than they originally were; that is, a rarefaction will be formed in EB. But as there is now less pressure in EB than in DE, the particles of air in the latter space will tend to move towards E B, and they themselves will be left wider apart than before; that is, the rarefaction will be transmitted from B E to E D, and in like manner will pass up the tube till it reaches A C. On arriving here, as the pressure in A C will be less than the pressure outside the tube, the air particles will crowd in from the exterior and give rise to a condensation. Thus, to recapitulate; the downward movement of the prong gives rise to a slight condensation in the tube below; this travels down the tube to B, where it is reflected as a pulse of rarefaction; this, rushing back, on reaching A is changed again to a pulse of condensation. Now if, while this has been going on, the fork has just made one complete vibration, the lower prong will now be coming down again as at first, and thus will cause an increase in the degree of condensation. The same cycle of change will take place as before, and will recur again and again, the degree of condensation and rarefaction, that is, the intensity of the sound, rapidly increasing to a maximum. To compare this with the experiment of the suspended weight:—the vibrations of the fork correspond to the properly timed impulses, and the air in the tube to the suspended body: and, just as in that experiment, the essential point was the proper timing of the impulses, so in this case the essential matter is, that the downward journey of the condensation, shall coincide with the downward movement of the prong. In order that this coincidence may occur each time, it is evident that the wave must travel down and up the tube, in exactly the same time that the fork makes one vibration; that is, while the fork makes one vibration the sound must travel twice the length of the tube. Moreover, every vibration of the fork gives rise to one sound wave; therefore, in order that a tube open at both ends may give its maximum resonance when excited by a fork, it must be half as long as the sound wave originated by that fork.

It will be seen that a certain amount of resonance is obtained if the tube is twice this length; for in that case, every *alternate* descent of the prong will coincide with a condensation below, and each *alternate* ascent with a rarefaction; but such resonance will evidently be much feebler. For intermediate lengths, the fork will soon be in opposition to the pulses in the tube, and thus no resonance can result.

Tubes closed at one end are termed stopped tubes; with these the case is somewhat different. Let A B, fig 33, represent a stopped tube, the lower prong of the tuning-fork above, being about to descend towards it. As we have already seen, this gives rise to a condensation A C, which travels down to B D. The air particles in B D, having no way of escape, save backwards, press upon those in C D, and thus the condensation is reflected back to C D, and finally to A C. From here some of the condensed particles escape into the external air, leaving the remaining particles slightly wider apart; that is, a slight rarefaction is formed. If while this has been taking place, the prong has reached its lowest position and is just returning, this movement will have the effect of increasing the rarefaction. This latter will then be transmitted down the tube to BD. On reaching this, there will be less pressure in B D than in D C, and

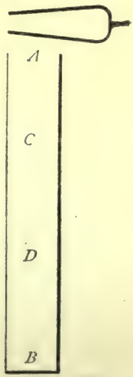


FIG. 33.

consequently the air particles in the latter will crowd into the former, causing a rarefaction in C D. In this way the rarefaction is transmitted back to A C. As the pressure of the external air will now be greater than that in A C, air particles from the former will crowd in, forming a condensation in A C. If at this moment, the prong is a second time beginning its descent, this condensation will be increased, and the same series of changes will take place as before. It is evident, therefore, that the sound wave must make two complete journeys up and down the tube, while the fork is executing one vibration; that is, in order that a stopped tube when excited by a fork, may give its maximum resonance, it must be $\frac{1}{4}$ as long as the sound wave originated by that fork.

We have already seen, that the length of the sound wave produced by a sounding body, may be ascertained by dividing the velocity of sound by the vibration number of that body; consequently it is easy to calculate the length of tube, either open or stopped, which will resound to a note of given pitch. The rule evidently is: divide the velocity of sound by the vibration number of the note; half this quotient will give the length of the open pipe, and one fourth will give the length of the stopped one. It is necessary that the tube should be of moderate diameter, or the rule will not hold good, even approximately.

The resonance of stopped tubes may easily be illustrated, by means of glass tubes, corked, or otherwise closed at one end. On holding a vibrating tuning-fork over the open end of a sufficiently long tube, held with its mouth upwards, and slowly pouring in water, the sound will swell out when the vibrating column of air is of the requisite length, the water serving the purpose of gradually shortening the column. For small forks, test tubes, such as are used in chemical work, are very convenient.

It is by no means necessary that the resounding masses of air should be in the form of a cylinder; this shape was selected for the sake of simplicity in explanation. Almost any shaped mass of enclosed air will resound to some particular note. Everyone must have noticed, that the air in a gas globe, vase, &c., resounds, when some particular sound is loudly sung near it. The following is an interesting method of optically illustrating this phenomenon. A, fig. 34, is a cylinder 3 or 4 inches in diameter, and 5 or 6 inches long, with an open mouth, B. The other end is covered with an elastic membrane, D, such as sheet india-rubber slightly stretched, thin paper, or membrane. At C is fastened a silk fibre, bearing a

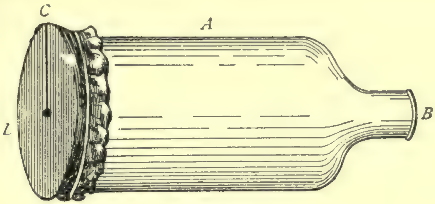


Fig. 34.

drop of sealing wax, hanging down like a pendulum against the membrane. If now anyone places himself in front of the aperture, and sings up and down the scale, on reaching some particular sound the pendulum will be violently agitated, showing that the membrane and the air within the bottle are vibrating in unison with that note.

Another simple experiment of the same kind can be performed with a common tumbler. Moisten a piece of thin paper with gum,

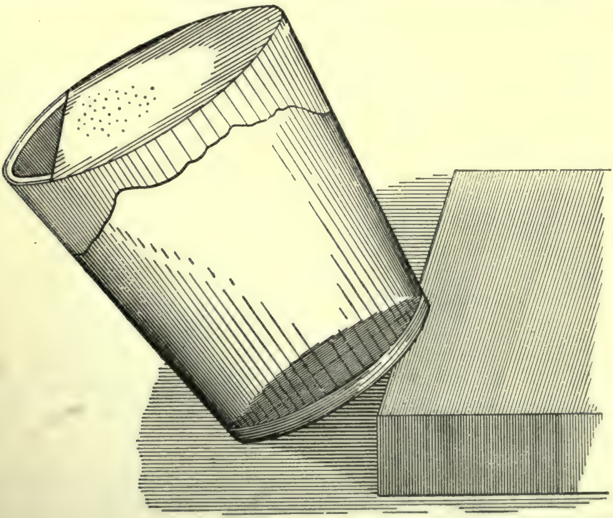


Fig. 35.

and cover the mouth of the tumbler with it, keeping the paper on the stretch. When dry cut away a part of the paper as seen in fig. 35. Put a few grains of sand, or any light substance on the cover.

and then tilt up the glass, so that the sand will nearly, but not quite, roll off. Having fixed the glass in this position, sing loudly up and down the scale. On reaching a certain note, the co-vibration of the air in the tumbler will set the paper and sand into violent vibration. By singing a sound of exactly the same pitch as that to which the air in the tumbler resounds, the sand may be moved when the singer is several yards away.

The phenomenon of resonance is taken advantage of, in the construction of resonating boxes. These are simply boxes (fig. 36), generally made of wood, with either one or two opposite ends open,

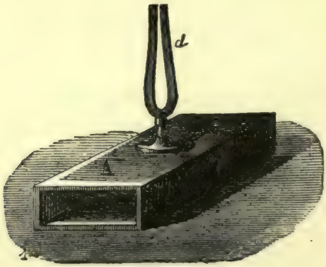


FIG. 36.

and of such dimensions, that the enclosed mass of air will resound to the tuning-fork to be attached to the box. Such boxes greatly strengthen the sound of the fork, by resonance; the vibration being communicated, through the wood of the box, to the air inside. It may be remarked here, that the sound of a fork attached to a resonance box of proper dimensions, does not last so long as it would, if the fork were held in the hand and struck or bowed with equal force; for in the former case it has more work to do, in setting the wood and air in vibration, than in the latter, and therefore its energy is sooner exhausted.

For forks having the vibration numbers in the first column of the following table, boxes having the internal dimensions given in the 2nd, 3rd, and 4th columns are suitable. The dimensions of the first four are, for boxes open at one end only; those of the last four, for boxes open at both ends. The fork is screwed into the middle of the top of the box. The dimensions are in inches.

Vibration No.	Length.	Width.	Depth.
128	22·2	11·6	6·1
256	11·5	3·8	2
384	7·3	3·2	1·8
512	5·4	2·7	1·5
640	8·8	2·7	1·4
768	7·8	2·3	1·3
896	6·2	2·1	1·1
1024	5·5	1·9	1

A Resonator is a vessel of varying shape and material, and of such dimensions, that the air contained in it resounds, when a note of a certain definite pitch is sounded near it. Resonators are most commonly constructed of glass, tin, brass, wood, or cardboard. The forms most often met with are the cylindrical, spherical, and conical. Their use is to enable the ear to distinguish a sound of a certain pitch, from among a variety of simultaneous sounds, of different pitches. The only essential, therefore, in the construction of a resonator is, that the mass of air which it encloses shall resound to the note which it is intended to detect. The best form for a resonator designed for accurate scientific work is the spherical, as it then reinforces only the simple sound to which it is tuned. The

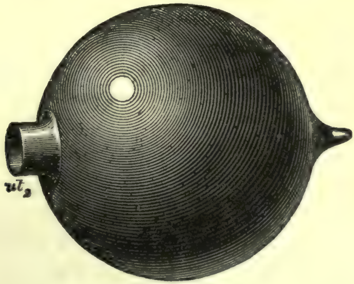


Fig. 37.

spherical resonators employed by Helmholtz in his researches, were of glass, and had two openings as shown in fig. 37. The opening on the left hand serves to receive the sound waves coming from the vibrating body, the other opening is funnel shaped and is to be inserted in the ear. Helmholtz caused this nipple to fit closely into the aural passage, by surrounding it with sealing wax, softening the latter by heat, and then gently pressing it into the ear. The resonator when thus used, has practically only one opening. In using these instruments, one ear should be closed, and the nipple of the resonator inserted in the other. On listening thus to simultaneous sounds of various pitches, most of them will be damped; but whenever a sound occurs of that particular pitch to which the resonator is tuned, it will be wonderfully reinforced by the co-vibration of the air in the resonator. In this way, anyone, even though unpractised in music, will readily be able to pick out that particular sound from a number of others. When, from the faintness of the sound to be detected, or from some other cause, any difficulty in hearing it is experienced, it is of advantage to alternately apply the resonator to, and withdraw it from the ear

A resonator, which is capable of being tuned to any pitch within the compass of rather more than an octave, has been used for some years by the writer. It is composed of three tubes of brass, sliding closely within one another. The innermost, fig. 38a, which is

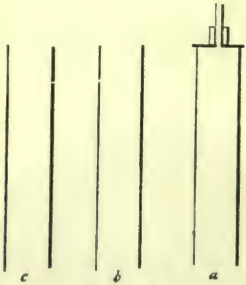


Fig. 38.

about four inches in length, and an inch or more in diameter, is closed at one end by a cap which is screwed on to the tube. In the centre of this cap is an aperture, about half an inch in diameter, which is closed by a perforated cork, through which passes a short piece of glass tube, the end of which is fitted to the ear. The resonator is thus a closed one, and its length can be increased by means of the sliding tubes, *b* and *c* (which are each about 4 inches long), from about 4 inches to 12 inches. It is best tuned approximately, by first calculating the length of stopped tube corresponding to a certain note, according to the method explained in the present chapter. Starting with the resonator at this length, by gradually increasing or diminishing its length, while that note is being sounded on some instrument, a point of maximum resonance will soon be obtained. In this way, the length of the resonator for all the notes within its range can be ascertained. The names of these notes may be conveniently written on a slip of wood, or engraved on a strip of metal, each one at a distance from the end equal to the length of the resonator, when tuned to the corresponding note. Then, in order to adjust the resonator to any note, it is only necessary to place the slip inside it, and gradually lengthen or shorten till the open end is coincident with the name of that note, as engraved on the slip. It is of great advantage to have two such resonators, both similarly tuned, and simultaneously apply one to each ear. If only one be used, the other ear should be closed.

For sounds high in pitch, glass tubes cut to the proper lengths, are very convenient. They may be made to taper at one end for insertion in the ear, or in the case of very high notes, left just as they are, and used as open tubes, being held at a small distance from the aural passage. It may be remarked here, that the tube of the ear is itself a resonator. The pitch of the note to which it is tuned, will of course vary in different persons, and may in fact be different for the two ears of the same person; it generally lies between $G^3 = 3{,}072$ and $E^3 = 2{,}560$.

SUMMARY.

Resonance or *Co-vibration* is the name given to the phenomenon of one vibrating body imparting its vibratory movement to another body, previously at rest.

To obtain the maximum resonance two conditions are essential:

(1) The two bodies must be in exact unison; that is to say, they must be capable of executing precisely the same number of vibrations in the same time.

(2) A certain period of time must be allowed for the exciting body to impress its vibrations on the other.

The phenomenon of resonance may be illustrated by means of tuning-forks, strings, &c., but partially confined masses of air are the most susceptible.

A *Resonance Box* is usually constructed of wood; it may be open at one or both ends, and must be of such dimensions that the enclosed mass of air will vibrate in unison with the tuning-fork to be applied to it.

A *Resonator* is an open vessel of glass, metal, cardboard, or other material, of such dimensions, that the mass of air contained in it resounds to a note of a certain pitch. Its use is, to assist the ear in discriminating a sound of this particular pitch, from a number of others at different pitches, all sounding simultaneously.

In order that a column of air, in a cylindrical tube open at both ends, may vibrate in unison with a given sound, the length of the tube must be approximately *one half* the length of the corresponding sound wave.

If the tube be closed at one end, its length must be *one-fourth* that of the sound wave.

In both cases the diameter of the tube should not exceed one-sixth the length.

CHAPTER VIII.

On the Quality of Musical Sounds.

Hitherto we have treated only of simple sounds, that is to say, each sound has been considered to be of some one, and only one particular pitch. This is, however, far from being the case with the great majority of musical sounds we hear. If such sounds are attentively examined, almost all of them will be found to be compound; that is, each individual sound will be found to really consist of a number of simple sounds of different pitch. Those readers, who are not already practically cognizant of this fact, are strongly recommended to convince themselves of it, by experiment, before proceeding further. Some persons, both musical and unmusical, find great difficulty in distinguishing the simple elementary sounds, that form part of a compound tone. Those who experience any such difficulty, will find it useful to go carefully through the following experiments.

Strike a note on the lower part of the key-board of a pianoforte, say $C_{|}$, in the Bass Clef. As the sound begins to die away, the upper octave of this note may, with a little attention, be readily distinguished. If the listener experiences any difficulty in recognising it, he will find it useful to lightly touch the C above (that is, the sound he is listening for) and let it die away before striking the Bass $C_{|}$. If he does not then succeed, a resonator tuned to the expected sound, or, better still, two such resonators, one for each ear, should be used. By alternately applying these to, and withdrawing them from, the ears, even the most untrained observer cannot but detect the wished-for sound. Next, strike the same Bass $C_{|}$ as before, but direct the attention to the G in the Treble staff an octave and a fifth above it. This is generally more easily recognised than the preceding, and is usually equally loud. When this has been clearly heard, strike the same note as before, and listen for the C' in the Treble staff, two octaves above it. More difficulty will perhaps be experienced in detecting this, but by the aid of properly tuned resonators, it will be heard sounding with considerable intensity. The next two sounds are better perceived as the tone is dying away; they are the E', two octaves and a major third above the sound struck, and the G' two octaves and a fifth above.

The student should vary this experiment by taking other notes, and listening to their constituent elements. These latter will always be found occurring in the above order: thus if the D in the Bass clef be struck the following sounds may be heard:—

s'
m'
d'
s
Key **D**. **d**

d$_{|}$

No sound intermediate in pitch between any of these will be detected. Further, these sounds are not aural illusions, but have a real objective existence, for they are capable of exciting corresponding sounds in other strings, by resonance. Thus, having softly pressed down any key, say (**d**), without sounding it, so as to raise the damper from the wires, strike sharply the octave below (**d$_{|}$**) and after a second or two, raise the finger from this latter, so as to damp its wires; the note (**d**) will be plainly heard, the corresponding wires having been set in vibration by resonance. This experiment will be found successful with all the constituent parts given above. For example, press down the $F'\sharp$ (**m'**) on the top line of the Treble staff, but without sounding it, and strike sharply the $D_{|}$ (**d$_{|}$**) two octaves and a major third below. Raise the finger from the latter after a second or two, and the wires of the former will be heard giving forth the $F'\sharp$ (**m'**).

All the constituent elements of a compound tone given above, are very prominent on an American Organ, and still more so on the Harmonium. They will be found to be in exactly the same order.

no sound intermediate between those given will occur. With the aid of resonators, it will be easy to detect still higher constituents than those mentioned above. It need scarcely be said, that, in the experiments with these instruments, only one reed should be vibrating at a time.

The constituent elements in the compound tones of the voice are more difficult to detect. It is advisable to begin with a good bass voice. All the constituents given above, may be heard after a little practice with the resonators. They are louder in some vowel sounds, as will be seen hereafter, than in others; the "a" sound as in "father," and the "i" as in "pine," are favourable ones to experiment with. After a little practice, the ear becomes practised in this analysis of sounds, and the resonators may be dispensed with to a great extent.

Before proceeding further, it will be best to explain the terms that are used in speaking of these constituents of a compound tone. On one system of nomenclature, the lowest element of a compound tone is termed the Fundamental; the next one (an octave above), the First Overtone; the next (a Fifth above that), the Second Overtone; the next (a Fourth above that), the Third Overtone; and so on. The constituent elements are also termed Partials; the lowest being termed the First Partial; the next, the Second Partial; the next, the Third Partial; and so on. Thus, taking ($\mathbf{d}_{\mathsf{l}}$) as the Fundamental or First Partial, the others will be,

5th Overtone	-	-	$\mathbf{s}^{\mathsf{l}}$	-	-	6th Partial.
4th ,,	-	-	$\mathbf{m}^{\mathsf{l}}$	-	-	5th ,,
3rd ,,	-	-	$\mathbf{d}^{\mathsf{l}}$	-	-	4th ,,
2nd ,,	-	-	$\mathbf{s}$	-	-	3rd ,,
1st ,,	-	-	$\mathbf{d}$	-	-	2nd ,,
Fundamental Tone		-	$\mathbf{d}_{\mathsf{l}}$	-	-	1st ,,

Each of these partials or overtones is a simple tone, that is, a sound of definite pitch, which cannot be resolved into two or more sounds of different pitch. A compound tone is a sound consisting of two or more simple tones.

By means of resonators, many higher partials than the six already mentioned can be detected. The following list contains the first twenty. The first column gives the order of the partials; the second and third, their names, calling the fundamental C_3 and $\mathbf{d}_3$ respectively; and the fourth gives the ratios of their vibrational numbers, to the fundamental, this latter being taken as 1.

ORDER.	NAME.	NAME.	VIB. RATIO.
XX	E^1	m^1	20
XIX	....	...	19
XVIII	D^1	r^1	18
XVII	....	...	17
XVI	C^1	d^1	16
XV	B	t	15
XIV	....	....	14
XIII	....	. .	13
XII	G	s	12
XI	....	..	11
X	E	m	10
IX	D	r	9
VIII	C	d	8
VII	....	....	7
VI	G_1	s_1	6
V	E_1	m_1	5
IV	C_1	d_1	4
III	G_2	s_2	3
II	C_2	d_2	2
I	C_3	d_3	1

Those un-named do not coincide exactly with any tone of the modern musical scale. VII is approximately $B\flat_1$ or **ta**$_1$.

It will be seen on inspecting the above table, that the partials occur according to a certain fixed law; viz., the vibrational numbers of the partials, commencing at the fundamental. are proportional to the numbers 1, 2, 3, 4, 5, 6, &c. Thus, theoretically, the above table may be indefinitely extended. Practically, the first twelve or more may be verified with a harmonium and a couple of resonators; those above, are best observed on a long thin metallic wire, or an instrument of the trumpet class, in which the higher partials are very prominent.

By experiments, similar to those which have been recommended in the case of the piano, it is easy to convince oneself, that nearly all the tones produced by stringed and wind instruments. are compound, and that the partials of which these compound tones consist, belong to the series given above; that is to say, though any one or more of these partials may be absent, no sound of any other pitch, than those given above, ever makes its appearance.

Instruments which produce only simple tones are comparatively rare. A tuning-fork, when struck on a hard substance, or when carelessly bowed, gives a compound tone, consisting of a fundamental and one or two very high overtones. When, however, it is mounted on a resonance box of proper dimensions and carefully bowed, the fundamental tone is so strengthened by resonance, that the resulting sound is practically free from overtones. The tones of flutes and of wide stopped organ pipes gently blown, and the highest notes of the piano, are nearly simple.

The relative intensities of the partials forming a compound tone vary very greatly in different instruments, and even in different parts of the same instrument; thus, on the lower part of a piano, the third partial is generally louder than the fundamental, while on the upper part, it is very much softer. In some voices, again, and with some vowel sounds, the third partial is painfully prominent, while in other voices and with other vowel sounds, it is only detected with difficulty. As a general rule, the farther the partial is from the fundamental, the less is its intensity. Taking the sound from a well-bowed violin as a model of tone, Helmholtz has given the following approximation to the relative intensities of its partials, the intensity of the fundamental being taken as 1.

PARTIAL.	INTENSITY.
VI	$\frac{1}{36}$
V	$\frac{1}{25}$
IV	$\frac{1}{16}$
III	$\frac{1}{9}$
II	$\frac{1}{4}$
I	1

Considering the loudness of the partials in many instruments, it may be a matter of surprise to some, that they are not more easily recognised. It should be remembered, however, that, as the partials of a compound tone all begin together, and usually continue with unvarying relative intensities till the tone ends, when they all terminate together, the ear has always been accustomed to consider it as a whole. Musical people especially, having been in the habit of directing all their attention to a tone as a whole, are often incapable of recognising the constituent parts, until their attention is directly called to them ; just as a person, after having had a clock ticking in his room for some time, ceases to notice the ticking unless something attracts his attention especially to it. Again, when one is thinking deeply, a remark made by another person is often not perceived : the nerves of hearing are doubtless excited, but the attention not being aroused, the sound is not perceived. When anyone has once become accustomed to listen for overtones, there is no difficulty whatever in hearing them ; in fact, they sometimes force themselves upon the ear of the practised listener when not wanted.

Most of the foregoing facts concerning partials have been known for centuries, but the phenomenon was regarded as little more than a curiosity, until Helmholtz proved that the quality of a musical tone depended upon the occurrence of partials. Before going into this matter, it will be necessary to show more exactly what is meant here by quality.

Many musical tones are accompanied by more or less noise ; thus, the tone of an organ-pipe is adulterated, as it were, more or less, by the noise of the wind striking the sharp edge of its embouchere ; the tone from a violin is mingled, more or less, according to the skill of the player, with the scraping noise of the bow against the strings ; the tone from the human voice is accompanied, more or less, with the noise of the breath escaping. Again, the sounds of some instruments differ from those of others, in that their intensities vary in different but regular ways. Thus in the piano and harp, the tones, after the wires are struck, immediately decrease regularly in intensity, till they die away ; while on the organ they continue with unvarying intensity, as long as they sound at all. All such peculiarities as the above are not included under the term quality, as we use it here. The following may be taken as a formal definition of the term quality, as employed below. If two tones perfectly free from noises, of precisely the same pitch, and of equal intensities, differ in any way from one another, then all those respects in which they differ are comprised under the term quality. Using the term quality in this sense, Helmholtz has shown that : *The Quality of a compound tone depends upon the number, order, and relative intensities of its constituent partials.*

Before stating the various methods by which this proposition has been proved, it may be advisable to explain its meaning a little more fully. In the first place, the proposition asserts, that the quality of a tone varies with the number of its component partials; thus if one tone consists of three partials, another of four, and another of six; then, each of these three tones will have a different quality from the other two. In the second place, the proposition declares that the quality of a tone varies with the order of its constituent partials; for example, suppose we have three tones, the first consisting, say, of the 1st, 2nd, and 3rd partials, the second of the 1st, 3rd, and 5th, and the third of the 1st, 3rd, and 6th, then each of these three tones will have a different quality from the other two. In the above cases, we have supposed the partials to be of the same relative intensities in each case. If, however, the relative intensities vary, the proposition affirms that the quality will vary also. Thus to take a simple case, suppose we have two tones each consisting of the 1st and 2nd partials, and that the two fundamentals are of the same intensity; then if the second partial of the one differs in intensity from the second partial of the other, the proposition asserts, that the quality of the one tone will differ from that of the other.

On reading the above propositions the following question at once suggests itself. Is the alleged cause, viz., the variation in the number, order, and relative intensities of the partials, sufficient to account for the observed effect, viz., the variation in quality? The variations in quality of tone are infinite; therefore, if the proposition be true, the variations in the number, order, and relative intensities must be infinite also. Now the number of variations in number and order of partials although very great are practically limited; but it is obvious that the relative intensities of the partials may vary in an infinite number of ways, and thus the above question must be answered in the affirmative. In the next place, it is easy to see that if the proposition be true, simple tones can have no particular quality at all, they must all resemble one another in this respect, from whatever source they come. On trial this will be found to be the case. We have already observed that these tones can be approximately obtained from tuning-forks mounted on appropriate resonance boxes, wide-stopped organ pipes and flutes gently blown, and the highest notes of the pianoforte. The tones from these four sources cannot be compared together very well, for the reason already referred to: the first mentioned being almost pure, the second and third being accompanied by characteristic noises, and the fourth having its peculiar variation of intensity; but they all agree in being gentle and somewhat dull. Moreover, they can be strictly compared in their own class; thus, for example, the tones from tuning-forks are all alike in quality; in selecting a tuning-fork, no one ever thinks of the quality of its tone.

There are two methods, by which the important proposition now under consideration may be proved,—the analytical, and the synthetical. The process in the former case is to take two sounds, which differ in quality, and by analysing them into their constituent partials (with or without the aid of resonators), show that these latter differ in the two cases, either in number, order, or in their relative intensities. Thus, if the student analyses a tone of rich and full quality, he will find the first six partials tolerably well developed, while on the other hand, in a tone of poor or thin quality, he will find most of them absent, or of much less intensity. Again, the metallic or brassy quality (as it is termed) of instruments of the trumpet class, he will find to be due to the clashing of very high partials, which are very prominent in such instruments, and which, as will be seen by referring to the table on page 72, lie very close together. As another illustration, the peculiar quality of the tones of the clarionet, may be accounted for, by the fact, that only the odd partials, the 1st, 3rd, 5th, &c., will be found to be present in the tones of this instrument. The student will find, in the analysis of the vowel sounds, a very instructive series of experiments. The differences in these sounds, must be simply differences in quality, according to our definition; and thus if the proposition under discussion be true, we ought to find corresponding differences in the number, order, or relative intensities of the partials present in the vowel sounds. On trial, this will be found to be the case. If, for example, the "a" as in "father" be sounded by a good voice, all the first six partials may be easily heard; but if the same voice gives the "oo" sound, scarcely anything but the fundamental will be detected.

The general process, in the synthetical method of proof, is, to take simple tones of the relative pitch of the series of partials, and, by combining these together in different numbers and orders, and with different intensities produce different qualities of tone. The first difficulty here, is to procure perfectly simple tones. These are best obtained from tuning-forks fitted with suitable resonating boxes. An elementary experiment can be conducted as follows. Select two tuning-forks, differing in pitch by an exact octave, and mount them on resonating boxes of the proper dimensions; set the lower one vibrating, by bowing it with a violin or double bass bow, and note the dull yet gentle effect of the simple tone produced. Now bow both the forks rapidly one after the other; the two simple tones will soon coalesce, and will sound to the ear as one tone, of the pitch of the lower one, but of much brighter quality than before. The effect of the higher fork, that is, of the second partial, will be strikingly seen, by damping it after both have been vibrating a second or two; the return to the original dull simple tone is very marked. This experiment may be varied very greatly, by the aid of four or five forks tuned to the first four or five partials. The following eight forks form a very serviceable series for these experiments.

C^{2}	=	1024.
$B\flat^{1}$	nearly	896.
G^{1}	=	768.
E^{1}	=	640.
C^{1}	=	512.
G	=	384.
C	=	256.
C_{1}	=	128.

Of course, the effect produced by these forks is only an approximation to the effect produced by the real partials, for, in the first place, the forks cannot very easily be all excited at the same instant; and again, their intensities can only be regulated in a very rough way. In an arrangement devised by Helmholtz for investigating the vowel sounds, these two difficulties were removed by the use of electro-magnets for exciting the forks, and by employing resonators at different distances, and with moveable openings, to regulate their intensities.

The way in which a tuning-fork is excited by an electro-magnet will be understood by a reference to fig. 39. Let A and B be the poles of an electro-magnet, and C and D the ends of the prongs of a tuning-fork between them. If now a current be sent through the electro-magnet, the poles A and B will attract C and D. Now if the current be stopped, A and B will cease to attract the prongs, which will therefore move towards one another again in consequence of their elasticity. Let the current again pass, and C and D will again be attracted. If we can thus alternately pass and stop the current, every time the prongs move forward and backward, the

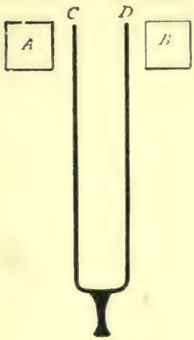


FIG. 39.

fork will continue to vibrate. Thus, if the current be intermittent, and the number of interruptions per second be the same as the vibration number of the fork, the vibration of the latter will be continuous. If the electro-magnet be powerful enough, the fork will also continue in motion, though the number of interruptions per second be $\frac{1}{2}$, $\frac{1}{3}$. $\frac{1}{4}$, &c., of the vibration number of the fork.

These interruptions of the current can be brought about by another fork, the vibration number of which is either the same, or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., of the first one. Let C (fig. 40) represent this second fork, and A, B the poles of an electro-magnet. To the upper prong a small wire is fastened which just dips into a little mercury con

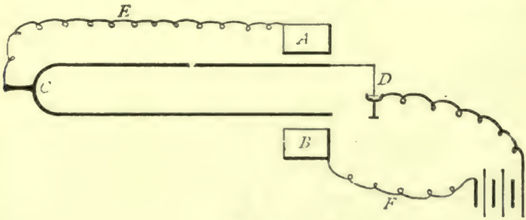


FIG. 40

tained in a cup D, when the fork is at rest. In this position the current from one end of the battery passes to the cup D, thence through the fork and the wire E to the electro-magnet A B, and then by wire F back to battery. But directly the current passes, the poles A and B attract the prongs, and thus the wire attached to the upper one is lifted out of the mercury in the cup D. The current is thus broken, A and B cease to attract, and the prongs return. But in so doing the wire again comes into contact with the mercury, the current is again set up, and A and B again attract the prongs. This alternate making and breaking of the circuit will thus be kept up, and the motion of the fork is rendered continuous.

If now the current from B (fig. 40) instead of passing directly back to the battery, be first led through the electro-magnet of the fork in fig. 39, it will be seen from what has been said above, that this fork also will be set in vibration. Further, the current before returning to the battery may be led through several such electro-magnets, furnished with tuning-forks; and if the latter have vibration numbers which are the same, or any multiples of that of the fork in fig. 40, they will be kept in vibration also. Now, the vibration numbers of overtones are multiples of the vibration number of the fundamental, and therefore only one such fork as that of fig. 40 is necessary, in exciting any number of forks such as that of fig. 39, if the latter are tuned to the series of partials and the former is in unison with the fundamental.

Fig. 41 shows the method by which Helmholtz obtained variations in intensity in his apparatus. D represents one of the tuning-

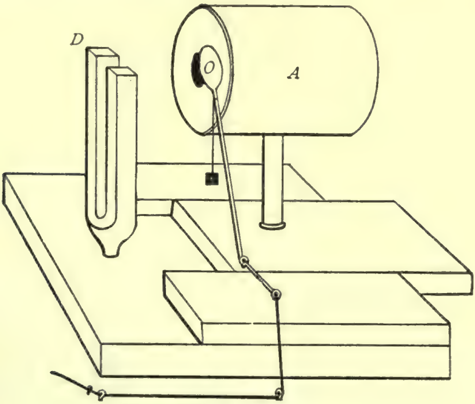


Fig. 41.

forks, kept in vibration by an electro-magnet, which is not shown in the figure. A is a resonator of suitable dimensions, the aperture of which can be closed by the cap, O. When thus closed the sound of the fork is almost inaudible, but, on gradually opening the aperture, the sound comes out with increasing loudness; the maximum being reached when the aperture is quite uncovered. In the figure, the resonator is shown, for the sake of distinctness, at a distance from the fork; when in use it may be pushed up as close to the fork as desired, by means of the stand working in the groove below. The cap covering the aperture of the resonator is connected, by means of levers and wires as seen in the figure, with one of the ten keys of a key-board, the other nine of which are in communication with nine other similar resonators each tuned to its own fork. These ten forks are of pitches corresponding to the ten partials of a compound tone.

Having thus the power of varying the number, order, and relative intensities of these ten simple tones, compound tones of any quality can be, as it were, built up.

In Chapter II we found that there are three elements that determine a sound wave, viz., its length, amplitude, and form. We have since found, that it is upon the length of a sound wave that the pitch of the resulting sound depends, and upon the amplitude that its intensity depends. The form of the wave being the only property remaining, it follows that it is upon this element that the quality of the sound depends. We have now to study the connection between these two.

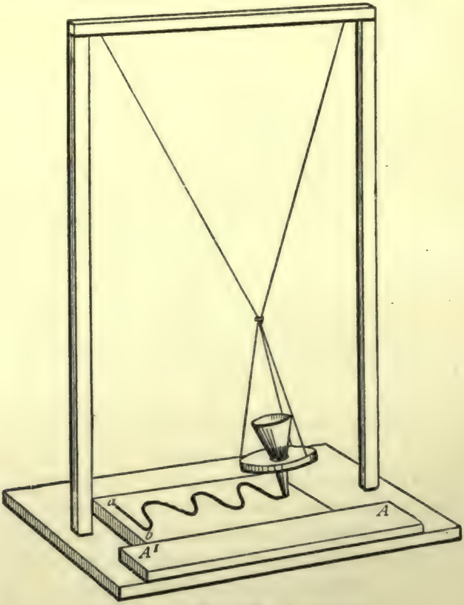


FIG. 42.

The simplest vibrational form is that made by a common pendulum, and is termed a pendular vibration. Suppose the bob of such a pendulum to be at the highest point of its swing to the left; as it swings to the right, its rate of motion becomes more and more rapid till it reaches its lowest position; during the latter half of its swing it gets slower and slower, till it reaches its extreme position on the right, when after a momentary rest, it begins its journey back. The first half of the journey is the exact counterpart of the second, the motion being accelerated in the first half at exactly the same rate that it is retarded in the second. It is easy to construct a pendulum, that shall write a record of its own motion, and thus to obtain a pictorial representation of pendular vibration. Fig. 42 shows a form of the instrument, which the student will have no difficulty in making for himself. The funnel below which rests in a ring of lead, is filled with sand. As it swings backwards and forwards, the sand escapes, leaving a straight ridge of sand on the board below, as seen at (*ab*). If, however, the board be at the same time uniformly moved along from A to A', the sand will be deposited along the wavy track seen in the figure. Such a tracing of a pendular vibration is seen on a larger scale in fig. 43. On comparing this tracing with that made

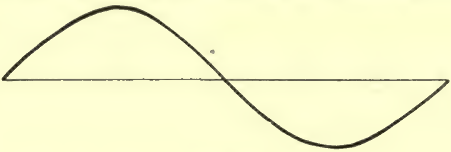


FIG. 43.

by a tuning-fork as described in Chapter IV it is found that they are of the same character: that is, a tuning-fork executes pendular vibrations. But a tuning-fork, as we have seen, gives simple tones. It seems, therefore, that simple tones are produced by pendular vibrations. Further experiment and observation confirm this, and we may take it as proved, that simple tones are always the result of pendular vibrations.

Now a compound tone is made up of partials: and partials are simple tones. Further, simple tones are due to pendular vibrations. It follows, therefore, that compound tones are due to combinations of pendular vibrations.

How are these pendular vibrations simultaneously conveyed through the air? Throw a stone into a piece of still water, and while the waves to which it gives rise are travelling outwards, throw another stone into the water. One series of waves will be seen to pass undisturbed through the other series. Let the tracing A*a*B*e*C, fig. 44 represent a wave of the first series, and A*d*B*b*C one of the second series, and let the dotted straight line, A*g*B*h*C,

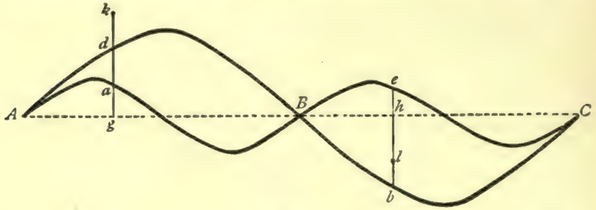


Fig. 44.

represent the surface of still water. In the first place consider the motion of a particle of water at g. The first wave would cause the drop to rise to a, and the second if acting alone would raise it to d. According to the fundamental laws of mechanics, each force will have its due effect and the drop will rise to the height k such that $ag + dg = gk$. Again, the drop at h, if under the influence of the first wave alone, would rise to e, but the second wave would depress it to b. Under these two antagonistic forces it falls to l, such that $hl = hb - he$. By ascertaining in this way, the motion of each point along the wave, we can, by joining all these points, determine the form of the compound wave made up of these two elementary ones.

The same mechanical laws apply to sound waves as to water waves. Thus if the two tracings A and B, in fig. 45, be the associated wave forms of two simple tones at the interval of an octave, then C, constructed from these, in the way just explained, will be the associated wave form produced by their union; that is, C is the associated wave form of a compound tone consisting of the first two partials. We have here supposed that A and B commence together, that is, in the same phase. If we suppose the curve B to be moved to the right until the point (1) falls under the point (2), and then compound these waves, we obtain a different resultant wave form, D. If B were displaced a little more to the right, another wave form would result. Helmholtz has shown experimentally that, when two sound waves are compounded in different phases, although waves of different forms are obtained, yet no difference can be detected in the resulting sounds; that is, the sounds corresponding to the forms C and D would be exactly alike.

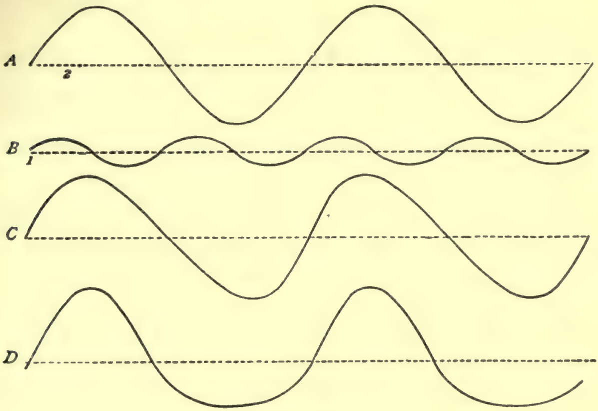


FIG. 45.

This fact seems to show, that the ear has not the faculty of perceiving compound tones as such, but that it analyses them into their constituent partials. If this be the case, it follows that all compound sounds are formed by the union of two or more simple tones. Now it has been proved by Fourier, that there is no form of compound wave which cannot be compounded out of a number of simple waves, whose lengths are inversely as the numbers 1, 2, 3, 4, 5, &c. Musically, this proposition means, that every compound musical sound may be resolved into a certain number of simple tones, whose relative pitch follows the law of the partial tone series.

According to the theory of Helmholtz, briefly referred to in Chapter III, this analysis is effected by the ear as follows:—"When a compound musical tone is presented to the ear, all those elastic bodies (that is, the radial fibres of the basilar membrane, and the corresponding arches of Corti) will be excited, which have a proper pitch corresponding to the various individual simple tones contained in the whole mass of tone, and hence by properly directing attention, all the individual sensations of the individual simple tones, can be perceived."

SUMMARY.

A *Simple Tone* is one that cannot be analysed into two or more sounds of different pitch.

A *Compound Tone* or *Clang* is a tone which is made up of two or more Simple Tones of different pitch.

Almost all the sounds employed in modern music are compound.

The Simple Tones that form part of a Compound Tone are termed *Partials* or *Partial Tones.* The lowest Partial of a Compound Tone is termed the First Partial; the next above, the Second; the next, the Third; and so on.

The First Partial of a Compound Tone is also called the *Fundamental Tone*, and the others, *Overtones;* thus the Second Partial is termed the First Overtone; the Third Partial, the Second Overtone; and so on.

In almost all the Compound Tones used in modern music, the vibration numbers of the Partial Tones, starting with the Fundamental, are in the ratios of

1 : 2 : 3 : 4 : 5 : 6 : 7 : &c.

Any one or more of these partials, however, may be absent in any particular tone. Thus, for example, the even numbered partials are absent in the tones of cylindrical stopped pipes.

The *Relative Intensities* of the partials of Compound Tones vary almost infinitely. As a general, but by no means universal rule, the higher the *order* of the partial, the less is its intensity; that is to say, the first partial is generally louder than the second; the second louder than the third, and so on.

Approximately Simple Tones may be obtained from carefully bowed tuning-forks mounted on suitable resonance boxes; or from flutes and wide stopped organ pipes, gently blown.

If two pure Musical Tones are of the same pitch and of equal intensities, all those respects in which they yet differ, are included under the term *Quality* or *Timbre.*

The *Quality* of a Compound Musical Tone depends upon the *Number*, *Order*, and *Relative Intensities* of its constituent partials.

Helmholtz has demonstrated this proposition by the Analysis and Synthesis of Compound Tones.

Just as pitch depends on wave length, and intensity on amplitude; so the quality of a tone (that is, the number, order, and relative intensity of its partials) depends on *wave form.*

A Simple Tone is the result of *pendular* sound waves or *pendular* vibrations, that is, of vibrations similar to those of a simple pendulum.

A Compound Tone is due to the combination of two or more pendular vibrations, or waves.

Every Compound Tone may be resolved into a certain number of Simple Tones, whose relative pitch follows the law of the partial series.

> Similarly, every compound wave may be resolved into a certain number of simple pendular waves, whose lengths are in the ratios of
>
> $$1 : \frac{1}{2} : \frac{1}{3} : \frac{1}{4} : \frac{1}{5} \text{ \&c.}$$

The form of the sound wave, therefore, determines the number, order, and relative intensities of the partials, that is, the quality of the resultant sound. On the other hand, given the quality, it is not possible to determine the corresponding wave form, since for any one such quality, there is an infinity of wave forms, due to the infinite number of relative positions or *phases* in which the constituent pendular waves may start.

The ear does not perceive a Compound Tone as such, but analyses it into its constituent pendular waves, each of these latter producing the sensation of a Simple Tone at its own particular pitch and intensity.

CHAPTER IX.

On the Vibrations of Strings.

A stringed instrument consists essentially of three parts, viz:—the string, catgut, or wire, to be set in vibration; some means of setting up this vibration; and a sound-board, or other resonant body, by means of which the vibratory movement is to be transmitted to the air.

The means by which strings are set in vibration vary in different instruments. They may be struck by a hammer, as in the pianoforte; bowed, as in instruments of the violin class; set in motion by a current of air, as in the case of the Æolian harp; or plucked, like the harp and zither. In this last case, the plucking may be done with the finger tips, as in the case of the harp and guitar, or by means of a quill or plectrum, as in the zither and harpsichord. In the case of bowed instruments, the particles of resin with which the bow is rubbed, catch hold of the portion of the string with which they are in contact, and pull it aside; its own elasticity soon sends it back, but being immediately caught up again by the bow, the vibrations are rendered continuous.

The vibrating string, presenting so small a surface, is capable of transmitting very little of its motion directly to the air. It is necessary that its vibrations should first be communicated to some body, which presents a much larger surface to the air. Thus, in the pianoforte, the vibrations of the wires are first transmitted, by means of the bridge and wrest-pins, to a sound-board; in the harp, the motion of the strings is communicated to the massive framework. In the violin, the vibratory movement of the strings is communicated by means of the bridge to the "belly." The bridge stands on two feet, immediately beneath one of which is the "sound post," which transmits the motion to the "back" of the instrument, the whole mass of air between the "back" and the "belly" thus being set in vibration.

The following simple experiment will illustrate the important part played by the sound-board or its substitute, in stringed instruments. Fasten one end of a string, 3 or 4 feet in length, to a heavy weight, and, holding the other end in one hand, let the weight hang freely. On plucking or bowing the string, scarcely any sound will be heard. Now attach the free end of the string to the peg at the left hand of the Sonometer (fig. 23), and let the weight hang freely over the pulley at the right hand. If the string be now plucked or bowed, a loud sound will be emitted.

We now proceed to study the conditions which determine pitch, quality, and intensity, in stringed instruments. As the tones produced by such instruments are rarely or never simple, it will be understood, that in investigating the laws relating to *pitch*, it is the *pitch* of the fundamental tone alone, that is considered.

If T denote the tension of a stretched string, and M its mass, it may be shown mathematically, that the velocity V, with which a transverse vibration will travel along it, will be—

$$V = \sqrt{\frac{T}{M}}$$

and if L denote the length of the string, it is evident that $\frac{2L}{V}$ is the time required for it to execute one complete vibration. Therefore if N denotes the number of vibrations the string performs in one second

$$N = 1 \div \frac{2L}{V} = \frac{1}{2L} \times V$$

Substituting the above value of V, we get

$$N = \frac{1}{2L}\sqrt{\frac{T}{M}}$$

From this formula we may deduce the following laws:—

(1). The tension of the string remaining the same, N (the vibration number) varies inversely as the length of the string.

(2). Other things remaining the same, N varies inversely as the diameter of the string.

(3). Other things remaining constant, N varies directly as the square root of the tension—that is, of the stretching force or weight.

(4). Other things remaining constant, N varies inversely as the square root of the density or weight of the string.

These statements can also be verified experimentally, without recourse to mathematics, by means of the Sonometer described in

Chapter IV. Thus, to prove the first law, stretch the wire by attaching any sufficient weight, as shown in fig. 23, and observe the pitch of the tone it then gives. Now place the movable bridge in the centre, pluck the half string and again note its pitch. Do the same with $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, of the wire. It will be found that the tones produced are as follows, calling the tone produced by the whole length, d_{l},

Whole string		..	d_{l}	..	1
$\frac{1}{2}$	,,	..	d	..	2
$\frac{1}{3}$	,,	..	s	..	3
$\frac{1}{4}$	,,	..	d^{l}	..	4
$\frac{1}{5}$	,,	..	m^{l}	..	5
$\frac{1}{6}$	,,	..	s^{l}	..	6

Now we already know, that the ratios of the vibration numbers of these tones are those given in the third column, and we at once see that these latter are the inverse of those in the first column. This experiment may be varied in an infinite number of ways. Thus, by placing the movable bridge so that the lengths of the string successively cut off, are,

$$1, \frac{8}{9}, \frac{4}{5}, \frac{3}{4}, \frac{2}{3}, \frac{3}{5}, \frac{8}{15}, \frac{1}{2},$$

it will be found, that these lengths give the notes of the diatonic scale, and the vibration ratios of the successive intervals of these from the tonic, we alread℘ ⁊ know to be

$$1, \frac{9}{8}, \frac{5}{4}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{15}{8}, 2,$$

which numbers are the former series inverted. Illustrations of this law may be seen in musical instruments with fixed tones, like the piano and harp, in which the strings, as every one knows, become shorter and shorter as the notes rise in pitch. In the guitar, violin, and other instruments with movable tones, variation in pitch is obtained, by varying the length of the vibrating portion of the string.

The second law may be verified, by stretching on the Sonometer, with equal weights, two wires of the same material, the diameter of one of which is, however, twice that of the other. The tones produced will be found to be an octave apart, the smaller wire giving the higher note, that is, the diameters of the wires being as 1 : 2, the vibration numbers of the tones produced are as 2 : 1. Illustrations of this law can be found in many musical instruments; thus, the second string of the violin being of the same length as the first, must be thicker in order that it may give a deeper tone.

To prove the third law, stretch a string on the Sonometer with a weight of, say 16lbs, and note the pitch of the resulting tone. Now stretch the same string with weights of 25lbs, 36lbs, and 64lbs successively, and observe the pitch of each tone. Calling the tone produced by the tension of 16lbs (d), those produced by the tensions of 25, 36, and 64lbs will be found to be (m), (s), and (d'), respectively. Now we have already ascertained, that the vibration numbers of d, m, s, d', are as 4 : 5 : 6 : 8 and these numbers are the square roots of 16, 25, 36, and 64. Examples of the application of this law are to be met with in the tuning of all stringed instruments. The violinist, harpist, or pianoforte tuner stretches his strings still more to sharpen, and relaxes the tension to flatten them.

The fourth law can be proved by stretching two strings of different densities, but of the same length and thickness, by the same weight. Now, by means of the movable bridge, gradually shorten the vibrating part of the heavier string, till it gives a note of the same pitch as the whole length of the lighter one. Now measure the length Z of the lighter string and the length Z_1 of the vibrating portion of the heavier one; it will be found that

$$Z : Z_1 :: \sqrt{D^1} : \sqrt{D} \qquad (\text{I})$$

D^1 and D being the densities of the heavier and lighter string respectively. These densities can be ascertained by weighing equal lengths of the two strings. Let N be the number of vibrations per second performed by the length Z_1 of the heavier string, then if N_1 be the number performed by the whole length Z of the same string, we know by the first law that

$$Z : Z_1 :: N : N_1$$

therefore from (I)

$$N : N_1 :: \sqrt{D^1} : \sqrt{D}.$$

Now as N also denotes the number of vibrations per second performed by the lighter string, this proves the law. As illustrations of the application of this law to musical instruments, the weighting of the lowest strings of the pianoforte by coiling wire round them.

may be mentioned. The density of the fourth string of the violin is increased in the same way.

The pitch of a string, stretched between two fixed supports, is materially affected by heat, especially if the string be of metal As the metal expands on heating and contracts on cooling, the tension becomes less, and the pitch is lowered in the former case, while the tension becomes greater and the pitch rises, in the latter. Heat also produces a difference in the elasticity of strings which acts in the same direction. Strings of catgut are also affected by moisture, which by swelling the string laterally, tends to shorten it, thus increasing the tension, and raising the pitch.

We pass on now, to discuss the conditions, which determine the *quality* of the tone produced by a stretched string.

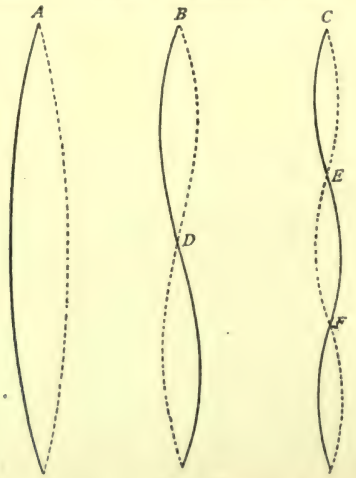


FIG. 46.

Fasten one end of an india-rubber tube, about 12 feet long, to the ceiling of a room, and taking the other end in the hand, gently move it backwards and forwards. It is easy after a few trials, to set the tube vibrating as a whole (fig. 46A). On moving the hand more quickly the tube will break up into two vibrating segments (fig. 46, B). By still more rapid movements, the tube can be made to vibrate in three (fig. 46, C), four, five, or more segments. Precisely the same results can be obtained, by fastening the tube at both ends, and agitating some intermediate point. The points

D, E, F (fig. 46), which seem to be at rest are termed "*nodes*" or "*nodal points*," and the vibrating portion of the tube BD, CE, or EF, between any two successive nodes is called a "*ventral segment.*"

To understand how these nodes are formed, let *a c*, fig. 47 (1) represent a string similar to that just referred to. By jerking the end *a*, a hump *a b* is raised, which travels to the other end. In fig. 47 (2) this hump has passed on to *b c*. In fig. 47 (3) it has been reflected, and is returning to the end *a*, but on the opposite side. While this has been going on, let us suppose another impulse to have been given, so as to produce the hump *a b*, fig. 47 (3). Now the hump *b c* is about to pass on to *a*, and in so doing, the point *b* must move to the left, but the hump *ab* is about to travel on to *c*, and in so doing must move the point *b* to the right. The point *b*, thus continually urged in contrary directions with equal forces, while the humps pass one another, remains at rest. Suppose that the hump takes one second to travel from *a* to *c* and back again: then it is evident that if an impulse is given every half second, the above state of things will be permanent, and the two parts *ab*, *bc* will appear to vibrate independently of each other, fig. 47 (4), the point *b* forming a node. A little reflection will show that, on the same supposition, if the impulses follow one another at intervals of one-third of a second, two nodes and three ventral segments will be formed, and so on. When therefore a string vibrates in 2, 3, 4 segments, each segment vibrates 2, 3, 4 times as rapidly as the string vibrating as a whole.

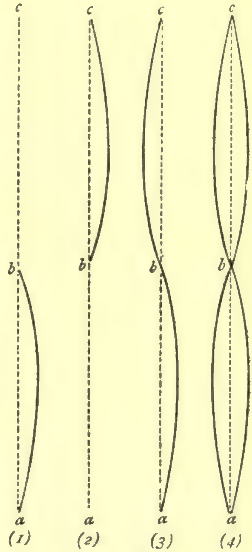


FIG. 47.

A tuning-fork may be used with great advantage, in setting up these segmental vibrations. One end of a silk thread is fastened to one of the prongs of the fork, the larger the better; the other end being either wound round a peg, or after passing over a pulley, attached to a weight. On bowing the fork, the string is set vibrating in one, two, three, or more segments, according to its degree of tension.

The two ends of a stretched string being at rest, it is evident that the number of ventral segments, into which it can break up, must be a whole number; it cannot break up into a certain number of ventral segments and a fraction of a segment. Any point of the string capable of being a node, can be made such, by lightly touching that point, so as to keep it at rest, and bowing or plucking at the middle of the corresponding ventral segment. Thus, if the string of the Sonometer be lightly touched at the centre, and bowed about $\frac{1}{4}$ of its length from the end, it will break up into two (or possibly six) ventral segments, with a node in the centre. Again, if the string be lightly touched at $\frac{1}{3}$ of its length from the end, and bowed about the middle of this third, it will vibrate in three segments (the other $\frac{2}{3}$ dividing into two) separated by two nodes. That the larger part of the string, in this experiment, does divide into two segments separated by a node, may be shown, by placing riders on the string, before it is bowed, one in the centre of this part, where the node occurs, and one in the middle of each of the two ventral segments. When the string is now lightly touched at $\frac{1}{3}$ of its length from the end, and bowed as before, the riders in the middle of the ventral segments will be thrown off, but that at the node will keep its place (fig. 48). This experiment may be repeated

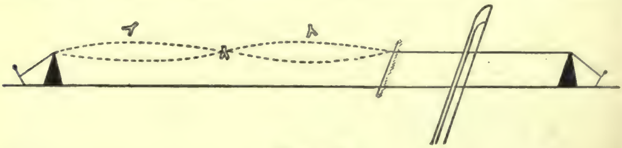


Fig. 48

with a larger number of nodes. Thus, suppose four nodes are required. Divide the string into five equal parts, as there will evidently be that number of segments, and at each of the four points of division, place a coloured rider, with a white one equidistant between each pair, and also between the last one and the end. Remove the coloured rider nearest the other end and lightly touch the point where it stood, with the finger. Draw the bow gently across the string, midway between this point and the end, and the white riders will fall off, while the coloured ones will remain at rest (fig. 49).

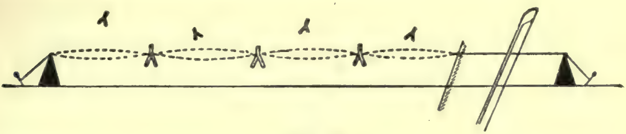


FIG. 49.

A stretched string can therefore vibrate as a whole, that is, with one ventral segment, or with two, three, four, five, six, or more ventral segments, but not with any intermediate fraction. Now if we call the note given forth by a string, when its whole length is vibrating ($\mathbf{d}_{|}$) ; we have already learnt, that when its two halves only are vibrating, we get the octave above, ($\mathbf{d}$) ; if it vibrate in three ventral segments we shall get the twelfth above, ($\mathbf{s}$); with four segments, ($\mathbf{d}^{|}$); with five, ($\mathbf{m}^{|}$); and so on. Only those notes belonging to the series 1, 2, 3, 4, 5, &c., can occur; no note intermediate between these can be produced. It will be at once observed, that this is the series of partial tones, and the idea at once suggests itself, that the occurrence of partials in the tones of stringed instruments is due to the fact, that a string not only vibrates as a whole, but *at the same time* in halves, thirds, quarters, &c., each segment giving rise to a simple sound of its own particular pitch.

The student may convince himself that this is really the case, by a variety of experiments. Thus, while a string is vibrating and giving forth a compound tone in which the fifth partial can be heard, lightly touch it with a feather at a point, distant $\frac{1}{5}$ of its length from the end; all the partials except the fifth, which has a node at this point, will rapidly die out, but this one will be plainly heard, showing that the string must have been vibrating in five segments. Again, touch the middle of the string in a similar manner, after setting it in vibration, and all the partials except those which require a node at this point will vanish, that is, only the second, fourth, sixth, &c., will remain. Further, pluck the string at its middle point; all the tones which require a node at this point must then be absent, only the first, third, fifth, seventh, &c., being heard. In this way, the presence or absence of any particular partial of a compound tone may be ensured.

The occurrence and relative intensities of partials on stringed instruments, depend upon :—

1st. The nature of the string.

2nd. The kind of hammer, bowing, plectrum, &c.

3rd. The place where the string is struck, bowed, or plucked.

With regard to the string, the more flexible it is, the more readily will it break up into vibratory segments, and therefore the greater will be the number of partials in its tones. Thick stout strings, such as are used for the lower notes of the harp, cannot from their rigidity break up into many segments, and therefore the fundamental will be louder than the other partials. On the other hand, thin strings of catgut, such as first violin strings, readily vibrate in many segments, so that their tones contain many partials. Still more is this the case with a long fine metallic wire, in which it is possible to hear some fifteen or twenty partials, the fundamental being very faint or even inaudible. The tinkling metallic quality of tone from such a wire, is due to the prominence of the high partials above the seventh or eighth, which lie at the distance of a tone, or less than a tone, apart. Again, the steel wires which give the highest notes in the pianoforte, being already so short, cannot readily break up into vibrating segments, and hence the highest tones of this instrument are nearly simple.

As we have seen, the three chief methods of setting strings in vibration are: by a blow from a hammer, by bowing, and by plucking. In the first method (employed, for example, in the pianoforte) the quality of the tone is largely affected by the nature of the hammer. If it is very hard, sharp, and pointed, the part of the string which is struck by it will be affected, and the hammer will have rebounded, before the effect of the blow has time to travel along the length of the wire. Thus small ventral segments will be formed, and prominent upper partials will be produced, the lower ones being feeble or absent. On the other hand, if a very soft, rounded hammer be used, the blow being much less sudden, the movement of the wire will have time to spread, and a powerful fundamental may be expected. On the pianoforte, both extremes are avoided, by covering the wooden hammers with felt, so that when they strike the wire, the rebound is not absolutely instantaneous; nevertheless the time during which the hammer and wire are in contact is extremely short. Similarly, in plucking; a soft, rounded instrument, such as the finger tip, gives a stronger fundamental and fewer high partials, than the harder and sharper quill, that used to be employed in the harpsichord.

The quality of the tone, given forth by a stretched string, depends largely upon the point at which it is struck, bowed, or plucked. We have already seen, that the point in question cannot be a node; it is more likely to become the middle of a ventral segment. All the partials, therefore, that require a node at that point, will be absent. Thus if the string be struck at the middle point, only the odd partials will be present in the compound tone produced: if the string be struck at a point, one third of its length from the end, the 3rd, 6th, and 9th partials will be absent. Again, if a string be struck at a point, one seventh, one eighth, or one ninth of its length from the end, the 7th, 8th, or 9th partials respectively, will be absent. Now, these are the first three dissonant partials of a compound tone, so that it improves the quality of tone to have them absent; and it is a curious fact, as Helmholtz observes, that pianoforte makers, guided only by their ears, have been led to place their hammers, so as to strike the strings at about this spot.

With regard to the quality of tone in the pianoforte, it will be found, that in the middle and lower region of these instruments, the tones are chiefly composed of the first six or seven partials, the first three being usually very prominent; in fact, the second and third are not unfrequently louder than the fundamental. As the first six partials form the tonic chord, the tones that have them well balanced, sound peculiarly rich. The result of pressing down the loud pedal should be noted. The idea usually entertained is, that by keeping the dampers raised from the wires, the tones are prolonged after the fingers are taken off the notes. This is true, but not the whole truth. For as the dampers are raised from all the wires, all the latter which are capable of vibrating in unison with the already vibrating wires, will do so. For example, if the loud pedal be depressed, and the $F_{|}$ in the Bass clef be struck, the F_2 wires an octave lower will be set vibrating in two halves; the $B\flat_3$ a fifth below that, in three parts; the F_3, two octaves below the note struck, in four parts; and so on, each section sounding forth the $F_{|}$. Again, the wires which were struck will not only vibrate as a whole, giving $F_{|}$, but in halves giving F, which will start the wires of the F digital, and will also set the $B\flat_2$ wires vibrating in three sections. Further, the original wires will vibrate in three segments producing the partial $C^{|}$, and this will start the wires corresponding to $C^{|}$ and C, and so on. It is easy to see, therefore, that when the loud pedal is held down, and a low note struck, the number of wires set vibrating is very great, giving an effect of increased richness. At the same time, the necessity of raising the pedal at every change of chord is very clearly seen. The result of pressing down the soft pedal is to slide the whole of the hammers along transversely, through a short distance, so that they strike only one of the two or three wires that are allotted to each note.

A great number of partials are usually present in the tones produced by the violin, at least the first eight being nearly always present. The peculiar incisiveness of tone is probably due to the presence of partials above the eighth, which, as will be seen from the table on page 72, lie very closely together. The violin has four strings tuned in fifths, the highest being tuned to $E^{|}$; the lower limit is therefore $G_{|}$. The viola or tenor violin, which is slightly larger than the above, has also four strings tuned in fifths, the highest being A ; the lower limit is consequently $C_{|}$. The violoncello has also four strings, each of which is tuned an octave lower than the corresponding one in the viola. The lower limit is therefore C_2. The double bass usually has only three strings, tuned in fourths, the highest being G_2; its deepest tone is thus A_3, only two notes below the violoncello, but its larger body of tone makes it seem of a deeper pitch than it actually is.

SUMMARY.

The three essentials of a stringed instrument are : (1), *the string* (2), *the means of exciting it* ; (3), *a sound-board or resonator.*

The vibration number of a stretched string varies

Directly as the square root of the *tension,*
Inversely ,, ,, ,, ,, *density,*
,, ,, *length,*
,, ,, *diameter.*

Stringed instruments flatten with rise of temperature, and *vice versa.*

Points of rest, or rather of least motion, in a vibrating string are termed *nodes.* The vibrating part of the string between two consecutive nodes is called a *ventral segment.* The middle of a ventral segment is sometimes referred to as an *antinode.*

The occurrence of partials in the tone of a stretched string, is due to the fact, that it vibrates, not only as a whole, but simultaneously also in halves, thirds, quarters, &c. ; each segment producing a simple tone or partial, of a pitch and intensity corresponding to its length and amplitude respectively.

The occurrence and intensities of these partials depend upon

(1). The nature of the string.
(2). The nature of the excitation.
(3). The position of the point where the string is excited.

To ensure the *absence* of any particular overtone, the string should be excited at that point where this overtone requires a *node* for its formation.

To favour the *production* of any particular overtone, the string should be excited at that point where this overtone requires an *antinode*.

CHAPTER X.

Flue-pipes and Reeds.

It will be found on examination, that in all wind instruments, the air contained in the tubes of such instruments is set in vibration, either by blowing against a sharp edge, at or near the mouth of the tube, as in the flute and the flue pipes of the organ; or by the vibration of some solid body placed in a similar position, as in the clarinet and the reed pipes of the organ. We shall proceed first, to investigate the conditions which determine pitch, quality, and intensity in the former class of instruments.

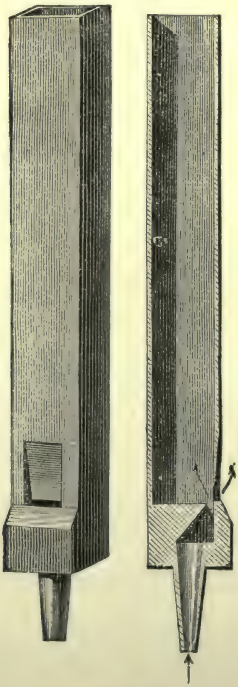


Fig. 50.

Fig 51.

As the type of instruments of this class, we may take an ordinary organ pipe. Such pipes are constructed either of metal or wood; the former being an alloy of tin and lead, or for large pipes, zinc; the latter, pine, cedar, or mahogany. Fig. 50 represents a wooden, and fig. 51 a metal pipe, both in general view and in section. They may be closed, or open at the upper end. Fig. 52 shows an enlarged section of the lower part. The air from the wind chest enters at (*a*) and passes into the chamber (*c*), the only outlet from which is the linear orifice at (*d*). The air rushing from (*d*) in a thin sheet, strikes against the sharp edge (*e*), and the column of air in the pipe is set in vibration. The precise way in which this sheet of air acts is not quite clear. Helmholtz says "The directed stream of air breaking against the edge, generates a peculiar hissing or rushing noise, which is all we hear when a pipe does not speak, or when we blow against the edges of a hole in a flat plate instead of a pipe. Such a noise may be considered as a mixture of several inharmonic tones of nearly the same pitch. When the air chamber of the pipe is brought to bear upon these tones, its resonance strengthens such as correspond with the proper tones of that chamber, and makes them predominate over the rest, which this predominance conceals." On the other hand, this thin sheet of air has been compared by Hermann Smith to an ordinary reed, and called by him an "aeroplastic reed." His theory is, that in passing across the embouchure (*ed*) the aeroplastic reed momentarily produces an exhaustive effect tending to rarify the air in the lower part of the pipe. This, by the elasticity of the air, soon sets up a corresponding compression, and these alternate rarefactions and condensations reacting upon the lamina, cause it to vibrate, and to communicate its vibrations to the air within the pipe.

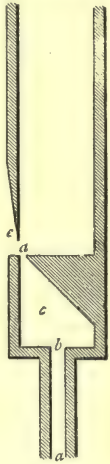


FIG. 52.

The pitch of the fundamental tone given forth by a pipe, depends upon its length; the longer the pipe the deeper the note. The reason of this has been already fully explained in Chapter VII. To recapitulate what is there stated and proved: The vibration number of the sound produced by an open pipe, may be found, by dividing the velocity of sound by twice the length of the pipe; that of a stopped pipe, by dividing by four times its length. In the latter case the *internal* length must be measured, as "length of pipe" really means, the length of the vibrating column of air.

The rule just given, although approximately true in the case of narrow pipes, cannot be depended upon, when the diameter of the pipe is any considerable fraction of its length. The following rule quoted from Ellis' "History of Musical Pitch" is much more accurate. Divide 20,080 when the dimensions are in inches, and 510,000 when the dimensions are in millimetres, by :—

(1). Three times the length, added to five times the diameter, for cylindrical open pipes.

(2). Six times the length, added to ten times the diameter, for cylindrical stopped pipes.

(3). Three times the length, added to six times the depth (internal from front to back), for square open pipes.

(4). Six times the length, added to twelve times the depth, for square stopped pipes.

As a matter of fact, however, the note produced by a stopped pipe is not exactly the octave of an open pipe of the same length: in fact, it varies from it by about a semitone.

The pitch of a pipe is also affected by the pressure of the wind. The above rule supposes this pressure to be capable of supporting a column of water $3\frac{1}{4}$ inches high. If this pressure be reduced to $2\frac{3}{4}$, the vibration number diminishes by about 1 in 300; if increased to 4, it rises by about 1 in 440. The pitch is also affected by the size of the wind slit and the orifice at the foot: by the shape and shading of the embouchure; and by the pressing in or pressing out of the edges of its open end, as by the "tuning cone."

As already stated, the velocity of sound in air, at 0° Centigrade, or 32° Fahrenheit, is 1,090 feet per second, increasing about two feet for every rise of temperature of 1° C. and about one foot for 1° F. The velocity of sound at any temperature may be more accurately determined from the formula

$$V = 1{,}090\sqrt{1 + at}$$

where t is the centigrade temperature and $a = \frac{1}{273}$, the coefficient of expansion of gases. Now, as the vibration numbers of the sounds emitted from stopped and open pipes may be approximately found by dividing the velocity of sound by four times and twice their lengths respectively, it is evident that such vibration numbers will vary with the temperature; the higher the temperature, the sharper the pitch, and *vice versa*. Furthermore, the length of the pipe itself varies with change of temperature, increasing with a rise and shortening again with a fall of temperature. This will obviously have a contrary effect on the pitch, but to a very much smaller extent; in fact, in wooden pipes the expansion is quite inappreciable. Thus the general effect of rise of temperature in organ pipes is to sharpen them.

It is evident, from the above, that the wooden pipes of an organ will sharpen somewhat more than the metal ones, for the same rise of temperature. Furthermore, it is found that small pipes become relatively sharper than large ones, under the same increment of heat; and not only is this the case, but the change takes place much more rapidly in small pipes than in large ones, and in open than in closed pipes. On the other hand, although metal pipes do not sharpen quite so much as wooden ones, they are affected much more rapidly. According to Perronet Thompson, diminution of atmospheric pressure sharpens the tones of pipes, and *vice versa.* He states that a fall of an inch sharpens the tuning C by a comma.

The lowest note producible in the largest organ is $C_4 = 16$, and is obtained from an open pipe about 32 feet long. This pipe together with those giving notes of lower pitch than $C_3 = 32$, are said to belong to the 32 foot octave. $C_3 = 32$ is produced by an open pipe about 16 feet long; hence, from

$C_3 = 32$	to the	B_3	above, constitutes the	16	foot	octave.
$C_2 = 64$	,,	B_2	,, ,,	8	,,	,,
$C_1 = 128$	,,	B_1	,, ,,	4	,,	,,
$C = 256$	,,	B	,, ,,	2	,,	,,
$C^1 = 512$	,,	B^1	,, ,,	1	,,	,,
$C^2 = 1024$	,,	B^2	,, ,,	$\frac{1}{2}$	,,	,,
$C^3 = 2048$	,,	B^3	,, ,,	$\frac{1}{4}$	,,	,,

Increase of intensity in the tones of an organ cannot be obtained by increase of force in blowing: for as we have just seen, a very slight increase in the wind pressure alters their pitch slightly, and still greater increase, as we shall presently see, would affect their quality also. Hence, increase of intensity on the organ has to be produced by bringing more pipes into action by means of stops, or by enclosing the pipes in a case, which can be opened or closed at pleasure, as in the swell organ.

We have now to turn our attention to the conditions that determine the occurrence of overtones, in the tones of organ pipes. Procure an ordinary open wooden or metal organ pipe and blow very gently into it. The fundamental tone of the pipe which we will call (**d**$_{|}$) will be produced. On gradually increasing the strength of the wind, a point will be reached, at which this note will vanish, and a note (**d**), an octave higher, will be heard. On blowing harder still, this (**d**) will cease, and a note (**s**) a fifth above will be given forth, and so on. All these notes **d**, **s**, **d**$^{|}$, **m**$^{|}$, &c., above the fundamental, which thus apparently make their appearance successively, are usually termed the harmonics of the pipe.

In order to understand how these tones are produced, let us turn back to page 60. We saw there how a condensation entering one end (*a*) of the tube (fig. 53), proceeds to the other end (*b*), and is there reflected as a rarefaction. Now suppose that at the moment this rarefaction starts back towards (*a*) another rarefaction starts from (*a*); what will happen when they meet in the centre? The wave from (*b*), if none other were present, would cause the particles of air in the centre (*c*) to move upwards; that from (*a*) would move them with equal force downwards. Under these circumstances the particles in the centre will remain at rest. But, just as in the case of the string, the two pulses of rarefaction will not interfere with one another; each will pursue its course to the end of the tube, where each will be reflected, as formerly explained, as a condensation. Now when these pulses of condensation meet in the centre of the tube, that which comes from (*b*), if it alone were present, would cause the air particles there to move downwards, while that from (*a*), would move them in the opposite direction. The result, as before will be, that the air particles in the centre will remain at rest; and comparing these pipes with the strings already studied, we see that under these circumstances, the middle of the tube becomes a "node," while the ends, being places of greatest vibration, correspond to the middles of "ventral segments." Further, as the impulses enter an open pipe, and are reflected at the ends, these points must *always* be places of maximum vibration, that is, must *always* correspond to the middle of ventral segments. But two ventral segments must necessarily be separated by a node: therefore, the above is the simplest way in which the column of air in an open tube can vibrate, and consequently this form of vibration must give the fundamental tone of the pipe. It may be represented by fig. 54 (A), in which the straight line in the centre shows the

Fig. 53.

position of the node, and the dotted lines give the *associated* wave form.

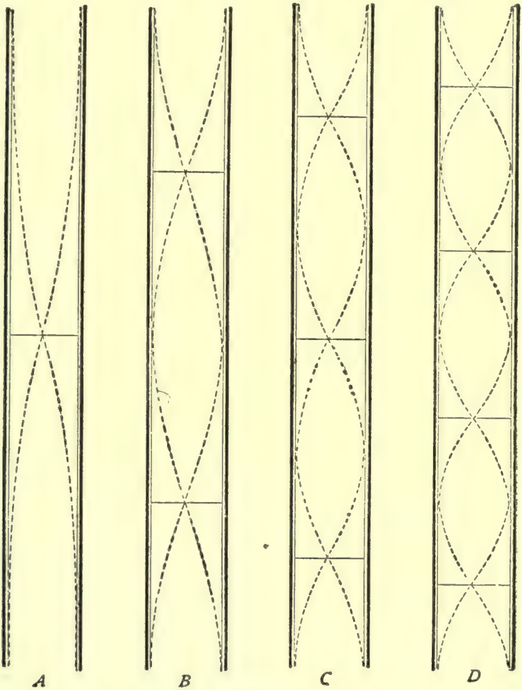


FIG. 54.

It is easy to show experimentally, that an open pipe which is giving forth its fundamental, has a node or place of least vibration in the centre, and two places of maximum vibration, one at each end. Let such a pipe be taken, the front of which must be of glass. Make a little tambourine, by stretching a piece of thin membrane over a little hoop. Place a few grains of sand on the membrane, which by means of a cord must then be gently lowered in a horizontal position into the sounding pipe. On entering it, the sand is at first violently agitated, but as the little tambourine descends, it becomes less and less disturbed, till at the centre, the sand remains quiet; on lowering it still more, the sand again begins to dance, becoming increasingly agitated as the bottom is approached.

Again, from what has been said above, it will be seen, that at the centre, where the node occurs, the air is alternately compressed and rarefied; compressed, when two condensations meet, and rarefied, when two rarefactions meet. This can also be experimentally verified; for if the pipe were pierced at the centre and the hole covered air-tight by a piece of sheet india-rubber, this latter being acted upon by the condensations and rarefactions, would be alternately pressed outwards and inwards. The organ pipe (fig. 55)

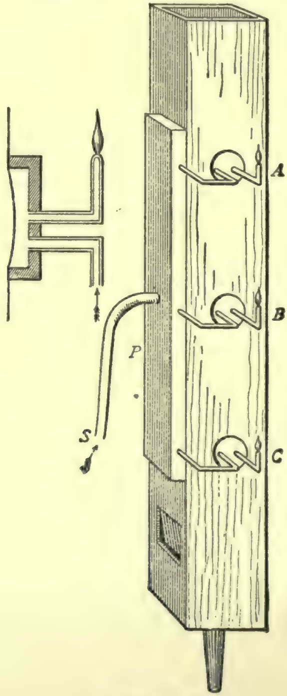


Fig. 55.

has been thus pierced at the centre B, and also at A and C, and the membranes covered by three little capsules (a section of each of which on an enlarged scale is shown at the left of the figure), from the cavities of which proceed three little gas jets, the gas being supplied by the three bent tubes which come from the hollow chamber P, which again is supplied by the tube S. Now on blow ing very gently into the pipe, so as to produce its fundamental, all three flames are agitated, but the central one most so. Turning down the gas till the flames are very small, and blowing again, the middle one will be extinguished, while the others remain alight.

Inasmuch as the two ends of an open pipe must, as we have shown, correspond to the middle of ventral segments, the next simplest way in which such a pipe can vibrate, is, with two nodes, as shown in fig. 54 B. In A there are two half segments, which are equivalent to one; in B there are two half segments and one whole one, equivalent to two segments; the rate of vibration in B will therefore be twice as rapid as in A. Accordingly, we find that the next highest tone to the fundamental, which can be produced from an open pipe, is its octave. The occurrence of the two nodes in B can be experimentally proved by the pipe of fig. 55, for if this pipe be blown more sharply, so as to produce the octave of the fundamental, the two flames A and C will be extinguished, while B will remain alight. C and D, fig. 54, represent the next simplest forms of vibration with three and four nodes respectively. The rate of vibration in (C) and (D) will obviously be three and four times respectively that in (A). Proceeding in this way, it will be found that the rates of all the possible modes of segmental vibration in an open pipe, will be as 1, 2, 3, 4, 5, &c., and this result, thus theoretically arrived at, is confirmed by practice; for we have seen that, calling the fundamental tone ($\mathbf{d}_{|}$); the harmonics produced from such a pipe are, **d**, **s**, $\mathbf{d}^{|}$, $\mathbf{m}^{|}$, &c., the vibration numbers of which are as 2, 3, 4, 5, &c.

We have hitherto supposed that each of these notes successively appears alone, but this is rarely the case, usually the fundamental is accompanied by one or more of these tones. When they are thus simultaneously produced, it is convenient to term them overtones, or, together with the fundamental, partials, as in the case of stretched strings. In order to explain the simultaneous production of these partials, we simply have to suppose the simultaneous occurrence of the segmental forms represented in fig. 54. We thus see that the notes obtainable simultaneously from an open pipe, are the *complete* series of partial tones, whose rates of vibration are as the numbers 1, 2, 3, 4, 5, &c.

Coming now to stopped pipes we have seen in Chap. VII, page 61, that a pulse of condensation entering a stopped pipe, travels to the closed end, and is there reflected back unchanged. On arriving at the open end, it is reflected back as a pulse of rarefaction, which on reaching the stopped end, is reflected unaltered. Now the closed end of a stopped pipe must always be a node, since no longitudinal vibrations of the air particles can occur there; and as we have seen above, the open end must be the middle of a ventral segment, therefore the simplest form in which the air column in a stopped pipe can vibrate, is that represented in (A), fig. 56. This form of

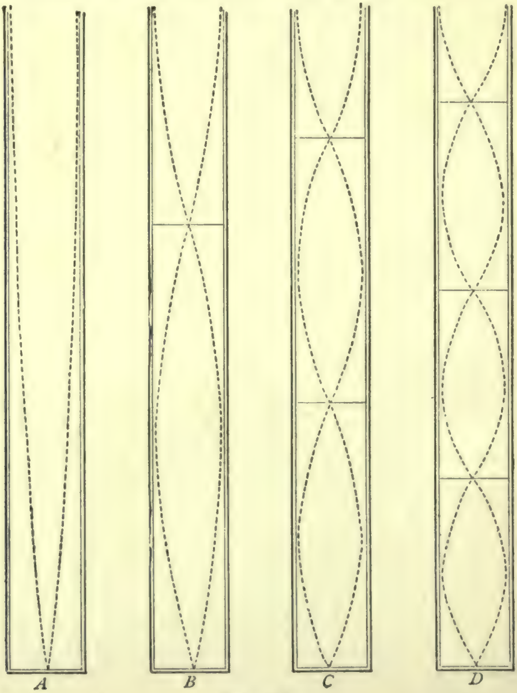


Fig. 56.

vibration must therefore produce the fundamental tone of the pipe. Comparing it with the simplest form in which the air in an open pipe can vibrate, A, fig. 54, it will be seen that the open pipe has two half segments, while the stopped has only one; consequently, if the pipes be of equal length as represented, the vibrating segment of the latter is twice as long as the former. Hence, as we have already seen, the fundamental tone of a stopped pipe, is an octave lower than that of an open one of the same length.

The next simplest way in which the air in a stopped pipe can vibrate, must be that in which two nodes are formed, and these must necessarily occur as shown in B, fig. 56, where the end of the pipe, as we have seen, forms one node, the place of the other being represented by the vertical line. In order to understand the formation of a node at this point, let fig. 57 represent a stopped pipe, and let *ab*, *bc*, *cd*, be each one-third of its length. Further, let it be supposed, that the pulses of condensation and rarefaction successively enter the open end, at intervals of time, each equal to that required for the pulse to travel from (*a*) to (*c*), that is, through two-thirds of the length of the tube. For the sake of simplicity, we will suppose, that the interval of time is one second, although of course it is really but a minute fraction of that period. First let a pulse of condensation C_1 enter the pipe. After the lapse of a second, that is at the beginning of the 2nd second, C_1 will be at (*c*) and the succeeding pulse of rarefaction R_1 will be just entering at (*a*). Neglecting R_1 for the present, let us see where C_1 will be at the beginning of the 3rd second; it will evidently have travelled through (*cd*) and back, and in fact will be at (*c*) again, but moving upwards. But by the supposition, another pulse of condensation C_2 is now entering the tube at (*a*) and therefore moving downwards. These two equal pulses of condensation will meet at (*b*) and the air particles here being solicited by C_1 to move upwards, and by C_2 to move downwards, will remain at rest. To return now to the pulse of rarefaction R_1, which at the beginning of the 2nd second was entering the tube at (*a*): at the beginning of the 3rd second it will be at (*c*), moving downwards: at the beginning of the 4th second it will be again at (*c*), but moving upwards. But, by the supposition, another pulse of rarefaction R_2 is now entering at (*a*). These two equal pulses will meet at (*b*), and the air particles there being solicited by R_1 to move downwards, and by R_2 to move upwards, will remain at rest. Thus the particles at (*b*) will be permanently at rest, that is, (*b*) will be a node. It will be noted, that it is perfectly allowable to consider, as we have done, the pulses of condensation and rarefaction separately; for we have already seen that two series of waves can cross, without permanently interfering with one another. It will be instructive, however, to consider the combined effects of the pulses of condensation C_1 and rarefaction R_1, at the end of the 2nd second, or what is the same thing, at the

FIG. 57

commencement of the 3rd. At this instant, as the student will perceive, both C_1 and R_1 will be at (*c*), C_1 moving up, and R_1 down. Now the effect of C_1 moving upwards is to swing the particles of air at this point upwards also, with a certain amount of force; and the effect of R_1 moving downwards, is to swing the same particles upwards also, with the same amount of force; C_1 and R_1 therefore, combine their forces to swing the air particles at (*c*) upwards. At the expiration of another second R_1 will be back again at (*c*) but moving now upwards, and C_2 will also be at the same point moving downwards. The air particles at (*c*) will now be swinging downwards, with the combined forces of R_1 and C_2. Thus it will be seen that (*c*) is a point of maximum vibration, that is, the middle of a ventral segment.

The half segment in (B) fig. 56 is seen to be one-third as long as the half segment in (A); therefore the length of the sound wave emitted by (B) must be one-third the length of that emitted by (A); that is, the note corresponding to the vibrational form (B), has three times the vibration number of that corresponding to (A).

The next simplest way in which the air column in a stopped pipe can vibrate is with three nodes, as represented in (C), fig. 56; the next simplest, with four nodes (D), the next with five, and so on. As the length of the half segment in (C) is one-fifth the length of that in (A), the wave length of the note corresponding to the vibrational form (C), must be one-fifth of that corresponding to (A), that is, its vibrational number is five times as great. Similarly in D, it is seven times as great. Summing up, then, we find theoretically that the vibration rates of the tones, which can be produced from a stopped tube, are as the odd numbers 1, 3, 5, 7, &c., no tone intermediate in pitch between these, being possible. This can be easily verified experimentally, by the aid of an ordinary stopped organ pipe. On blowing very gently, the fundamental, which we may call (**d**$_1$), is heard; on gradually increasing the force of the blast, a point is reached at which this fundamental ceases, and the (**s**) an octave and a fifth above, springs forth; still further increase the wind pressure, and this gives place to the (**m**^{1}) two octaves and a major 3rd above the fundamental. By no variation in the blowing can any tone intermediate in pitch between these be obtained; and the vibration numbers of these three notes (**d**$_1$), (**s**), and (**m**^{1}), are as 1, 3, and 5, and thus the results obtained above are corroborated experimentally.

As before observed, sounds thus successively obtained from a pipe, by variation in the wind pressure, may be conveniently termed harmonics; the terms "partials" and "overtones" being used when they are simultaneously produced. For example, if the air in a stopped pipe were simultaneously vibrating in the forms (A), (B), and (C), fig. 56, we should obtain from it a compound tone consisting of the first three odd partials, that is, the 1st, 3rd, and 5th.

With regard to the open organ pipe, the fundamental is never produced alone; according to the dimensions and shape of the pipe, it is accompanied by, from two to five, or more overtones. As a rule, the overtones are more prominent in narrow than in wide pipes, and in conical, than in cylindrical ones. The shape of the pipe has a great influence on the production of partials. The conically narrowed pipes found in some organ stops, which have their upper opening about half the diameter of the lower, have the 4th, 5th, and 6th overtones proportionally more distinct than their lower ones. Stopped wooden pipes of large diameter, when softly blown, produce sounds which are nearly simple. Such tones are sweet and gentle, but tame and monotonous. A greater pressure of wind, or a reduction in the diameter of the pipe, developes the 3rd and 5th partials.

The great body of tone in the organ is produced by wide open pipes, forming the "principal stops." The tones they produce, owing to the deficiency of upper partial tones, are somewhat dull; they lack character, richness, and brilliancy. Long before Helmholtz had shown that richness of tone is due to the occurrence of well-developed upper partial tones, organ builders had learnt how to supply such tones artificially, by means of smaller pipes, tuned to the pitch of these partials, forming what are termed mixture stops. As an example of such mixture stops, the "sesquialtera" may be mentioned, which originally consisted of three pipes to each digital, the smaller two producing tones, a twelfth and a seventeenth, above the fundamental of the larger one, thus reinforcing the 3rd and 5th partials. The sesquialtera is now often made with from three to six ranks of open metal pipes. The smaller ranks are usually discontinued above middle C as they become too shrill and prominent, larger pipes sounding an octave lower, being sometimes substituted.

In the flute, the tone is produced, as in the organ pipe, by directing a current of air against a thin edge, the edge in this case being the side of a lateral aperture near the end of the tube. In the older form, that of the flageolet, there is an arrangement very similar to that of the ordinary organ pipe, and the air is simply blown in. Variations in pitch are effected, in the first place, by opening apertures in the side, and thus practically altering the length of the pipe; and secondly, by so increasing the wind pressure, as to bring out the first harmonic to the exclusion of the fundamental, all the tones thus springing up an octave. The quality of its tone is sweet but dull, owing to the want of upper partials. When very softly blown, it gives tones that are all but simple.

Reed Instruments.

Two kinds of reed are used in musical instruments, the free reed and the beating reed. Fig. 58 shows the construction of the former.

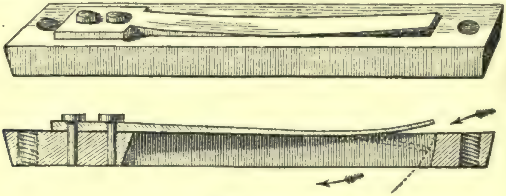


Fig. 58.

It consists of a thin narrow strip of metal called a "tongue" fastened by one end to a brass plate, the rest of the tongue being free. Immediately below the tongue, there is an aperture in the brass plate, of the same shape, and very slightly larger than the tongue itself. Thus the tongue forms the door of the aperture, capable of swinging backwards and forwards in it. If a current of air be driven upon the free end of the tongue, the latter is set vibrating to and fro in the aperture between its limiting positions A and B, fig. 59. When in the position A, the current of wind

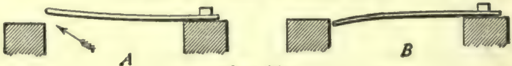


Fig. 59.

passes through, but when the tongue reaches the position B, the current is suddenly shut off; only when the tongue resumes the position A, can the air again pass. As the vibrations of the tongue are periodic, a regular succession of air pulses are thus produced, giving rise to a musical sound, precisely as in the case of the Syren.

The action of the beating reed is similar to that of the free reed; in fact, the beating reed only differs from the free reed, in having its tongue slightly larger than the aperture, so that it beats against the plate, in closing the aperture, instead of passing into it.

The reed is used in its simplest form in the harmonium, American organ, and concertina. In the harmonium and concertina, the current of air is forced through the reeds by means of a bellows. In the American organ, the bellows so act, as to form a partial vacuum below the reeds, the external air being thus drawn through them.

The pitch of the sounds, obtained from such reeds evidently depends upon the vibration rate of the reed itself. This again depends upon the size and thickness of the reed, and the elasticity of the material of which it is composed. Harmonium reeds are usually sharpened by gently filing or scraping the free end, and flattened by applying the same operation to the part of the tongue near the fixed end. A rise of temperature, diminishes the rate of vibration, as the tongue expands and its elasticity is diminished. The pitch is also somewhat affected by the force of the wind.

The tones obtained from reeds such as the above, are very rich in overtones. All the series of partials up to the sixteenth, or even higher, may be distinctly recognised in any of the lower notes of the harmonium; in fact, the undue prominence of the higher partials is one of the drawbacks of this instrument.

In order to understand this wealth of partials in reed tones, we must turn back to Chap. VIII. We saw there, that Fourier has proved mathematically, that every form of wave may be analysed into a number of simple waves, whose lengths are inversely as the numbers 1, 2, 3, 4, 5, &c. Now it is plain, that the more abrupt or discontinuous the compound wave, the greater will be the number of its constituent simple waves. The compound sound wave resulting from the vibration of a reed, is highly discontinuous; since the individual pulses must be separated by complete pauses during the closing of the apertures. Hence the number of its constituent simple waves will be correspondingly large, that is, the compound tone produced by a vibrating reed is made up of a very large number of partial tones. The harder and more unyielding the tongue, and the more perfectly it fits its aperture, the more discontinuous will be the pulses, and consequently the more intense and numerous the overtones.

The compound tone of the reed, being thus overburdened by the intensities of its upper partials, it becomes an advantage to soften these latter, or what comes to the same thing, to strengthen the fundamental, without at the same time strengthening the overtones. This can be done, by placing the reed at the mouth of an open pipe, the fundamental tone of which is of the same pitch as the fundamental tone of the reed. This latter tone will then be greatly reinforced by the resonance of the pipe. The other partial tones of the reed will also be strengthened, but to a much less extent; for the force necessary to produce segmental vibration, increases rapidly as the number of segments increases. The higher partials of the reed are therefore practically unsupported by the associated pipe. It is evident that the pipe associated with a reed, may be selected to resound to one of the overtones of the reed, instead of o the fundamental, the resulting tone being in this case of quite different quality to the above. The form of the pipe may also vary, producing other changes in the quality of tone produced. It is thus that the varieties of reed pipes in the organ are obtained.

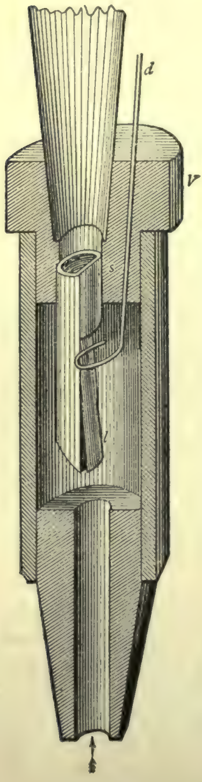


Fig. 60.

Fig. 60 shows how the reed is inserted in the organ pipe. *V* is the socket in which the lower end of the pipe is fixed, *l* is the beating reed, which is tuned by increasing or diminishing its effective length, by means of the movable wire *d*, sliding in the block, *s*.

The reed instruments in use in the orchestra, may be classified into the wood wind instruments, which have wooden reeds, and the brass wind instruments, which have cupped mouth-pieces. The chief instruments of the former class are the Clarinet, the Hautbois or Oboe, and the Bassoon. In these instruments the proper tones of the reeds themselves are not used at all, being too high and of a shrill or screaming quality; the tones employed are those depending on the length of the column of air in the tube, as determined by the opening or closing of the apertures. The vibration of the air column thus controls the yielding reed, which is compelled to vibrate in sympathy with it.

The Clarionet or Clarinet has a cylindrical tube terminating at one end in a bell. At the other end is the mouth piece, which is of a conical shape, and flattened at one side so as to form a kind of table for the reed, the opposite side being thinned to a chisel edge. The bore of the instrument passes through the table just mentioned, which moreover is not quite flattened, but slightly curved away from the reed, so as to leave a thin gap between the end of the reed and the mouth piece. The Clarinet has thus only a single reed, and that a beating one.

The tube is pierced with eighteen holes, half of which are closed by the fingers, and half by keys. The lowest note is produced by closing all the apertures and blowing gently. By opening successively the eighteen apertures, eighteen other notes may be obtained at intervals of a semitone ; and thus the lower scale, of one-and-a-half octaves, is obtained. By increase of wind pressure, or by opening an aperture at the back of the tube, the pitch of the tube is raised a twelfth; in fact the instrument acts like a stopped tube, increased wind pressure bringing out, not the second, but the third of the ordinary harmonic series.

The quality of tone on the Clarinet is very characteristic, and is due to the fact that only the odd partials, 1, 3, 5, 7, &c., are present in its tones; just as in the case of stopped organ pipes. In fact the Clarinet must be considered as such a pipe, stopped at the end where the reed is placed; for it is here that the greatest alternations of pressure occur; that is, as we have seen above, this point must be a node.

The Oboe and Bassoon have conical tubes expanding into bells. The reed in each is double and formed of two thin broad spatula-shaped plates of cane in close approximation to one another. Variations in pitch are obtained as in the flute, by varying the effective length of the tube, by means of apertures closed by the fingers or keys. Like the flute also, the first harmonic is the octave, so that increase of wind pressure raises the pitch by that interval. The partials present in the tones of these instruments, are those of the complete series, 1, 2, 3, 4, 5, &c.

In instruments with cupped mouth pieces, the lips of the player, which form the reed, are capable of vibrating at very different rates, according to their tension, form, &c. A very simple type of this class of instrument may be obtained, by placing a common glass funnel into one end of a piece of glass tubing, a few feet long, and half an inch or so in diameter. The tones, which can be obtained from such a tube, by varying the tension and form of the lips and the force of the wind, are those of the complete partial series; the lowest ones are, however, very difficult to obtain. Thus no note can be produced on such an instrument, but such as belong to the series of partials, or harmonic scale, as it is sometimes termed.

All instruments with cupped mouth-pieces are constructed on the same principle as this primitive instrument; that is, they are tubes without lateral apertures, the notes producible upon them being the harmonics of the tube. Now, as will be seen from the table on page 72, there are various gaps in this harmonic scale, as compared with the diatonic and chromatic scale, and accordingly it will be found that the most important departures of brass instruments from the rude type selected above, have been made for the purpose of supplying these missing notes.

The chief instruments of this class are the French Horn, Trumpet, and Trombone. The French Horn consists of a conical twisted tube of great length, expanding at the larger end into a bell. The fundamental, which is a very deep tone, is not used. As will be seen on reference to the table on page 72, the higher harmonics (for example, those from the seventh upwards) form an almost unbroken scale. To supply the missing notes, the hand is thrust into the bell to a greater or less extent, thus lowering the pitch of the note which is being produced at the time. These instruments are also frequently supplied with keys, which vary the effective length of the tube, and thus produce the missing tones, but at some expense of the quality of tone.

The Trumpet supplies the notes which are wanting to complete its scale in a much more effective manner. An U shaped portion of the tube is made to slide with gentle friction, upon the body of the instrument, so that the tube can thus be lengthened or shortened, within certain limits by the player.

The Trombone is simply a bass trumpet, and in principle is the same as the above. In these brass instruments, the tension, &c., of the lips only determines which of the proper tones shall speak, the actual pitch of the tone being almost entirely independent of the tension itself

The vocal organ, or larynx, is essentially a reed instrument. The reed itself is a double one, and consists of two elastic bands, called the Vocal Chords, or Ligaments, which stretch from front to back across the larynx. When they are not in action, these ligaments are separated by a considerable aperture. By means of muscles inserted in the cartilages to which the vocal ligaments are attached, these latter can be brought close together with their edges parallel. The air from the lungs acting upon them, while in this position, sets them in vibration, in the same way as the air from a bellows operates upon a reed. Variations in pitch are effected by varying the tension of the vocal ligaments. This is effected by the contraction of certain muscles, which act on the cartilages to which the ligaments are attached in such a way as to stretch these latter to a greater or less extent. The density of the vocal ligaments also seems to be variable. According to Helmholtz "much soft watery inelastic tissue lies underneath the proper elastic fibres and muscular fibres of the vocal chords, and in the breast voice this probably acts to weight them and retard their vibrations. The head voice is probably produced by drawing aside the mucous coat below the chords, thus rendering the edge of the chords sharper and the weight of the vibrating part less, while the elasticity is unaltered."

As in other reed instruments, the tones of the human voice are very rich in overtones. In a sonorous bass voice, it is easy to detect the first seven or eight, and by the aid of resonators even more. When a body of voices are heard together, close at hand, singing *forte*, the shrill overtones are only too prominent. These overtones are very largely modified in intensity by the size and shape of the nasal cavity and the pharynx, also by the varying size and shape of the mouth and position of the tongue. Hence, when the vocal ligaments have originated a compound tone rich in partials, the varying features just mentioned, may reinforce now one set of partials, and now another, in very many different ways, thus producing the endless variety of qualities found in the human voice.

The following is a very instructive experiment in connection with this subject, showing how the reinforcement of particular partials by the resonance of the mouth cavity modifies the quality. Strike an ordinary C tuning-fork, and hold the ends of the vibrating prongs close to the open mouth, keeping the latter in the position required for singing "ah." Notice how the quality of the fork is affected. Now do the same again, but put the mouth in the position required for sounding "oo." Observe the change of quality. A looking glass will be required in order to see that the fork is in the right position. Once more repeat the experiment; but this time, while the vibrating fork is held in position, move the mouth from the position "ah" to the "oo" position, at first gradually, and then rapidly; the corresponding change in the quality of the tone of the fork is very striking, For a detailed account of the Larynx and of Voice production, the student is referred to Behnke's "Mechanism of the Human Voice."

SUMMARY.

To find approximately the vibration number of any given flue-pipe; divide the velocity of sound by twice the length of the pipe for open, and by four times its length for stopped pipes.

The pitch of an open pipe is not *exactly* the same as that of a stopped pipe of half its length.

The pitch of a flue pipe is *sharpened* by a *rise* of temperature.

Wooden pipes sharpen more than metal ones for the same increase of temperature.

Nodes are produced in flue pipes by the meeting of two rarefactions or of two condensations travelling in opposite directions: consequently it is at the nodes that the greatest variations in density occur.

The open end of a pipe is always an *antinode*.

,, closed ,, ,, ,, ,, a *node*.

When the air column in an *open* pipe vibrates with one node only, that is as a whole, the fundamental (say $\mathbf{d}_1$) is produced; when with two nodes only, that is in two halves, the 1st Harmonic ($\mathbf{d}$); when with three nodes only, the 2nd Harmonic ($\mathbf{s}$); and so on.

When the air column in a *stopped* pipe vibrates with one node only, that is as a whole, it gives the fundamental (say $\mathbf{d}_1$); when with two nodes, that is in three thirds, its 1st Harmonic ($\mathbf{s}$); when with three nodes, its 2nd Harmonic ($\mathbf{m}^1$); and so on.

The tones produced by flue pipes are compound, because of the fact, that the air column vibrates simultaneously, as a whole and in aliquot parts, each part producing an overtone of a pitch corresponding to its length.

Stopped pipes only give the partials of the *odd* series, 1, 3, 5, &c.

The flute is a flue pipe of variable length.

The pitch of a reed is *lowered* by a *rise* of temperature.

The sounds produced by reeds are rich in overtones.

The fundamental tone of a reed-clang may be strengthened relatively to its overtones, by placing over the reed, a pipe which is in unison with that fundamental.

The *Clarinet*, *Oboe*, and *Bassoon* are stopped pipes, in which the pipe governs the reed, that is to say, the tones produced depend on the varying length of the pipe, and not upon the reed. The

Clarinet is a *cylindrical stopped* pipe, and therefore its tones consist of only the *odd* partials.

The *Oboe* and *Bassoon* are *conical stopped* pipes, and their partials follow the ordinary series.

In instruments, with cupped mouth pieces, such as the French Horn, Trumpet, &c., the *lips* of the player form the reed. Changes of pitch were originally brought about, by successively developing their different harmonics. Their partials belong to the complete series.

The Human Voice is essentially a reed instrument (the *Vocal Ligaments* or *Chords*) with a resonator (the *Mouth*, *Pharynx*, and *Nasal Cavities*) attached. The Vocal Ligaments originate a compound tone, rich in partials, the relative intensities of which are profoundly modified by the ever varying resonator, thus producing the almost infinite variety in quality of tone, which is characteristic of the Human Voice.

CHAPTER XI.

On the Vibrations of Rods, Plates, &c

As the musical instruments treated of in the present chapter are of comparatively less importance than those already studied, the principles which they involve will be more briefly touched on. We shall first consider the

Vibrations of Rods or Bars.

A Rod is capable of vibrating in three ways (the last however being of little importance, musically speaking), viz.

1. Longitudinally.
2. Laterally.
3. Torsionally.

1. Longitudinal vibrations again may be classified according as the rod is

(*a*) Fixed at both ends.
(*b*) Fixed at one end only.
(*c*) Free at both ends.

(*a*) The Longitudinal vibrations of a rod or wire, fixed at both ends, may be studied on the monochord, by passing briskly along the wire, a cloth, which has been dusted with powdered resin. The sound produced is much higher in pitch than that obtained by causing the wire to vibrate transversely. On stopping the wire at the centre, and rubbing one of the halves, the upper octave of the sound first heard, is emitted. When the wire is stopped at one third its length, and this third excited, the fifth above the last is heard; and so on. Thus, as in the case of transverse vibrations, the vibration number varies inversely as the length of the wire. On altering the tension, the pitch will not be found to have varied; that is, the pitch is independent of the tension.

The longitudinal vibrations of a wire fixed at both ends, somewhat resemble those that take place in an open organ pipe. In both cases, the time of a complete vibration is the time taken by a pulse to move through the length of the wire or pipe, and back again. In the case of the latter, we have seen, that the vibration number of the note produced by any given pipe, may be ascertained by dividing the velocity of sound in air, by twice the length of the pipe. Conversely, if we know the vibration number of the pipe, we can ascertain the velocity of sound in air, by multiplying this number by twice the length of the pipe. This principle may be employed to determine the velocity of sound in other gases. Thus, fill and blow the pipe with hydrogen, instead of air, and ascertain the pitch of the note produced: its vibration number multiplied by twice the length of the pipe, will give the velocity of sound in hydrogen. Or we may proceed thus; blow one pipe with air, and another with hydrogen, the latter pipe being furnished with telescopic sliders, so that its length can be altered at pleasure. Now while both pipes are sounding, gradually lengthen the pipe, till both are in unison. When this is the case, let l denote the length of the air sounding pipe, and l_1 the length of the other: then if V be the velocity of sound in air, and V_1 its velocity in hydrogen, it is evident that—

$$\frac{V_1}{V} = \frac{l_1}{l}$$

from which V_1 may be readily calculated. In this way, the velocity of sound in the various gases has been ascertained.

Now from what has been said above, it will be seen, that this same method may be applied, in order to ascertain the velocity of sound in solids. For example, suppose we wish to ascertain the velocity of sound in iron. Stretch some twenty feet of stout iron wire between two fixed points, one of which is movable: a vice, the jaws of which are lined with lead answers very well. Rub the wire with a resined piece of leather, and gradually shorten it till the sound produced is in unison with a C tuning-fork, the vibration number of which is, say, 512. When the unison point is reached, measure the length of wire: say it is $16\frac{1}{2}$ feet. Then the time of a complete vibration is the time required for the pulse to run through $2 \times 16\frac{1}{2} = 33$ feet of the wire. But there are 512 vibrations or pulses per second: therefore sound travels along the iron wire at the rate of $512 \times 33 = 16{,}896$ feet in a second. Generally, let l denote the length, in feet, of a wire or rod fixed at both ends, and n the vibration number of the note it emits when vibrating longitudinally; then, if V denote the velocity of sound in the substance of which the rod or wire is composed,

$$V = 2\,l\,n.$$

The overtones of a wire fixed at both ends follow the ordinary series, 1, 2, 3, 4, 5, &c., the wire vibrating in two segments, with a node in the centre to produce the first overtone, and so on.

(*b*) The longitudinal vibrations of rods fixed at one end, present considerable analogy with those in stopped organ pipes. Thus the vibration number varies inversely as the length of the rod, as may be easily shown by fixing varying lengths of rod in a vice, and exciting them with a resined cloth. Again, the time required for a complete vibration, is the time during which a pulse makes two complete journeys up and down the rod. Thus, these vibrations may be used to ascertain the velocity of sound in any substance, the method of proceeding being similar to that explained above, but the formula will be

$$V = 4\,l\,n.$$

The partials obtainable from these rods, are, like those of a stopped organ pipe, the odd partials of the complete series, 1, 3, 5, 7, &c.; the first overtone requiring a node, at a point one-third the length of the rod from the free end; the second at one-fifth of the length, and so on.

The only musical instrument in which this kind of rod vibration is utilized is Marloye's harp. It consists of a series of wooden rods of varying lengths, vertically fixed on a sound-board below. The rods are excited, by rubbing up and down with the resined fingers.

(*c*) In rods or tubes free at both ends, the simplest longitudinal vibrations are set up, when the tube is clasped or clamped at the centre, and excited by longitudinally rubbing either half: the simplest form of vibration is therefore, with one node in the centre. Rods so treated are analogous with open organ pipes. For example, the vibration number varies inversely as the length of the rod; and the time of a complete vibration is the same as that required for a pulse to run to and fro over the rod; so that here again the velocity of sound in the substance of which the rod is composed, may be ascertained by multiplying the vibration number of the note produced, by twice the length of the rod.

As just stated, the simplest form in which these rods can vibrate, is with one node in the centre; the next simplest, as in the case of the open organ pipe, is with two nodes; the next, with three, and so on; the partials produced being those of the complete series, 1, 2, 3, 4, 5, &c.

2. Coming now to the lateral vibrations of rods, we find these may also be classified according as the rods are,—

(*a*) Fixed at both ends,
(*b*) Fixed at one end only,
(*c*) Free at both ends.

(*a*) A rod fixed at both ends, vibrates laterally in exactly the same manner as a string; that is, it may vibrate as a whole, forming one ventral segment; or with a node in the centre, and two ventral segments; or, with two nodes, and three ventral segments, &c. The relative vibration rates are, however, very different, as may be seen from the following table:—

segments	1,		2,		3,		4.
vibration rates	9	:	25	:	49	:	81
	or 3^2	:	5^2	:	7^2	:	9^2.

(*b*) In laterally vibrating rods fixed at one end, the vibration number varies inversely as the square of the length. Chladni endeavoured to utilize this fact, in ascertaining the vibration number of a musical sound. He first obtained a strip of metal of such a length, that its vibrations were slow enough to be counted. Suppose, for example, a strip is taken, 36 inches long, and that it vibrates once in a second. Reducing it to one-third that length, according to the above law, it will vibrate nine times per second. Reducing it to six inches, it will make 36 vibrations per second; to three inches, 144 vibrations; to one inch, 1,296 vibrations. By a little calculation, it is easy to find the vibration number of any intermediate length.

The relative rates of vibration of the partial tones of such rods, are very complex; the second partial is more than two octaves above the fundamental, and the others are correspondingly distant from one another. Examples of instruments, in which these lateral vibrations of rods fixed at one end are utilized, may be fonnd in the musical box and the bell piano.

(*c*) The simplest mode in which a rod free at both ends can vibrate laterally, may be experimentally observed by grasping a lath some six feet long, with both hands, at about one foot from either end, and striking or shaking it in the centre. It will be found that there are two nodes, as shown in fig. 61, A. The next

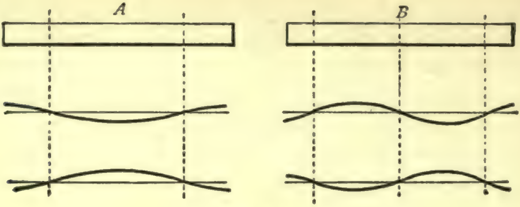


FIG. 61

simplest is with three nodes, fig. 61, B. The tones, corresponding to these divisions, rise very rapidly in pitch, thus:

Number of nodes	2,	3,	4,	5,	6,	7.
Approximate vib. rates	9 :	25 :	49 :	81 :	121 :	169.
or	3^2 :	5^2 :	7^2 :	9^2 :	11^2 :	13^2.

The Harmonicon is an example of an instrument, in which the lateral vibrations of rods free at both ends are utilized; but the most important member of this class is the Tuning-fork. This instrument, generally constructed of steel, may be considered as derived from a straight bar, such as that depicted at the lower part of fig. 62, by folding it in two, at the middle. The tone of the bent

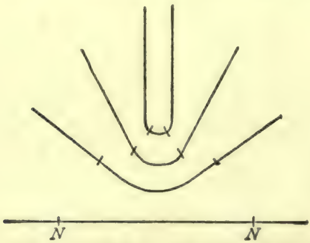


FIG. 62.

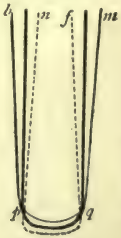


FIG. 63.

bar will be somewhat flatter than the original straight one, and the nodes, which in the straight bar were near the two ends, will have approached very close together in the bent one. Fig. 62 shows by the short marks, this gradual approach of the nodes, as the bar is more and more bent; and fig. 63, by its thin and dotted lines, represents the two extreme positions of the fork, while sounding its fundamental. When the prongs are at their extreme outward position *b m*, the portion between the nodes *p* and *q* rises; when they are closest together, at *n f*, this same portion descends. Thus while the prongs move horizontally the portion between p and q vibrates vertically. To this portion there is usually welded or screwed, an elongated piece of steel, which shares this vertical motion, and does duty as a handle. When this handle is placed upon a sound-board, its vertical vibrations are communicated to it; a larger body of air is set in motion, and thus the sound of the fork augmented. A tuning-fork does not divide like a straight bar into four vibrating segments with three nodes; its second complete form of vibration, which corresponds to the first overtone, is with four nodes, two at the bottom and one on each prong. In some forks, examined by Helmholtz, the relative rates of the fundamental and first overtone, varied from 1 : 5·8 to 1 : 6·6. The overtones of tuning-forks are consequently very distant from the fundamental and from one another; the first overtone, as we see from the above, being more than two octaves above the fundamental. The rates of vibration of the whole series of overtones, starting with the first overtone, are approximately as 9, 25, 49, 81, &c., that is, as the squares of the odd numbers, 3, 5, 7, 9, &c.

These high overtones are very evanescent, and soon leave the fundamental tone pure and simple. This is especially the case, as already observed, when the fork is mounted on a resonance chamber, tuned to its fundamental. The fork should either be struck with a soft hammer, or carefully bowed. Striking with a hard metallic substance, favours the production of the higher partials, for the reason given in the case of pianoforte strings. Large forks, when too rapidly bowed, produce very powerful overtones. The best method of keeping tuning-forks in continuous vibration, is by means of electro-magnets, as already described in Chap. VIII.

The pitch of a tuning-fork is only very slightly affected by heat. The effect of increase of temperature on a fork, is to slightly flatten it; for the fork itself expands, and its modulus of elasticity is lowered on heating; both of these causes combining to lower the pitch. The variation with temperature is only about one vibration in 21,000, for each degree Fahrenheit. Forks are also little affected by ill usage. A slight amount of rust is imperceptible in its influence on pitch; and with a very large amount, such as could only occur through great carelessness, the error is never likely to exceed 1 in 250. Rust about the bend has a much greater influence over the pitch, than at the ends. Tuning-forks are perhaps most injured, by wrenching or twisting of the prongs, such as might occur through a fall, or by screwing or unscrewing them in and out of resonance boxes.

VIBRATIONS OF PLATES.

Though not of much importance in reference to music, these vibrations are of much interest, on account of the beautiful method by which their forms are analysed. The plates usually employed are constructed of either metal or glass, the metal being usually brass. Any regular shape may be adopted, the most common being the circular and square forms. The plate is firmly fastened at the centre or some other point, to a stand; and the vibrations are best set up, by bowing the edge of the plate with a double-bass bow. The rate of vibration of a circular plate is directly proportional to the thickness, and inversely proportional to the square of the diameter.

A node can be formed at any desired point, by touching that point firmly, while bowing. By thus successively touching various parts of the plate, a variety of notes of different pitches, corresponding to its overtones, may be obtained, the plate vibrating

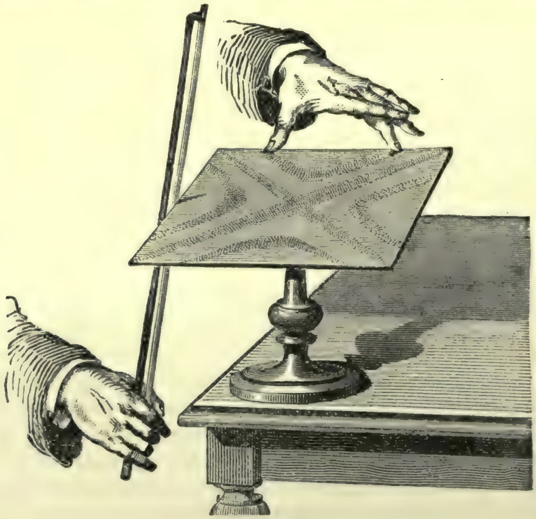
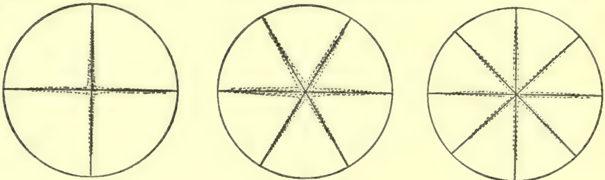


FIG. 64.

differently for each note. About 100 years ago, Chladni discovered the method of rendering these different vibration forms visible, by strewing sand lightly and evenly over the plate before bowing. When a plate, thus treated, vibrates, the sand being violently agitated over the vibrating segments, is rapidly jerked away from these parts, and arranges itself along the nodal lines (fig. 64).

The simplest way in which a circular plate can vibrate, is in four segments (fig. 65, A); the next simplest in six segments (fig. 65, B);

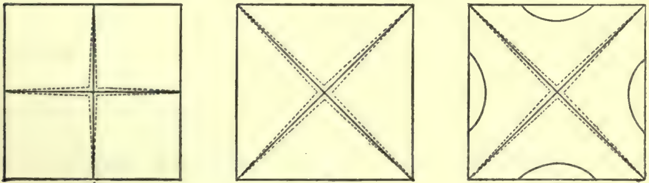


A B C

FIG. 65.

the next in eight (fig. 65, C); and so on. Much more complicated figures, with nodal circles, may be obtained by stopping the plate at appropriate points and bowing accordingly.

Figure (66, A) shows the simplest way in which a square plate can vibrate, and (fig. 66, B) gives the next simplest form; the note



A B C

FIG. 66.

produced in the latter case being the fifth above that produced in the former. The sand figures become very complicated and beautiful as the tones rise in pitch; (fig. 66, C) representing one of the least complex. Adjacent segments are always in different phases; that is, while one is *above* its ordinary position, the adjacent ones are *below* it. This can be proved experimentally, as will be subsequently shown.

BELLS.

Theoretically, a bell vibrates in the same way as a plate fixed at the centre. The simplest way in which it can vibrate is with four nodal lines, the tone thus produced being the fundamental. The next simplest form of vibration is with six nodes, the next with eight, and so on; an odd number of nodes never being produced. The corresponding vibration rates are as follows:—

Number of nodes	4, 6, 8, 10, 12,
Relative vibration rates	4 : 9 : 16 : 25 : 36,
or	$2^2 : 3^2 : 4^2 : 5^2 : 6^2$.

Practically, however, owing among other things, to unavoidable irregularities in the casting, no church bell ever has a single fundamental, or only one series of overtones. To this fact is due, the well-known difficulty in ascertaining the precise pitch of such a bell; the discords and throbbings that are heard, even in the best sounding bells, when the listener is close to them, may be put down to the same cause. There are no absolute points of rest in a vibrating bell, for the fundamental is never produced alone; but it is easy to explore the surface of the sounding bell with a light ball suspended from a thread, and thus find the places of least and greatest motion, the ball being violently dashed away from the latter.

Membranes.

These, in the form of side drums, bass drums, and kettle drums, are used in the orchestra rather to mark the rhythm, than to produce a musical sound; although it is true, the last mentioned are approximately tuned to two or three notes of the 16 foot octave.

They may be studied by exciting them sympathetically, by means of organ pipes, and analysing the vibration forms produced, by scattering sand over them, as in the case of plates. The nodal lines are circles and diameters, or combinations of these.

Summary.

Rods vibrating *longitudinally* and (1) free or (2) fixed at *both* ends are analogous with *open* organ pipes: their vibration numbers are inversely as their lengths, and they give the complete series of partial tones.

When (3) fixed at *one* end only, they are analogous with *stopped* organ pipes and give only the odd series of partials.

In (1) and (2) the time required for a complete vibration is the same as the time taken by a pulse to move along the whole length of the rod, and back again; in (3) the time required for a complete vibration is twice this time. These facts being known, it is possible to determine the velocities of sound in various solids, just as a knowledge of the similar fact in the analogous case of organ pipes, renders it easy to ascertain the velocities of sound in different gases.

Rods vibrating *laterally* and (1) fixed at *both* ends may vibrate in 1, 2, 3, 4, 5, &c., segments. If (2) free at *both* ends they can vibrate only in 3, 4, 5, 6, &c., segments. Reckoning the two fixed ends in the former case as nodes, the vibration rates of the segments are as follows:

Number of nodes	2,	3,	4,	5,	6, &c.
Vibration rates	3^2,	5^2,	7^2,	9^2,	11^2.

When fixed at *one* end only, the vibration number varies inversely as the square of the length, and the overtones are very distant from one another and from the fundamental.

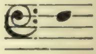
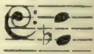
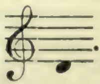
The 1st overtone of the tuning-fork is more than two octaves above the fundamental, and the 2nd overtone more than an octave above the 1st.

The pitch of the tuning-fork is very slightly affected by ordinary changes of temperature.

CHAPTER XII.

Combination Tones.

In the preceding chapters, musical sounds, whether simple or compound, have been considered singly, and the phenomena they present, so studied. When two or more such sounds are heard simultaneously, other phenomena usually occur. In the present chapter, we proceed to study one of these.

When two musical tones, either simple or compound, are sounded together, new tones are often heard, which cannot be detected when either of the two tones is sounded by itself. For example: press down the keys corresponding to the notes C^2 and $A^{|}$,  on the harmonium, and blow vigorously. On listening attentively, a tone may usually be heard, nearly coinciding with $F_{|}$,  which will not be heard at all, when either of the two notes above is separately sounded. Again, sound the two notes $B\flat_2$ and $F_{|}$  loudly on the same instrument. With attention, a tone will be heard almost exactly coinciding with D, . Raise either of the fingers, and this tone will vanish.

These tones, which make their appearance when two independent tones are simultaneously sounded, have been termed by some authors, Resultant Tones, by others Combination Tones. The independent tones, which give rise to a combination tone, may conveniently be termed its generators.

Two varieties of combination tone are met with: in the one, the vibration number of the combination tone is equal to the difference between the vibration numbers of its generators; in the other, it is equal to their sum. The former are consequently termed Difference or Differential Tones, and the latter, Summation Tones.

Differential Tones.

These tones have been known to musicians for more than a century. They appear to have been first noticed in 1740 by Sorge, a German organist. Subsequently, attention was drawn to them by Tartini, who called them "grave harmonics," and endeavoured to make them the foundation of a system of harmony.

As already stated, the vibration number of a differential tone, is the difference between the vibration numbers of its generators. It is easy therefore to calculate what differential any two given generators will produce. For example, two tones, having the vibration numbers 256 and 412 respectively are sounded simultaneously, what will be the vibration number of the differential tone produced? Evidently 412 — 256, that is 156.

Further, if the two generators form any definite musical interval, the differential tone may be easily ascertained, though their vibration numbers may be unknown. For example, what differential will be produced by two generators at the interval of an octave? Whatever the actual vibration numbers of the generators, they must be in the ratio of 2 : 1. Therefore the difference between them must be the same as the vibration number of the lower of the two generators; that is, the differential will coincide with the lower generator. Or shortly it may be put thus:—

$$\text{generators}\left\{\begin{array}{l}\mathrm{d}^{\prime} = 2\\ \mathrm{d} = 1\end{array}\right.$$

$$\text{Differential Tone, } \mathrm{d} = 1$$

Again, what differential will be produced by two tones at the interval of a Fifth? The vibration numbers of two tones at the interval of a Fifth are as 2 : 3, difference = 3 — 2 = 1. Therefore the vibration number of the differential will be to the vibration number of the lower of the two tones as 1 : 2; that is, the differential will be an octave below the lower generator, or briefly,

$$\text{generators}\left\{\begin{array}{l}\mathrm{s}^{\prime} = 3\\ \mathrm{d}^{\prime} = 2\end{array}\right.$$

$$\text{Differential Tone, } \mathrm{d} = 1$$

In the following Table, the last column shows the Differentials produced by the generators given in the second column.

INTERVAL.	GENERATORS.	RELATIVE VIB. RATES.	DIFFERENCE.	DIFFL. TONES.
Octave......	d^2 / d^1	2 / 1	1	d^1
Fifth.......	s^1 / d^1	3 / 2	1	d
Fourth......	d^1 / s	4 / 3	1	d_1
Major Third	m^1 / d^1	5 / 4	1	d_1
Minor Third	s^1 / m^1	6 / 5	1	d_1
Major Sixth	l^1 / d^1	5 / 3	2	f
Minor Sixth	d^2 / m^1	8 / 5	3	s
Tone.......	r^1 / d^1	9 / 8	1	d_2
Semitone....	d^2 / t^1	16 / 15	1	d_2

It is evident from the above, that when the two generators are at a greater interval apart than an octave, the differential tone lies between them; when they are at the exact interval of an octave, it coincides with the lower generating tone; but when at a less interval than an octave it lies below this latter, and the smaller the interval, the lower relatively will be the differential.

Any of the partials of compound tones may act as generators, if sufficiently powerful. Thus, if $\left\{\begin{matrix} d^1 \\ m \end{matrix}\right.$ be two compound generators, we see from the above table that the fundamentals d^1 and m may give rise to the differential tone s_1; but the 2nd partials d^2 and m^1, if sufficiently powerful, may also generate the differential **s**; or the partial m^1 and the fundamental d^1 may produce differential d_1; and so on. It is only in rare cases, however, that the overtones will be strong enough to produce audible differentials.

The one condition necessary for the production of differential tones, is, that the same mass of air be simultaneously and powerfully agitated by two tones; that is, the tones must be sufficiently loud. As the intensity of the generators increases, so does that of the differential, but in a greater ratio. The condition just referred to, is best satisfied on the Double Syren of Helmholtz (fig. 22), two circles of holes in the same chamber being open. The differentials produced by this instrument are exceedingly powerful.

Two flageolet fifes, blown simultaneously by two persons, also give very powerful differentials. The latter may be approximately ascertained from the table given above, but allowance must be made for the tempered intervals. Thus, if the tones G^3 and F^3 be loudly blown, the differential produced will be very nearly that given in the table, viz., F, three octaves below.

Differential Tones are very conspicuous on the English Concertina: in fact, so prominent are they, that their occurrence forms a serious drawback to the instrument. They may be plainly heard also on the Harmonium and American organ: especially when playing in thirds on the higher notes. Two soprano voices singing loudly, will produce very audible differentials. Owing to the evanescent character of its tones, it is difficult to hear differentials on the pianoforte, but they can be detected even on this instrument by a practised ear. Differential tones may be easily obtained also from two large tuning-forks, which should be struck sharply. Two singing flames are also well adapted for producing these tones.

Not only do two generating tones give rise to a differential, but this differential may itself act as a generating tone, together with either of its generators, to produce a second differential tone; and this again may in its turn act as a generator in combination with one of the original generators, or with a differential, to produce a third; and so on.

The differential tone z_1 which is generated by two simple or compound tones x and y, is termed a differential of the first order. If x and z_1 or y and z_1 generate a differential z_2, this is said to be of the second order; and so on. Differential tones of the second order are usually very faint, and it requires exceedingly powerful tones to make differential tones of the third order audible: in fact, the latter are only heard under very exceptional circumstances.

To determine what differentials of the second and third order can be present, when two tones at any definite interval are loudly sounded, we proceed as before. For example, let the two generators be $\left\{ \begin{smallmatrix} m' \\ d' \end{smallmatrix} \right.$. The relative rates of vibration being $\left\{ \begin{smallmatrix} 5 \\ 4 \end{smallmatrix} \right.$, the relative vibration rate of the differential of the first order $= 5 - 4 = 1$. Subtracting this from the generators 5 and 4 we obtain the relative vibration rates of the differentials of the second order viz. $5 - 1 = 4$ and $4 - 1 = 3$, this latter only being a new tone. Again, subtracting this 3 from the higher generator, we get another new tone $5 - 3 = 2$, a differential of the third order. Thus, omitting duplicate tones we have—

$$\text{generators} \left\{ \begin{array}{l} m' = 5 \\ d' = 4 \end{array} \right.$$

$$\begin{array}{llll} \text{Differential of} & \text{1st order,} & d_{|} & = 1\ (= 5 - 4) \\ \quad ,, & \text{2nd} \quad ,, & s & = 3\ (= 4 - 1) \\ \quad ,, & \text{3rd} \quad ,, & d & = 2\ (= 5 - 3) \end{array}$$

The 2nd, 3rd, 4th, and 5th columns of the following table show the differentials of the 1st, 2nd, 3rd, and 4th order which may be produced by the tones in the 1st column.

INTERVAL.	DIFF. OF 1st. ORDER.	2ND ORDER.	3RD ORDER.	4TH ORDER.
Fourth $\left\{ \begin{array}{l} d' = 4 \\ s = 3 \end{array} \right.$	$1 = d_{\vert}$	$2 = d$		
Major 3rd $\left\{ \begin{array}{l} m' = 5 \\ d' = 4 \end{array} \right.$	$1 = d_{\vert}$	$3 = s$	$2 = d$	
Minor 3rd $\left\{ \begin{array}{l} s' = 6 \\ m' = 5 \end{array} \right.$	$1 = d_{\vert}$	$4 = d'$	$2 = d$ $3 = s$	
Major 6th $\left\{ \begin{array}{l} l' = 5 \\ d' = 3 \end{array} \right.$	$2 = f$	$1 = f_{\vert}$	$4 = f'$	
Minor 6th $\left\{ \begin{array}{l} d^2 = 8 \\ m' = 5 \end{array} \right.$	$3 = s$	$2 = d$	$6 = s'$ $1 = d_{\vert}$	$7 = \overset{*}{ta}{}'$ $4 = d'$
Tone $\left\{ \begin{array}{l} r' = 9 \\ d' = 8 \end{array} \right.$	$1 = d_2$	$7 = \overset{*}{ta}$	$2 = d_{\vert}$ $6 = s$	$5 = m$ $4 = d$ $3 = s_{\vert}$

*Nearly.

It will be seen from the above, that in general a complete series of tones may be produced, corresponding to the complete series of partial tones, 1, 2, 3, 4, &c., up to the generators. It will be noticed also, that the same tones may occur with compound tones, as differential tones of their upper partials.

Though combination tones are generally subjective phenomena, yet on some instruments, as for example, the Double Syren and the Harmonium, they are objective, or at any rate partly so. As a proof of this fact, it is found that differential tones on these instruments, may be strengthened by resonance. Thus, sound loudly G^1 and D^2 on a harmonium, and tune a resonator to the differential G. By alternately applying the resonator to, and withdrawing it from the ear, while the generating notes are being sounded, it is easy to appreciate the alternate reinforcement and falling off of the G.

It was formerly thought, that differential tones were formed by the coalescence of beats (see next Chap.), a supposition which was supported by the fact, that the number of beats generated by two tones in a second, is identical with the vibration number of the differential tone they generate. That this is not the cause of Differential Tones, will be seen from the following considerations:

1st. Under favourable circumstances, the rattle of the beats and the differential tone may be heard simultaneously.

2nd. Beats are audible, when the generating tones are very faint, in fact, they may be heard even when the generating tones are inaudible. Differentials, on the other hand, invariably require tolerably loud generators.

3rd. This supposition offers no explanation of the origin of the analogous phenomenon of Summation Tones.

Finally, Helmholtz has offered a theory of the origin of Differential Tones, which satisfactorily explains all the phenomena of both Differential and Summation tones. This theory is difficult to explain, without such recourse to mathematics, as would be unsuitable to a work like this. We must be content, therefore, to state it in general terms as follows:

When two series of sound-waves simultaneously traverse the same mass of air, it is generally assumed that the resultant motion of the air particles is equal to the algebraic sum of the motions, that the air particles would have had, if the two series had traversed the mass of air, independently of one another. This, however, is only strictly true, when the amplitudes of the sound-waves are very small, that is, when the air particles oscillate only through very small spaces. When the amplitudes of the waves are at all considerable in proportion to their length, secondary waves are set up, which on reaching the ear give rise to Combination Tones. The higher octave of the fundamental tone, which may be frequently heard, when a tuning-fork is sharply struck, has a similar origin.

SUMMATION TONES.

Helmholtz, as already mentioned, worked out the theory just referred to, mathematically, and proved that two tones with given vibration numbers, may not only produce a third tone, having its vibration number equal to their difference, but also another tone equal to their sum. To this latter sound, the term "Summation Tone" is applied.

It is not difficult to satisfy oneself experimentally of the reality of the summation tones on such an instrument as the Harmonium or American Organ; indeed, these tones are much louder than is generally supposed. Thus if F_2 and $C_{|}$ be selected, the summation tone will be $A_{|}$, which with careful attention may generally be detected; the following table gives in the last column the summation tones that may be produced by the generators in the second column.

INTERVAL.	GENERATORS.	RELATIVE VIB. RATES.	SUM.	SUMMATION TONES.
Octave.....	d $d_{\|}$	2 1	3	s
Fifth......	$s_{\|}$ $d_{\|}$	3 2	5	m
Fourth.....	d $s_{\|}$	4 3	7	ta*
Major 3rd..	$m_{\|}$ $d_{\|}$	5 4	9	r
Minor 3rd..	$s_{\|}$ $m_{\|}$	6 5	11	f*
Major 6th..	$l_{\|}$ $d_{\|}$	5 3	8	f
Minor 6th..	d $m_{\|}$	8 5	13	l*

*Approximately.

SUMMARY.

A *Combination* or *Resultant* Tone is a third sound, which may be heard, when two tones of different pitch are *simultaneously* sounded, and which is not heard, when either of these two tones is sounded *alone*.

The two tones which give rise to a Combination Tone are termed its *generators*.

There are two kinds of Combination Tone—

(1). The *Differential* Tone: the vibration number of which is the *difference* of the vibration numbers of its generators;

(2) the *Summation* Tone: the vibration number of which is the *sum* of these vibration numbers.

Differential Tones may be of various *orders*.

A *Differential of the* 1*st order* is that which is produced by two independent tones or generators.

A *Differential of the* 2*nd order* is that which is produced by the Differential of the 1st order, and either of the generators.

A *Differential of the* 3*rd order* is that which is produced by the Differential of the 2nd order, and either of the foregoing tones; *i.e.*, either the Differential of the 1st order, or one of the generators.

A *Differential of the* 4*th order* is that which is produced by the Differential of the 3rd order and either of the foregoing tones; and so on.

Differential Tones are *not* the result of the coalescence of beats.

CHAPTER XIII.

On Interference.

We have now to consider another of the phenomena which may occur, when two musical sounds are heard simultaneously; and in the present chapter, we shall suppose the two sounds in question to be *simple tones*.

Let the horizontal dotted straight lines in fig. 67, represent surfaces of still water; and let two series of waves of equal length

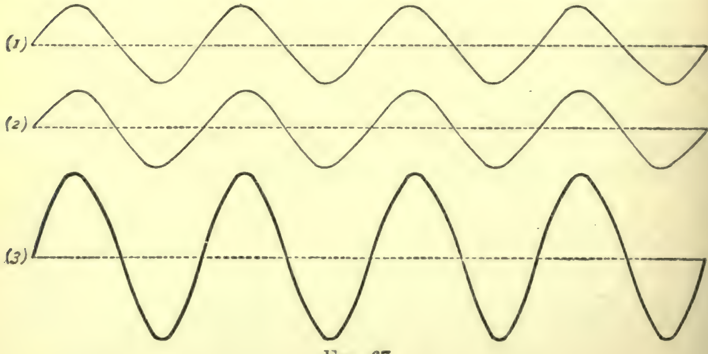


Fig. 67.

and amplitude be, at the same moment, passing from left to right. Let the curved line (1) represent in section the form, that the waves would have, if those of the first series alone were present; and let (2) represent, in the same way, the form, that the water would assume, if the second series of waves alone were passing. Let us also, in the first place, suppose the two series of waves to coincide, so that crest falls on crest, and trough on trough; that is, let them both be in the same *phase*, as represented in (1) and (2). Under these circumstances, each series will produce its full effect, independently of the other, as explained in Chap. VIII, pp. 82 & 83; crest being added to crest, and trough to trough, to produce a wave (3) of the same length as each of the coincident waves, but of twice the amplitude of either.

Now, let us suppose, that these two series of waves come together in such a way, that the crests of one exactly coincide with the troughs of the other: in other words, let them be in opposite *phase* as represented in fig. 68 (4) (5). In this case, by the use of the

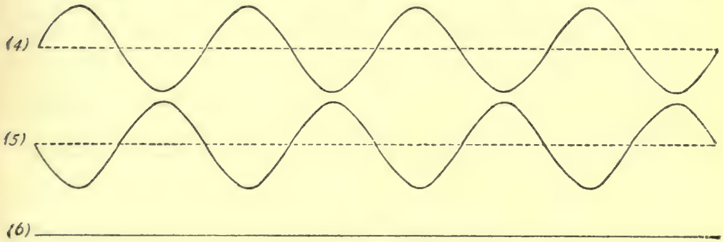


Fig. 68.

same kind of reasoning as employed in Chap. VIII, we find that, as the drops of water are solicited in opposite directions, by equal forces, at the same time, the result is no wave at all, fig. 68 (6). We have supposed here, that the waves in both series have the same amplitude. If they have different amplitudes, it is evident from the above, that, 1st. when the two series are in the same phase, the amplitude of the resultant wave is equal to the sum of the amplitudes of the constituents; 2nd, when the two series are in opposite phases, the amplitude of the resultant wave is equal to the difference of the amplitudes of the constituents. Further, it is evident, that if they are neither in the same nor opposite phases, the amplitude of the resultant wave will be intermediate between these two limits.

Now we may take the curves (1) (2) (4) (5) of figs. 67 and 68, as the associated waves of two simple sounds, and therefore at once deduce the following results. 1st, Two sound waves of the same length and amplitude, and in the same phase, produce a resultant wave of the same length, but twice the amplitude of either wave. 2nd, Two sound waves of the same length and amplitude, but in opposite phase, destroy one another's effects, and no wave is produced. 3rd, Two sound waves of the same length but different amplitudes, will produce a wave of the same length as either wave, but having an amplitude equal to the sum or difference of their amplitudes, according as the waves are in the same or opposite phase. 4th, If the two sound waves are not exactly in the same or opposite phase, the amplitude of the resultant wave will be intermediate between these limits.

If one sound wave have twice the amplitude of another, the intensity of the tone produced by the one will be four times that produced by the other, since intensity varies as the square of the amplitude. It follows therefore from the above, that when two simple tones of the same pitch and intensity are sounded together, the two may so combine as to produce 1st, a simple tone of the same pitch, but of four times the intensity of either of them; 2nd, silence; or 3rd, a simple tone of the same pitch, but intermediate in intensity between these two limits; according as their sound waves come together in the same, opposite, or intermediate phases.

The fact that two sounds may so interfere with one another as to produce silence, strange as it may seem at first, can be demonstrated experimentally, and is a special case of the general phenomenon of "Interference of waves." The only difficulty in the experimental proof, is to obtain sound waves of equal length and intensity, and in exactly opposite phase. Before explaining the way in which this difficulty can be overcome, we shall take the following supposititious case.

Let A and B (fig. 69) be two tuning-forks of the same pitch, and let us consider only the right hand prongs A and B. Now if these

FIG. 69.

prongs are in the same phase, that is, both swinging to the right and left, at exactly the same times, and if they are exactly a wave-length apart, it is evident that the two series of waves passing along A C, originated by their oscillation, will exactly coincide, condensation with condensation, and rarefaction with rarefaction, as represented by the dark and light shading. The same thing will occur, if the distance between A and B be two, three, four, or any whole number of wave-lengths. But suppose the distance from A to B were only half a wave-length, as represented in fig. 70; evidently, the condensations from the one fork will coincide with the rarefactions from the other, and thus the air to the right of B

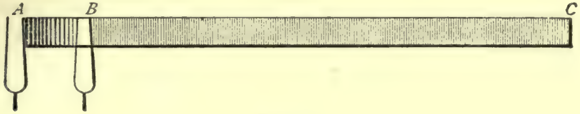


FIG. 70.

will be at rest, as indicated by the uniform shading. Precisely the same thing would occur, if A and B were three or any number of half wave-lengths apart.

Sir John Herschel made use of this principle in the construction of the apparatus shown in fig. 71. The tube *o f*, which should be

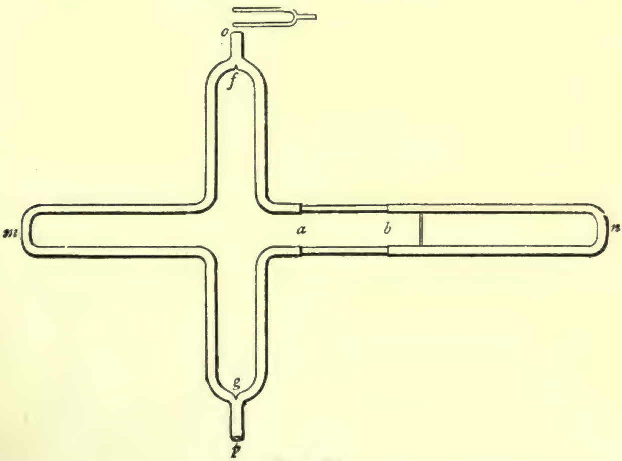


FIG. 71.

longer than represented in the figure, divides into two at *f*, the one branch being carried round *m*, and the other round *n*. These two branches again unite at *g*, to form the tube *g p*. The U shaped portion *n b*, which slides air-tight by telescopic joints over the main tube *a b*, can be drawn out, as shown in the figure. When a vibrating fork is held at *o*, the sound waves produced, divide at *f*, and pass along the two branches, reuniting at *g*, before reaching the ear of the observer at *p*. Now if the U shaped portion is pushed home to *a*, the waves through both branches reach the ear together; but if it be gradually pulled out, a point is reached at which the sound disappears altogether. From what has been said above, it will be seen, that this is the case, when the right hand branch is half a wave-length longer than the left hand branch, that is, when $a\ b$ is equal to one fourth of a wave-length. Thus, this instrument may be used, not only to demonstrate the phenomenon of Interference, but also for roughly ascertaining the wave-length, and hence the pitch of a simple tone.

The vibrating plate (fig. 72) is a very convenient instrument with which to illustrate the phenomenon of interference. In the brief

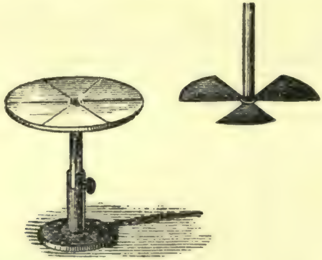


Fig. 72.

description of this instrument given in Chap. XI, it was stated that adjacent sectors are always in opposite phase; that is, while one sector is moving upwards, the adjacent ones are moving downwards. If this be the case, it follows, that the sound waves originated above two adjacent sectors are in opposite phase, and thus interfere with one another, to diminish the resultant sound. Accordingly, if the hand be placed above any vibrating sector, the sound is not diminished, but increased. Still more is this the case if the cardboard or wooden sectors, on the right of fig. 72, be held over the segments of the plate when vibrating as shown in the figure; interference being then completely abolished, the remaining segments sound much more loudly. Thus, by sacrificing a part of the vibrations, the remainder are rendered more effective.

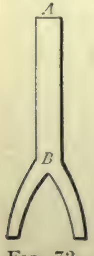


Fig. 73.

The effect of the interference of adjacent sectors may be rendered visible, by the additional apparatus shown in fig. 73. A B is a tube which branches into two at the bottom, and is closed at A by a membrane, upon which a few grains of sand are scattered. Holding the ends of the branches over adjacent segments, the membrane is unaffected, and the sand remains at rest, for the condensation which enters at one branch and the rarefaction which simultaneously enters at the other, unite at B to neutralize one another's effects. When, however, the ends are held over alternate sectors, the sand is violently agitated, showing that they are in the same phase.

It is easy to illustrate the phenomenon of Interference, with no other apparatus than an ordinary tuning-fork. Let *o*,*o*, fig. 74

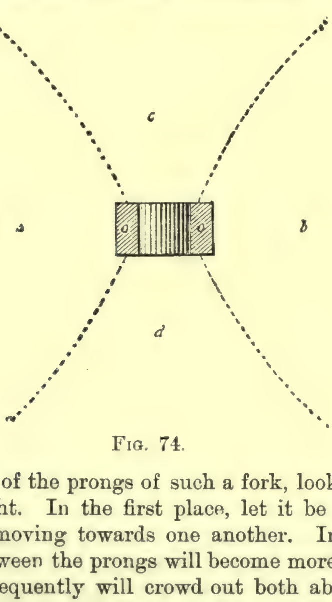


FIG. 74.

represent the ends of the prongs of such a fork, looked down upon, as it stands upright. In the first place, let it be supposed, that these prongs are moving towards one another. In this case, the particles of air between the prongs will become more closely packed together, and consequently will crowd out both above and below, giving rise to condensations both in *c* and *d*. At the same time, in consequence of the inward swing of the prongs, the air particles to the left and right, sharing this movement, will be left wider apart than at first; that is, rarefactions will be formed at *a* and *b*. Now, let it be supposed that the prongs are making an outward journey; a partial vacuum will then be formed between them, and the air rushing in from without, will cause rarefactions at *c* and *d*; while *o* and *o*, pressing on the air at either side, will at the same time, give rise to condensations at *a* and *b*. Thus, we see, that as long as a tuning-fork is vibrating, four sets of waves are proceeding from it, two issuing in directly opposite directions from between the prongs, and two, also in directly opposite directions, at right angles to the first mentioned; and as we have just seen, the waves that issue from between the prongs are in the opposite phase, to those that proceed at right angles to them; that is, whenever there are condensations at *c* and *d*, there are rarefactions at *a* and *b*, and *vice versa*. Now, each of these four sets of waves, in passing outwards from its source, will of course spread in all directions; and therefore, the adjacent waves will meet along four planes, represented by the dotted lines in the figure. Along these lines, therefore, the interference must be total; that is, any air particle in any one of them, which is urged in any direction by the waves in *c* or *d*, will be urged in the opposite direction, with precisely equal force, by the waves from *a* or *b*; that is, it will remain at rest. Consequently the dotted lines are lines of silence, the maximum of sound being midway between any two of them. If the vibrating fork were large enough, and a person were to walk round it in a circle, starting from one of these points of maximum intensity, he would find, that the sound gradually diminished as he approached the dotted line, where it would be *nil*. After passing this point, the sound would increase to the maximum, then diminish again, and so on; four points of maximum, and four of minimum intensity occurring during the circuit.

To verify all this, strike a tuning-fork, and then hold it with the prongs vertical, and with the back of one of them parallel to the ear. Note the intensity of the sound, and then quickly revolve the fork half-way or a quarter-way round: the intensity is unaltered. Now strike the fork again, and after holding it as at first, turn it one-eighth round, so that it is presented corner-wise to the ear; the sound will be all but extinguished. Again strike the fork, and holding it to the ear as at first, revolve it slowly: the four positions of greatest intensity and the four interference positions are readily perceived. To vary the experiment, again strike the fork, and rotate it rapidly before the ear: the effect is very similar to the beats, to be studied presently.

These experiments are much more effective, and the results can be demonstrated to several persons at once, when a resonator is used. For an ordinary C^{l} tuning-fork, a glass cylinder closed at one end, about $\frac{3}{4}$ inch in diameter, and between six and seven inches long, is very convenient. If not of the exact length to resound to the fork, a little water may be gradually poured in, as described in Chap. VII. When the vibrating fork is held with the back of one prong parallel to the top of the resonator, or at right angles to this portion, the sound of the fork is much intensified; but when held with the corner of the prong towards the resonator, the sound dies out. The effects produced when the fork is revolved, are precisely the same as those above mentioned, but much intensified by the resonance of the tube.

These experiments with resonators may be varied in many ways. Thus, first hold the vibrating fork with the edge of one of the prongs towards the resonator; no sound is heard. Now, keeping the fork still in this position, move it along horizontally for a short distance, so that only the lower prong is over the resonator; the sound will now burst forth, for the side of the resonator cuts off the waves issuing from between the prongs, which before interfered with those from the outside of the lower prong.

Again, tune two such resonators as the above to any tuning-fork, and arrange them at right angles to one another, as represented in fig. 75. Now hold the vibrating tuning-fork in such a position,

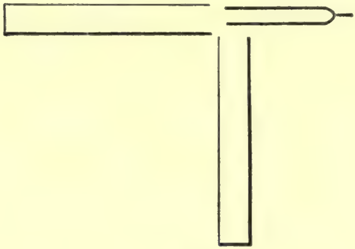


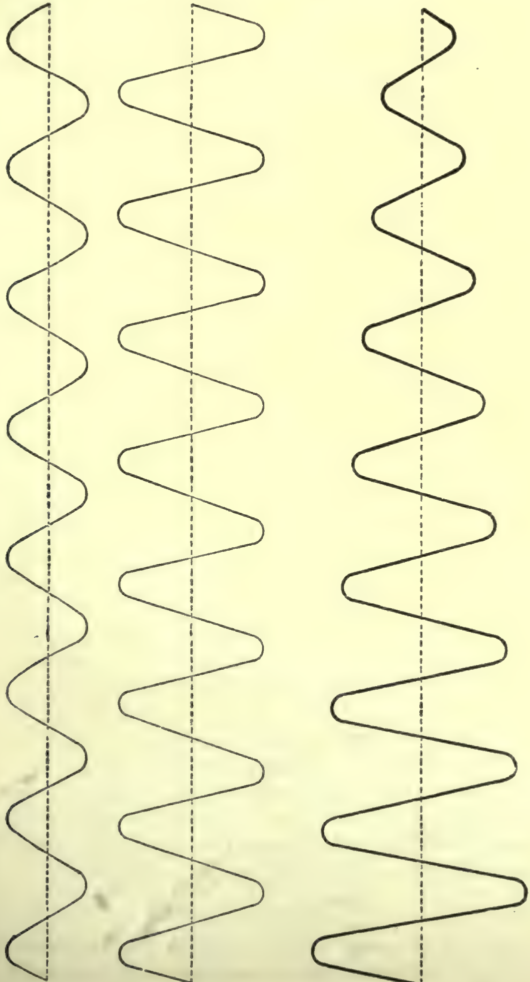
Fig. 75.

that, while the back of one of the prongs is presented to one resonator, the space between them is presented to the other. Under these circumstances, very little sound will be heard, for, from what has been already said, it will be seen, that the waves proceeding from the two resonators will always be in opposite phase, and thus will neutralize one another's effects. If, however, while the fork is vibrating, we slide a card over the mouth of one of the resonators, the other resonator will produce its due effect, and the sound will burst forth.

It is found, that two similar organ pipes placed together on the same wind chest, interfere with one another; the motion of the air in the two pipes taking place in such a manner, that as the wave streams out of one, it streams into the other and hence an observer at a distance hears no tone, but at most the rustling of the air. For this reason, no reinforcement of tone can be effected in an organ, by combining pipes of the same kind, under the conditions just referred to.

We pass on to consider the case of the interference of two simple tones, which differ slightly in pitch. Let two tuning-forks, standing close together, side by side, be supposed to commence vibrating together in exactly the same phase; and for the sake of simplicity, we will suppose their vibration numbers to be very small, viz., 15 and 16 respectively. Now, although these two forks may start in exactly the same phase, that is, the prongs of each may begin to move inward or outward together, this coincidence can evidently not be maintained, since their vibration rates are different. The flatter fork will gradually lag behind the other, till, in half a second, it will be just half a vibration behind, having performed only $7\frac{1}{2}$ vibrations while the other fork has performed 8. At the end of half a second, therefore, the two forks will be in complete opposition; the prongs of the one fork moving one way, while those of the other fork are moving in the opposite. After the lapse of another half second, the flatter of the two forks will be exactly one complete vibration behind the other, and consequently the forks will be in exact accordance again, as they were at first. These changes will evidently recur regularly every second. Thus, assuming as we have done, that the forks are in exactly the same phase at the commencement, we find that, at the beginning of each successive second, the sound-waves from the two forks coincide, condensation with condensation, or rarefaction with rarefaction, to produce a sound-wave of greater amplitude than either; but at the half seconds, the two series of sound waves will interfere, the condensation of one with the rarefaction of the other, to produce a sound-wave of less amplitude, or even, if the amplitudes of the two waves are equal, to produce momentarily, no sound wave at all. These changes in the amplitude of the resultant wave will evidently be gradual, so that the effect on the ear will be as follows: at the commencement, a sound of considerable intensity will be heard; during the first half second, its intensity will diminish, till at the exact half second, it is at a minimum, or may even be *nil;* during the next half second the intensity will increase, till at the beginning of the next second the sound has the same intensity as at first. Precisely the same changes will occur during each successive second; so that a series of crescendos and diminuendos, or swells, will be heard, one crescendo and one diminuendo being produced in the present supposed case, every second.

These alternations of intensity, which are perceived whenever two tones of nearly the same pitch are sounded together, are commonly termed *Beats.*



A B C

FIG. 76.

In order to obtain a clearer insight into this matter, let us suppose the forks in the above case to commence vibrating, as before, in exactly the same phase, and let us consider the waves produced during the first half second. As their vibration numbers are assumed to be 16 and 15, the sharper fork will have originated exactly 8, and the flatter fork exactly $7\frac{1}{2}$ waves during this period. Let the 8 equal associated waves of fig. 76 B, represent the former, as if they alone were present: and let the $7\frac{1}{2}$ associated waves of fig. 76 A, represent the latter, on the same supposition. The two series are placed one above the other, instead of being superposed, for the sake of distinctness. The forks are supposed to be at the right hand side of the figure, the waves travelling towards the left; thus the first pair of waves originated, are now on the extreme left, the next pair immediately behind these, and so on. In accordance with the supposition, the two series of waves (which, it may be noted, are not represented as of equal amplitude) commence in exactly the same phase, but in consequence of their difference in length, this exact accordance becomes less and less in succeeding waves, till at length, those on the extreme right are in exactly opposite phase. Now when the two forks are simultaneously sounding, their sound-waves combine or interfere, to produce a resultant wave, the associated wave form of which we can obtain, by compounding the two associated wave forms, A & B, in the manner before described. The thick curved line of fig. 76 C has been thus obtained; and we see from it, that the two original sound-waves coalesce, to produce a resultant sound-wave, which at first has an amplitude equal to the sum of the amplitudes of its constituent waves, but that the amplitude gradually diminishes, till in half a second, it is only equal to the difference of the amplitudes of its constituents. It is easy to see from the figure, that during the next half second, the amplitude of the resultant wave will gradually increase, till at the beginning of the next second, it will again have reached its maximum. These alternations in the amplitude of the resultant waves, produce of course in the resultant sound, corresponding alternations of intensity, which, as already mentioned, are termed Beats, and which may be represented in the ordinary musical way by crescendo and diminuendo marks—

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It is evident from fig. 76, that half a beat is formed by the interference of the waves there represented, that is, in half a second. Therefore when two sounds, the vibration numbers of which are 15 and 16 respectively, are heard together, 16 — 15 or 1 beat per second will be heard; that is, the number of beats per second, is equal to the difference of the vibration numbers. It is true, that 16 vibrations per second would not produce a musical sound, but that in no way affects the above reasoning. For suppose the vibration numbers of the forks to have been 160 and 150; the figure will represent the waves originated in one-twentieth of a second. Consequently in this case half a beat will be formed in one-twentieth of a second, or one beat in one-tenth of a second; that is, 10 = (160 — 150) beats per second. It is evident, therefore, that *the number of beats per second, due to two simple tones, is equal to the difference of their respective vibration numbers.*

For purposes of experimental study, wide stopped organ pipes are well adapted for the production of beats between simple tones; for when such pipes are gently blown, the fundamentals only are heard, or at most, accompanied by very faint third partials. If two exactly similar pipes be used, the tones produced will of course be in unison. To obtain beats, the pitch of one may be slightly lowered by shading the embouchure; or better still, one of the pipes instead of being permanently closed at the top, may be stopped by a movable wooden piston, or plug, working air-tight in the pipe. After the pipes have been brought into unison, the pitch of the one may be varied to any desired extent, by moving the wooden piston, which alters the length of the vibrating air column. If the plug be moved very slightly from its unison position, very slow beats may be obtained, each beat lasting for a second or more. The crescendo and subsequent diminuendo of the beat is then very perceptible. By gradually moving the plug farther and farther from its unison position, the beats follow one another more and more rapidly, till at last they cease to be separately distinguishable.

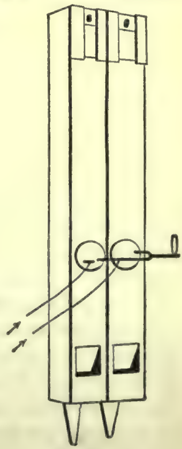


Fig. 77.

The interference of two such organ pipes as the above, may be rendered visible by the use of the manometric flame apparatus shown in fig. 55. Instead of each tympanum having its own flame however, the outlet pipes from the two tympana unite into one (fig. 77), with a single flame at the end.

Thus, when the vibrating air columns are in the same phase, the india-rubber membranes will vibrate simultaneously in the same direction, so as to expel the gas with greater force, and thus produce a very elongated flame. On the other hand, when they are in opposite phase the membranes will move simultaneously in opposite directions, and thus, neutralizing one another's effects, their movements will be without influence on the flame. The latter will therefore rise and fall with the beats, of which indeed, they are the optical expression. By the aid of a rotating mirror, the separate vibrations of the flame may also be observed as explained on page 5.

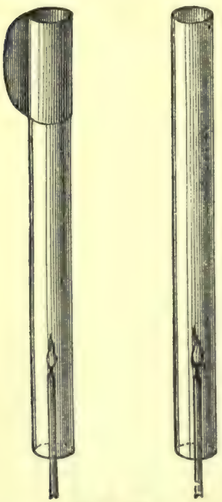


Fig. 78.

Instead of the organ pipes referred to above, two of the singing flames described in Chap. I, fig. 3, may be used; but in this case, beats from overtones, as well as from the fundamentals, will, in all probability, be heard. In order to vary the pitch, one pipe should be supplied with a sliding tube, as shown in the left hand pipe of fig. 78.

Two unison tuning-forks may be used to produce beats of varying rapidity. The pitch of one may be lowered by attaching pieces of bees-wax to its prongs, or, if the forks be large, by fastening a threepenny piece, by means of wax, to each prong. With large forks, these beats also may be optically expressed. A pencil of light from the lamp L (fig. 79), passes through the lens I, and then strikes against a little concave mirror fastened to one prong of the fork T. From this mirror it is then reflected to a similar mirror attached to the fork T', and is finally received on the screen A. When the forks are at rest, only a spot of light appears on the screen; but if one fork is set vibrating, this spot lengthens out, to form a vertical line of light. Now let both forks vibrate together with equal amplitudes, and in the first place suppose them to be in unison: if they are in exactly the same phase, the line of light will be twice as long as at first; if in opposite phase, the line will be reduced to a spot: in any intermediate phase, the line will have a length intermediate between these two extremes. In the next

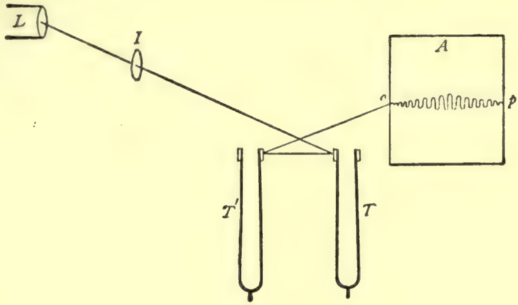


FIG. 79.

place, suppose the forks are not in exact unison; as we have already seen, they will be at one moment in the same phase, then gradually diverge till in opposite phase, and again gradually converge to the same phase. The line of light will vary coincidentally; at one moment being of considerable length, then gradually shortening till but a mere spot, and then lengthening again. The beats, of which this alternate lengthening and shortening is the optical expression, will at the same time be heard.

If the beam of light in the above, instead of falling on a screen, be received on the revolving mirror of fig. 3, the separate vibrations will, as it were, be visible, and will appear as represented in fig. 79 *o p*, in which the varying amplitude of the sinuosities corresponds to the varying intensity of the resultant sound.

If no better apparatus be at hand, beats may be studied on the pianoforte, by loading one of the two wires of a note with wax, and then striking the corresponding key; or they may be observed by stretching two similar strings on a violin, and after bringing them into unison, throwing one more or less out of tune; but in these cases, as the tones are compound, the matter is complicated by the beats of the overtones.

If two tuning-forks are nearly, but not quite in unison, and the vibration number of one of them is known, it is easy to ascertain the vibration number of the other, by counting the beats between them, provided we know which is the sharper of the two. For example, suppose we have a standard C fork producing exactly 512 vibrations per second, and on sounding it with another fork, we find that in half a minute, 90 beats are counted. Now 90 beats per half minute, is at the rate of three beats per second; but we know that the number of beats generated by two sounds, is equal to the difference of their vibration numbers; therefore the vibration number of the fork under trial, must be either $512 + 3$ or $512 - 3$; that is, either 515 or 509, according as it is sharper or flatter than the standard,—a matter, which the ear of the musician can easily decide.

It is found by experience, that beats which occur at the rate of from 2 to 5 per second, are the most easily counted. Beyond five beats in a second, there is considerable difficulty in counting, owing to their rapidity; and below two beats in a second, there is also a difficulty, owing to the length of time occupied by each loudness. For ascertaining the pitch of instruments in the way just described, cases of tuning-forks are constructed consisting each of twelve forks, the vibration numbers of which increase by four vibrations per second, from 412 to 456 for A, and from 500 to 544 for C^{I}. To show the method of using them, we will take the following case. It was desired to ascertain the pitch of a certain piano. In a preliminary trial, by sounding the C^{I} with each of the forks, it was found that it produced with the 536 fork, from 2 to 3 beats per second, and with the 540 fork, beats at a somewhat slower rate. The former was first taken, and the beats produced by it with the pianoforte C^{I}, carefully counted for 30 seconds. The number was found to be 75, which is at the rate of $\frac{75}{30} = 2\frac{1}{2}$ beats per second. Therefore the vibration number of the note in question was $536 + 2\frac{1}{2} = 538\frac{1}{2}$. To verify this, the 540 fork was sounded with the pianoforte C^{I}; 44 beats were now counted in 30 seconds, that is $\frac{44}{30} = 1\frac{1}{2}$ beats per second, nearly. This gives the same result as before, viz., $540 - 1\frac{1}{2} = 538\frac{1}{2}$.

It is possible, however, to ascertain the vibration number of a musical sound by means of beats, independently of any previously ascertained standard. This will be seen from the following considerations. Suppose we have two forks, one of which gives the exact octave of the other. Let us further suppose, that it is possible to count the number of beats per second produced, when they are sounded together, and let the number be, say 100. What will be the vibration numbers of the forks? Now, in the first place, it is evident that, whatever they are, the difference between them must be 100; since the number of beats per second, produced by two sounds, is equal to the difference of their vibration numbers. In the second place, the vibration number of the higher fork must be twice that of the lower, since they are an octave apart. Thus the problem reduces itself to finding two numbers, one of which is double the other, the difference between them being 100. Now 200 and 100 are the only numbers which satisfy these conditions, and therefore the vibration numbers of the forks will be 200, and 100, respectively.

To put this in a general way

Let x denote the vibration number of the lower fork, then $2x$ will denote the vibration number of the higher fork, therefore if n denote the number of beats per second produced by them

$$2x - x = n$$

that is $$x = n$$

Therefore, if two sounds are exactly an octave apart, the number of beats they generate per second, will be the vibration number of the lower sound.

But when two sounds, at the interval of an octave, are heard together, no beats at all are perceived. How is this difficulty to be overcome? Let us suppose we have two forks A and Z, an octave apart, A being the lower one. Tune another fork B slightly sharper than A, so that it produces with it, not more than 4 beats per second; tune another fork C sharper than B, and making with it about 4 beats per second; tune another fork D in the same manner, to beat with C; and so on, till we get a fork within 4 beats of Z. Now count accurately the number of beats between A and B, B and C, C and D, and so on up to Z; add these all together, and the total will evidently be the number of beats between A and Z.

Instruments constructed on the above principle are called Tonometers, of which there are two varieties: the Tuning-fork Tonometer and the Reed Tonometer.

The Tuning-fork Tonometer was invented by Scheibler, who died in 1837. One of his instruments, which still exists, consists of 56 forks, each of which produces four beats per second with the succeeding one. Therefore, between the lowest and the highest forks, there are 55 sets of four beats; that is, $55 \times 4 = 220$, which, by the above, must be the vibration number of the lowest fork, 440 being that of the higher one.

In Appun's Tonometer, the tuning-forks are replaced by reeds. Although better adapted to all purposes of lecture illustration than the Tuning-fork Tonometer, the Reed Tonometer has two serious drawbacks, viz.: the reeds do not retain their pitch with accuracy, and their variation with temperature is unknown.

The method of using the Tonometer is similar to that above described, in the case of the standard forks. In the experienced hands of Mr. Ellis, the tuning-fork Tonometer has given results at least equal in accuracy to those obtained by means of any other counting instrument (see table on page 39).

SUMMARY.

When two series of sound waves of the same lengths and amplitudes, traverse simultaneously the same mass of air:

(1) If the waves of the one series are in exactly *the same phase* as those of the other, resultant waves are produced of the same length, but of *double* the amplitude ;

(2) If the waves of the one series are in exactly the *opposite phase* to those of the other, the result is,—*no wave* ;

(3) If the waves of the one series are neither in the same phase as, nor in opposite phase to, those of the other, the amplitude of the resultant waves will be intermediate between the two limits given above, viz., no amplitude at all, *i.e.*, silence, and twice the amplitude of the constituent waves.

When two simple sounds of the *same pitch and intensity* are simultaneously produced the result is

(1) *Silence* ; or

(2) A sound of the same pitch as, but of four *times the intensity* of, either ; or

(3) A sound, intermediate in intensity between these two limits,

according as the corresponding sound waves are in (1) *opposite* phase, (2) the *same* phase, or (3) any relative position intermediate between these two.

When two sounds differing slightly in *pitch* are simultaneously produced, the flow of sound is disturbed by regular recurring *throbs* or *alternations in intensity*, termed *beats*. These beats are due to the alternate coincidence and interference of the two systems of waves.

If the two tones be of equal intensity, the maximum intensity of the beat, will be *four times* that of either sound heard separately, the minimum intensity being zero.

The number of beats per second, due to simple tones, is equal to the *difference* of their vibration numbers.

This fact is the principle of the Tonometer, of which there are two varieties,

(1) The *Tuning-fork* Tonometer;

(2) The *Reed* Tonometer.

CHAPTER XIV.

On Dissonance.

Having studied, in the preceding Chapter, the causes and characteristics of beats, we now proceed to inquire into the effects they produce, as they become more and more rapid.

Slow beats in music are not altogether unpleasant; in low tones, and in long sustained chords they often produce a solemn effect: in higher tones, they impart a tremulous or agitating expression; accordingly, modern organs and harmoniums usually have a stop, which, when drawn, brings into play a set of pipes or reeds, so tuned, as to beat with another set, thus imitating the trembling of the human voice and of violins.

When, however, the beats are more rapid, they become unpleasant to the ear. In studying this matter, it will be best to begin with simple tones. Select two C^{I} tuning-forks, and gradually throw them more and more out of tune, by sticking wax on the prongs of one of them, as described in the last Chapter. Sound the forks together after each addition of wax, and note the effect of the increasing rapidity of the beats. It will be found, that when they number five or six per second, the effect begins to be unpleasant, and becomes harsher and more jarring, as they grow more and more rapid. Of course the beats soon become too rapid to be counted by the unaided ear, but their rate can easily be ascertained by subtracting the vibration numbers of their generators. When the beats amount to about 32 per second,, though they are too rapid to be individually discriminated, yet the resultant sound has the same harsh jarring intermittent character, that it has had all along, only much more disagreeable. The two tones are now at the interval of a semitone, about the worst discord in music, and no one, who tries the above experiment, and notes carefully the effect of the beats, as the interval between the forks increases from unison to a semitone, can doubt that the discord here arises from these beats.

Now, gradually increase the interval between the forks still more; the rapidity of the beats of course increases, but the resultant sound becomes less and less harsh; till finally, when the beats number about 78 per second, all the harshness vanishes. At this point, the interval between the forks is rather less than a minor third. The interval at which the dissonance thus disappears, has been termed the *Beating Distance.*

The fact just alluded to,—that all Discord or Dissonance between musical tones arises from beats,—is one of Helmholtz's most important discoveries. In order to thoroughly convince himself of its truth, the student must proceed step by step. In the first place, as we have seen, beats are reinforcements and diminutions of intensity, which are due to the interference of two separate sound waves. Now this being the case, if such reinforcements and diminutions can be made to occur in the case of a single sound, then, not only should beats be heard, but the harsh jarring we call discord, which is supposed to be due to beats, should be heard also.

This was put to the test of experiment by Helmholtz, in the following way. A little reed pipe was substituted for the wind conduit of the upper box of his Syren (see page 33), and wind driven through this reed pipe. The tone of this pipe could be heard externally, only when the revolution of the disc brought its holes before the holes of the box, and so opened an exit for the air. Hence, allowing the disc to revolve, while air was being driven through the pipe, an intermittent sound was obtained, which sounded exactly like the beats arising from two tones sounded at once.

By means of a perforated disc and multiplying wheel, similar to that shown in fig. 19, the same thing may be still more easily demonstrated. One circle of holes on the disc is sufficient, but they should be larger than shown in the figure. One end of the india-rubber tube is held opposite to the circle of holes, just as in the figure, but the other end is to be applied to the ear. On the other side of the disc and opposite to the end of the india-rubber tube, a vibrating tuning-fork is held, the necessary intermittence of tone being brought about by the revolution of the disc.

In either of the above ways, intermittent tones may be obtained, and this intermission gives them all exactly the same kind of roughness, that is produced by two tones which beat rapidly together. Beats and intermissions are thus identical, and both, when succeeding each other fast enough, produce a harsh discordant jar, or rattle.

Two questions now suggest themselves; first, why should such an intermittent sound—why should rapid beats—be unpleasant? and secondly, why should beats cease to be unpleasant when they become sufficiently rapid?

With regard to the first question, beats produce intermittent excitement of certain auditory nerve fibres. Now any excitement of a nerve fibre deadens its sensibility, and thus during a continuance of the excitement, the excitement itself deadens the sensibility of the nerve, and in this way protects it against too long and too violent excitement. But during an interval of rest, the sensibility of the nerve is quickly restored. Therefore if the excitement instead of being continuous is intermittent, the nerve has time to regain its sensibility more or less, during the intervals of rest; thus the excitement acts much more intensely than if it had been continuous, and of the same uniform strength.

In the analogous case of light, for example, every one must have experienced the unpleasant sensation of walking along the shady side of a high picket fence, with the evening sun shining through. Here the fibres of the optic nerve are alternately excited and at rest. During the short intervals of rest, the nerve regains more or less its sensibility, and thus the excitements due to the sunlight are much more intense than they would have been, had the irritation been continuous; for in this case, the continuous irritation would have produced a continuous diminution in the sensibility of the nerve. It is precisely the same cause, which renders the flickering of a gas jet, when water has got into the pipe, so unpleasant.

An intermittent tone is to the nerves of hearing, what a flickering light is to the nerves of sight, or scratching to the nerves of touch. A much more unpleasant and intense excitement is produced than would be occasioned by a continuous tone. The following simple experiment is instructive on this point. Strike a tuning-fork and hold it farther and farther from the ear, till its tone can *just not* be heard. Now if the fork, while still faintly vibrating, be revolved, it will become audible. For as we have seen, during its revolution, it is brought into positions such, that it alternately can and cannot transmit its sound to the ear, and this alternation of strength is immediately perceptible to the ear. As Helmholtz has pointed out, this fact supplies us with a delicate means of detecting very faint tones. For if another tone of about the same intensity, but differing very slightly in pitch, be sounded with it, the intensity of the resulting sound, as we saw in the last chapter, will alternate between silence and four times the intensity of the original sound, and this increase of intensity will combine with the alternation to render it audible.

With regard to the second question, "why should the beats cease to be unpleasant, when they become sufficiently rapid"? we must again have recourse to the analogous phenomenon of light. If a carriage wheel be revolved slowly, we can see each of the spokes separately; on revolving more quickly, they merge together into a shadowy circle. Again the singing flame of fig. 3 is all but extinguished two or three hundred times per second, but to the unaided eye it appears stationary. When the alternations between irritation and rest follow one another too quickly, they cease to be perceived, and the sensation becomes continuous. So in the case of sound, after the exciting cause has ceased to act, a certain minute interval of time is necessary for the excited nerve to lose its excitement; and, when the beats succeed one another so rapidly, that there is not this interval between them, then the cessations and reinforcements, that is, the beats, become imperceptible.

In our first experiment, we began with two C^{I} forks in unison, and on gradually increasing the interval between them, we found that the harshest discord was obtained, when they produced about 32 beats per second, and that, when their vibration numbers differed by about 78, the two tones were just beyond beating distance: that is the 78 beats so coalesced as to be imperceptible. Now these numerical results apply only to this region of pitch. If we select another pair of tones in a different part of the musical realm, the general result will be the same, but the numbers will not be those above: that is to say, the discord will become harsher and harsher as the beats increase up to a certain point, but the number of beats per second at this point will not be 32; and similarly, the discord will become less and less after this, and finally vanish, but the number of beats per second at the beating distance, will not be 78.

The harshness of a dissonance therefore, does not depend upon the rapidity of beats alone: it depends also upon the position of the beating tones in the musical scale. This will be evident from the following examples—

INTERVAL.	TONES.	VIB. NOS.	NO. OF BEATS PER SEC.
Semitone....	C^{1} B	512 480	32
Tone........	D C	288 256	32
Major Third	E_{1} C_{1}	160 128	32
Fifth........	G_{2} C_{2}	96 64	32

The number of beats produced in each of these four intervals is 32 per second, and therefore if harshness of discord depended on rapidity of beats alone, these intervals should be equally discordant. But as every one knows, they are not; in fact, if, as we suppose, the tones are simple, the last two will have no trace of harshness whatever. Thus the number of beats per second, necessary to produce a certain degree of discord, varies in different parts of the scale of musical pitch, diminishing as we descend, and increasing as we ascend. Similarly the Beating Distance becomes greater, as we get lower in pitch, and contracts as we go higher.

The following are Mayer's determinations of the beating distance between Simple Tones, in various parts of the musical scale. The first column gives the name of the Simple Tone; the second, its vibration number; the third, the number of beats generated between the simple tone given in the first column, and another simple tone at Beating Distance; the fourth, the Beating Distance approximately expressed in musical language,—in other words, this column shows the smallest consonant interval in the region of the tone given in the first column. It is difficult to fix the points of greatest discord, but we should probably be not far wrong in placing it at somewhat less than half the Beating Distance; or throughout the greater part of the scale, at about a semitone.

Tone.	Vib. No.	Beating Dist.	Nearest Consonant Interval.	Duration of Sensation.
C_2	64	16	Major 3rd.	$\frac{1}{16}$ of a sec.
C_1	128	26	Minor 3rd.	$\frac{1}{26}$ „
C	256	47	Minor 3rd, less $\frac{1}{4}$ Semitone.	$\frac{1}{47}$ „
G	384	60		$\frac{1}{60}$ „
C^1	512	78	Minor 3rd, less $\frac{1}{2}$ Semitone.	$\frac{1}{78}$ „
E^1	640	90		$\frac{1}{90}$ „
G^1	768	109		$\frac{1}{109}$ „
C^2	1024	135	Tone or Second	$\frac{1}{135}$ „

Two other points may be noticed before leaving this table. In the first place, it appears, that a very high number of beats—more than 100 per second—may be appreciable to the ear without coalescing. In order to convince oneself that this is true, it is only necessary to hear the following four intervals successively between simple tones, and note that, though it soon becomes impossible to discriminate the separate beats, yet, the harsh jarring effect is the same throughout.

SIMPLE TONES.	VIBRATION NOS.	BEATS PER SEC.
$\begin{cases} C \\ B_1 \end{cases}$	$\begin{cases} 256 \\ 240 \end{cases}$	16
$\begin{cases} C^1 \\ B \end{cases}$	$\begin{cases} 512 \\ 480 \end{cases}$	32
$\begin{cases} C^2 \\ B^1 \end{cases}$	$\begin{cases} 1024 \\ 960 \end{cases}$	64
$\begin{cases} C^3 \\ B^2 \end{cases}$	$\begin{cases} 2048 \\ 1920 \end{cases}$	128

Secondly, the time during which a sensation of sound will endure, after its cause has ceased to act, varies for sounds of low and high pitch. For since 16 beats per second in the region of C_2 coalesce, it is only reasonable to conclude that the sensation of each of these beats remains for $\frac{1}{16}$ of a second. Similarly the duration of sound in the region of C_1 is $\frac{1}{26}$ of a second, and so on, as given in the last column of the above table. If this conclusion be correct, it seems to afford an explanation of the fact, that the Beating Distance becomes greater as we descend in the scale.

To sum up, then, as far as we have gone: Dissonance between two simple tones, is due to Beats: taking two Simple Tones in unison with one another, and gradually altering the pitch of one of them, the harshness of the dissonance increases with the rapidity of the beats, up to a certain point; beyond that point it diminishes, until finally, all harshness—all dissonance—vanishes when the two tones are at a certain distance apart: and finally, the number of beats per second which produces the greatest dissonance, and the Beating Distance both vary as we ascend and descend in the musical scale.

It would seem from the above, that however much we widen the interval between two simple tones beyond the beating distance, they never again become dissonant, for being now beyond that distance, it is plain they can no longer beat. On putting the matter to the test of experiment, however, it is found that this is not the case; there are certain intervals, beyond the beating distance, which do beat. For example, if two forks be tuned, one to C, and the other to B, or to C'♯ or thereabouts, beats will be heard when they are sounded together, although they are far beyond the Beating Distance.

This fact, though at first sight inconsistent with the foregoing, is, in reality, not so; for the beats in question are not produced by the two simple tones, but by one of them and a differential tone generated by them. The following figures show this—

$$\left.\begin{array}{r} B = 480 \\ C = 256 \\ \hline \text{Differential Tone, } 224 \\ \text{and } 256 - 224 = 32 \text{ beats.} \end{array}\right\} \quad \text{or} \quad \left\{\begin{array}{r} C'\sharp = 540 \\ C = 256 \\ \hline 284 \text{ Diffl. Tone.} \\ \text{and } 284 - 256 = 28 \text{ beats.} \end{array}\right.$$

C and B generate a Differential, the vibration number of which is 224, and this with the tone C will produce 256 — 224 = 32 beats per second; similarly C and C'♯ generate the Differential 284, which with C gives 284 — 256 = 28 beats per second; and on reference to the table on page 159, we see that both 32 and 28 beats per second, are well within beating distance at this part of the musical scale.

Again, if we sound together two forks, one tuned to C and the other tuned only approximately to G, beats may be heard, but only when the forks are vigorously excited. Thus taking C = 256, G should be 384: let the G fork, however, be mistuned to 380, then

380 — 256 = 124 Differential of 1st order
256 — 124 = 132 ,, 2nd ,,

and these two differential tones will produce 132 — 124 = 8 beats per second. These beats, however, will be faint, inasmuch as the differential tone of the second order is itself very weak.

With other intervals beyond the beating distance, no dissonance will be heard between simple tones. Two forks, forming any interval between a minor and a major third for example, in the middle or upper part of the musical scale, produce no roughness when sounded together; the interval may sound strange to musical ears, but there is no trace of dissonance.

To sum up, therefore: if the interval between two simple tones be gradually increased beyond the beating distance, no roughness or dissonance will be heard, till we are approaching the Fifth; and only then, if the tones are sufficiently loud to produce a Differential of the second order: on still further widening the interval, beats may be heard in the neighbourhood of the octave, due to a Differential of the first order.

Thus all dissonance between simple tones will be found on examination to be due to beats, generated, either by the simple tones themselves, by one of the simple tones and a Differential, or by two Differentials.

Before inquiring into the causes of dissonance between Compound Tones, it will be as well to call to mind the fact, that a single compound tone may and often does contain dissonant elements in itself. Let us take the compound tone C_2, for example: Inasmuch as its fundamental has the vibration number 64, the difference between the vibration numbers of any two successive partials must be 64. By reference to the accompanying table of partials, and to the table on page 159, we see that the intervals between the first 7 partials are greater than the Beating Distance, but that the intervals between the partials above the 7th are less than the Beating Distance. For, take the 8th and 9th partials, which are C^1 and D^1 respectively, the number of beats produced by these two simple tones is 64 and we know by the table on page 159 that the number of beats necessary to concord, in this part of the musical scale is 78; therefore a certain amount of roughness, due to these 64 beats will result. The dissonance gets worse as we ascend; for example, the number of beats per second between the 15th partial, B^1, and the 16th, C^2, is of course 64, which forms a very harsh dissonance in this part of the scale. As we have already seen, the partials of the tones of most instruments, become weaker and weaker, the farther they are from the fundamental; so that in general, these very high partials are not strong enough to produce any appreciable roughness, but this is by no means always the case. If the note C_2 be sounded on the Harmonium or American Organ, especially with such a stop as the bassoon, it is quite easy to detect the jarring of these higher partials, and by means of a resonator tuned to a note intermediate between any two of them, the beating of those two is perceptibly increased. The same jarring effect may be readily perceived in the tones of the Trombone and Trumpet; in fact, it is this beating that gives to the tones of these instruments, their peculiar penetrating or braying character; a discontinuous sensation, as before observed, producing a much more intense

XVI	C^2
XV	B^1
XIV	*
XIII	*
XII	G^1
XI	*
X	E^1
IX	D^1
VIII	C^1
VII	*
VI	G
V	E
IV	C
III	G_1
II	C_1
I	$C_2 = 64$

*Out of Scale.

effect than a continuous one of equal strength. For precisely the same reason, the tones of a powerful bass voice are apt to partake of this strident quality.

Coming now to the subject of dissonance between two compound tones, we shall find that beats may arise;

(1) *Between the Fundamentals themselves;*

(2) *Between the Fundamental of one Tone and an overtone of the other;*

(3) *Between overtones;*

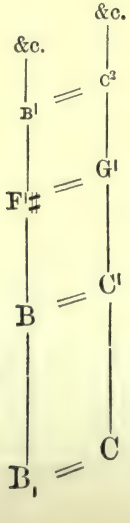
(4) *From the occurrence of Differentials;*

(5) *From the occurrence of Summation Tones.*

To take these causes of beats one at a time;

(1) *Beats arising between Fundamental Tones.*

Inasmuch as these Fundamental tones are simple, all the conclusions above as to simple tones, at once apply to them. But when such beats arise between the fundamentals of two compound tones, the dissonance will in general be harsher, than between two simple tones of the same pitch, for in the former case each pair of overtones may beat also. Supposing for example, the two fundamentals to be $B_{\prime}$ and C, the following diagram shows the dissonant overtones.



The harshness of the beats between each pair of overtones in the above, must be estimated, from the conclusions we arrived at before, in the case of simple tones, for these overtones *are* simple tones; but in estimating the total harshness of the whole combination, it should be remembered that for ordinary qualities of tone, the intensity of the partials becomes less and less, as we go farther from the Fundamentals (a fact roughly indicated in the above by the use of smaller type for the upper partials); and therefore the intensity of the beats in the above, will become less and less as we ascend.

(2) *Beats arising between the Fundamental of one tone and an overtone of the other.* As an example, we may take the common dissonance—

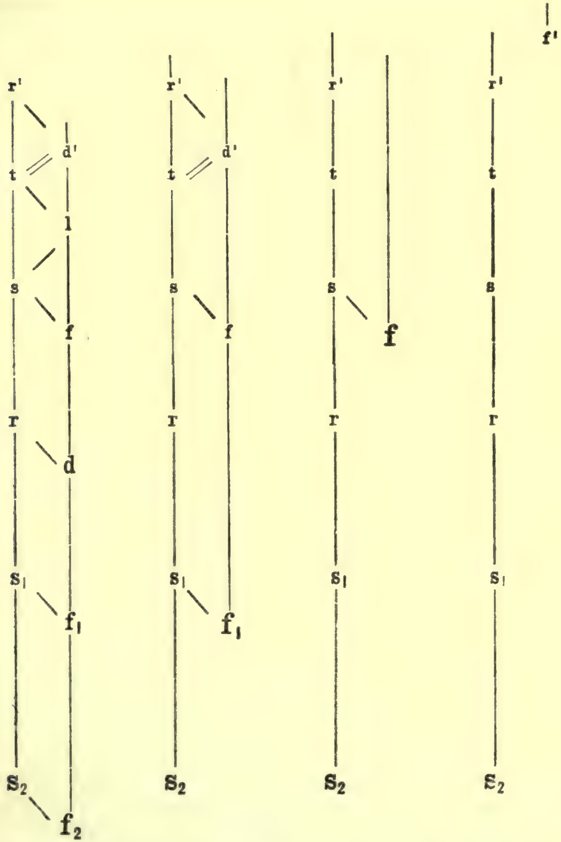
$$\left\{\begin{matrix}\mathbf{f}\\ \mathbf{s}_{|}\end{matrix}\right. \quad \text{or} \quad \left\{\begin{matrix}\mathrm{F}\\ \mathrm{G}_{|}\end{matrix}\right.$$

This interval, when sounded between *simple* tones, is quite free from harshness; the tones are far beyond beating distance, and no differential is near enough to produce beats. When, however, it is sounded between ordinary *compound* tones, beats are generated by the fundamental **f** and the 2nd partial of $\mathbf{s}_{|}$, thus:—

The following dissonances, between compound tones, although often called by the same name, are very different indeed in their degree of dissonance.

No. 1.	No. 2.	No. 3.	No. 4.
$\left\{\begin{matrix}\mathbf{s}_2\\ \mathbf{f}_2\end{matrix}\right. \left\{\begin{matrix}\mathrm{G}_2\\ \mathrm{F}_2\end{matrix}\right.$	$\left\{\begin{matrix}\mathbf{f}_{|}\\ \mathbf{s}_2\end{matrix}\right. \left\{\begin{matrix}\mathrm{F}_{|}\\ \mathrm{G}_2\end{matrix}\right.$	$\left\{\begin{matrix}\mathbf{f}\\ \mathbf{s}_2\end{matrix}\right. \left\{\begin{matrix}\mathrm{F}\\ \mathrm{G}_2\end{matrix}\right.$	$\left\{\begin{matrix}\mathbf{f}^{|}\\ \mathbf{s}_2\end{matrix}\right. \left\{\begin{matrix}\mathrm{F}^{|}\\ \mathrm{G}_2\end{matrix}\right.$

To render this evident, it is only necessary to set forth the partials of each tone, thus:—

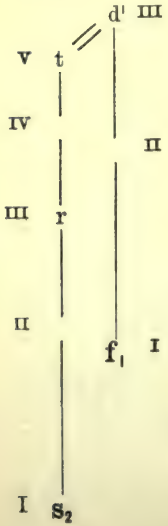


No. 1, *Primary*. No. 2, *Secondary*. No. 3, *Tertiary*. No. 4, *Quarternary*.

In setting out the above, we do not go above the 6th partial, inasmuch as the partials above this point are in general too weak to have any influence on the subject under discussion.

In No. 1, not only do the fundamentals beat, but every pair of overtones also, while above the 3rd pair, there is a perfect galaxy of dissonances. In No. 2, the Fundamentals are beyond beating distance, but there are beats between one of them (**f_1**) and the 1st overtone of the other (s_1). The harshness of this dissonance will consequently chiefly depend on the intensity of this overtone, which will vary in different instruments, and even in different parts of the same instrument. Thus, in the lower notes of the piano, the 1st overtone is not unfrequently louder than the fundamental itself. On other instruments, however, and in general, its intensity is not so great; in a well bowed violin, for example, it is only about one fourth as loud. Further, the beating between the 3rd pair of partials, and between the 5th pair of No. 1, is wanting in No. 2. Thus, on the whole, this latter dissonance is much less harsh than No. 1. The dissonance in No. 3 is of a very mild character, for the Fundamental (**f**) beats only against the 4th partial (**s**) and as a general rule, the 4th partial is comparatively weak. In No. 4 there is no beating whatever, unless the 7th or 8th partial is audible, and even then it would be very slight.

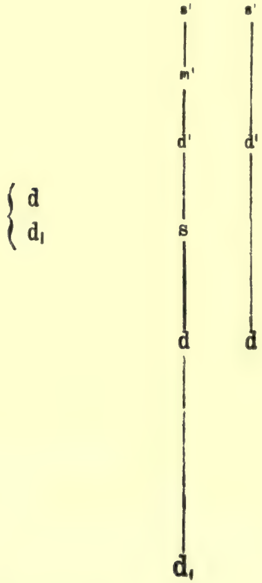
The late Mr. Curwen proposed to distinguish dissonances such as Nos. 1, 2, 3, and 4 above, by terming them respectively Primary, Secondary, Tertiary, and Quaternary dissonances. Thus, in Primary dissonances the fundamentals themselves beat, while in Secondary, Tertiary, and Quaternary dissonances, the Fundamental of the one tone beats respectively with the 2nd, 4th, and 8th partials of the other.



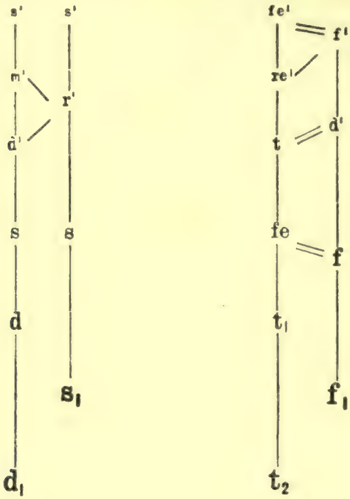
The above conclusions must of course be modified for the tones of instruments, which have not the complete series of partials up to the 6th. For example, the tones of stopped organ pipes, and of clarionets are wanting in the even partials, and therefore a secondary dissonance between such tones, is of a very mild character indeed, the only beating which occurs, arising from a 5th and a third partial, as shown in the accompanying sketch.

(3) *Beats between the overtones of Compound Tones.* In studying these beats, we shall for the reasons stated above, take into consideration, the first six, and only the first six partials; and the student must continually bear in mind the fact, that, in general, the intensity of these partials rapidly diminishes as we go farther and farther from the fundamental.

If we limit ourselves to intervals not greater than an octave, we shall find, that the only interval entirely free from these partial beats, is the Octave itself, thus :—



In all intervals smaller than the Octave, it will be found that two or more of the first six partials beat with one another. To take a couple of examples: In the Perfect Fifth a 3rd partial beats against 4th and 5th partials ; and in the Diminished Fifth 2nd, 3rd and 4th partials come within beating distance of 3rd, 4th, 5th and 6th ; thus :—



$\left\{\begin{matrix} s_{\mathsf{I}} \\ d_{\mathsf{I}} \end{matrix}\right.$
Perfect 5th

$\left\{\begin{matrix} f_{\mathsf{I}} \\ t_2 \end{matrix}\right.$
Diminished 5th

As other examples we may take the interval $\left\}\begin{matrix} d \\ se_{\mathsf{I}} \end{matrix}\right.$ or $\left\}\begin{matrix} C \\ G\sharp_{\mathsf{I}} \end{matrix}\right.$ and its inversion $\left\}\begin{matrix} se_{\mathsf{I}} \\ d_{\mathsf{I}} \end{matrix}\right.$ or $\left\}\begin{matrix} G\sharp_{\mathsf{I}} \\ C_{\mathsf{I}} \end{matrix}\right.$. It will be more convenient in this case to give the vibration numbers of the partials, rather than to express them in musical notation. Taking C or **d** = 256, the vibration number of $G\sharp_{\mathsf{I}}$ or $\mathbf{se}_{\mathsf{I}}$ = 200, and the partials are as follows:

Partials of C =	256		512	768	1024	1280
,, $G\sharp_{\mathsf{I}}$ =	200	400	600	800	1000	1200
				32	24	80

Partials of $G\sharp_{\mathsf{I}}$ =		200	400		600	800
,, C_{I} =	128	256	384	512	640	768
			16		40	32

On reference to the table on page 159, it will be found, that the beats that are not calculated above, are beyond beating distance; but the 32, 24, and 80 beats per second of the first interval, and the 16, 40, and 32 of the second, form very harsh dissonances; and moreover in the second interval, the 16 beats per second will usually be very prominent, as they are between a 2nd and a 3rd partial.

The Interval $\Big\}{se_1 \atop f_1}$ or $\Big\}{G\sharp_1 \atop F_1}$ and its inversion $\Big\}{f_1 \atop se_2}$ or $\Big\}{F_1 \atop G\sharp_2}$ are of frequent use in music. Taking $G\sharp_1 = 200$ as before, the vibration number of F will be $170\frac{2}{3}$; therefore

Partials of $G\sharp_1 =$	200 ,	400 ,		600 ,	800 ,	1000
,, $F_1 =$	$170\frac{2}{3}$,	$341\frac{1}{3}$,	512 ,	$682\frac{2}{3}$	$853\frac{1}{3}$,	1024
	$29\frac{1}{3}$	$58\frac{2}{3}$		$82\frac{2}{3}$	$53\frac{1}{3}$	24

Partials of $F_1 =$		$170\frac{2}{3}$,	$341\frac{1}{3}$,		512 ,	$682\frac{2}{3}$
,, $G\sharp_2 =$	100 ,	200 ,	300 ,	400 ,	500 ,	600
		$29\frac{1}{3}$	$41\frac{1}{3}$		12	$82\frac{2}{3}$

From the table on page 159 we see that the $29\frac{1}{3}$, $58\frac{2}{3}$, and $82\frac{2}{3}$ beats per second are slightly within or just on the verge of the beating distance. In the first Interval, the $53\frac{1}{3}$ beats per second will produce a harsh discord, as will also the 24 per second, though as they only arise between soft 5th and 6th partials, they have no very great intensity. The inversion is the worse of the two, the $41\frac{1}{3}$ beats per second between a 3rd and a 2nd partial, forming a bad and somewhat prominent dissonance, which is made still worse by the 12 and and $82\frac{2}{3}$ beats per second higher up.

As already remarked, the above results must be modified for Compound Tones, which do not possess all the first six partials.

(4) *Beats arising between Compound Tones, through the occurrence of Differentials.* The Fundamentals being Simple Tones, the conclusions we arrived at concerning the beats due to the Differentials, generated by simple tones, in the former part of this chapter, at once apply to them. It only remains to ascertain the effects, due to the Differentials, generated by the Overtones.

Differential Tones are only produced when the generators are pretty loud, therefore we shall not go beyond the 2nd or 3rd partial. as those above rarely have any considerable intensity. Moreover it will not be necessary to consider any differentials except those of the 1st order; differentials of the 2nd order seldom or never occuring between overtones. Let us consider the differentials of the 1st order generated between the partials of two compound tones, the fundamentals of which have, as we will suppose, the vibration numbers 200 and 304. Then the numbers in the first horizontal line of the following table are the partials of the former tone, and those in the first vertical column those of the latter. At their intersections are found the differences of these numbers; that is, the vibration numbers of the differentials due to them.

	200	400	600
304 608	104 408	96 208	296 8

If we arrange these tones in the order of their pitch, omitting the 8, which of course produces no tone at all, we have the groups:—

104	208	304	408
96	200	296	400
8	8	8	8

The difference between each pair is 8, and this is the only number of beats per second, which will be produced by all these differential tones; for the difference between any other two of the above numbers, gives too great a number of beats per second to be perceptible at the pitch of the tones that produce them.

Now let us ascertain the number of beats per second that will be generated by direct action between these same partials:

Partials of lower tone	200	400	600
,, higher ,,		304	608
Beats per second		96	8

Ninety-six beats per second, at the pitch of 400, is far beyond beating distance; 8 beats per second therefore is the number due to the direct action of overtones, that is, the same number which we found to arise from the action of the differentials. A like result will be obtained whatever numbers are selected for the fundamentals, so that in general, "dissonance due to combination tones produced between overtones, never exists, except where it is already present by virtue of direct action among the overtones themselves."

Thus we may pass on to the last cause of beats between compound tones, viz:—

(5) *Beats due to Summation Tones.* Summation Tones, though certainly not very loud, are much louder than they are commonly supposed to be. On the Harmonium and American Organ, Summation Tones, generated by any pair of tones on the lower half of the key-board, may be readily heard without the use of resonators, and therefore cannot but have some effect on the resulting sound.

From the table at the end of Chap. XII, page 134, we see that the fundamentals $\left\}\begin{smallmatrix}\mathbf{d}\\ \mathbf{d}_{1}\end{smallmatrix}\right.$ generate the Summation Tone ($\mathbf{s}$) which coincides with the 3rd partial of the lower tone, so that in the case of the Octave, no new element is introduced.

In the Fifth $\left\}\begin{smallmatrix}\mathbf{s}_{1}\\ \mathbf{d}_{1}\end{smallmatrix}\right.$, the Summation Tone ($\mathbf{m}$) is a new tone introduced between the 2nd and 3rd partials of the ($\mathbf{d}_{1}$), but it forms no dissonance with them.

In the Fourth $\left\}\begin{smallmatrix}\mathbf{d}\\ \mathbf{s}_{1}\end{smallmatrix}\right.$, the Summation Tone (approximately **ta**) comes very near the Beating Distance with the 2nd partial of **d**. For, taking $\mathbf{d} = 400$, then $\mathbf{s}_{1} = 300$, the Summation Tone $= 300 + 400 = 700$, and 2nd partial of $\mathbf{d}^{1} = 800$; therefore the Summation Tone and ($\mathbf{d}^{1}$) will produce $800 - 700 = 100$ beats per second, which, as may be seen from the table on page 159, is just beyond beating distance at this pitch.

In the Major Third $\left\}\begin{smallmatrix}\mathbf{m}_{1}\\ \mathbf{d}_{1}\end{smallmatrix}\right.$, the Summation Tone ($\mathbf{r}$) will dissonate with the 2nd partials ($\mathbf{d}$) and ($\mathbf{m}$) of both tones; the same is true in the case of the Minor Third, but the dissonance is harsher: for take $\mathbf{s}_{1} = 300$; then $\mathbf{m}_{1} = 250$, the Summation Tone is $300 + 250 = 550$, and the 2nd partials ($\mathbf{s}$) and ($\mathbf{m}$) are 600 and 500 respectively. Thus the number of beats per second is $600 - 550 = 50$, and $550 - 500 = 50$, which, at this pitch, is less than the number due to the whole tone.

In the Major Sixth, $\left\}\begin{smallmatrix}\mathbf{l}_{1}\\ \mathbf{d}_{1}\end{smallmatrix}\right.$, the Summation Tone ($\mathbf{f}$) dissonates at the interval of a tone, with the 3rd partial of the ($\mathbf{d}_{1}$). The Minor Sixth, $\left\}\begin{smallmatrix}\mathbf{d}\\ \mathbf{m}_{1}\end{smallmatrix}\right.$, is better in this respect, for take $\mathbf{d} = 400$, then $\mathbf{m}_{1} = 250$, Summation Tone $= 650$, which with the 3rd partial $\mathbf{t}$ ($= 750$) will produce $750 - 650 = 100$ beats per second, which at this pitch is only just on the borders of the Beating Distance.

The Summation Tone, when present, renders a primary dissonance between Compound Tones harsher than it otherwise would be. Take $\}\,{}^{\mathbf{r}_{\mathsf{I}}}_{\mathbf{d}_{\mathsf{I}}}$ for example: let $\mathbf{d}_{\mathsf{I}} = 80$ then $\mathbf{r}_{\mathsf{I}} = 90$ and the Summation Tone will be $80 + 90 = 170$, a tone about midway between the 2nd partials, **d** (= 160) and **r** (= 180).

Summary.

Beats are the source of all discord in music.

Starting with two simple tones in unison; if one of them be put slightly out of tune, slow beats will be heard, which are not very unpleasant, as long as they do not exceed one or two per second. On increasing the interval between the two tones, the beats gradually become more and more rapid, and at length form a harsh dissonance. If this interval be gradually increased, a point is finally reached, where all dissonance vanishes. The interval at which the dissonance *just* disappears, is termed the *Beating Distance.*

The harshness of any particular dissonance, depends *partly* upon

(1) *the rapidity of the beats*, and partly upon

(2) *the region of Pitch in which the dissonance lies.*

Similarly, the Beating Distance for Simple Tones varies in different parts of the realm of pitch, from a *Tone* at $C^2 = 1024$ to a *Major Third* at $C_2 = 64$.

Dissonance may arise between Simple Tones beyond Beating Distance, from the *occurrence of Differentials.*

A Compound Tone may be dissonant or harsh in itself, if it contain very high and loud partials.

Dissonance between *Compound Tones* may arise,

(1) From beats between fundamentals,

(2) ,, ,, the *fundamental* of one tone and an *overtone* of the other,

(3) From beats between *overtones* only,

(4) From beats due to *Differential Tones*,

(5) ,, ,, ,, *Summation Tones.*

CHAPTER XV.

The Definition of the Consonant Intervals.

We have seen in Chap. V, how, by means of the Double Syren, it may be proved, that, for two sounds to be at the exact interval given in the first column below, their vibration numbers must be in the exact ratio of the numbers given in the second column.

Interval.	Ratio.
Octave	2 : 1
Fifth	3 : 2
Fourth	4 : 3
Major Third	5 : 4
Minor Third	6 : 5
&c.	&c.

If the vibration numbers are not in the exact ratio given above, the interval will be perceptibly out of tune. This fact had been ascertained long before the instrument just referred to was invented, by the actual counting of the vibration numbers. Ingenious, but unsatisfactory theories, of a more or less metaphysical nature (among which, that of Euler held sway for many years), were devised to account for this remarkable fact. Its true explanation, as given below, is due to Helmholtz.

We commence as usual with Simple Tones, and first with the *Octave*. Let two Simple Tones be sounded together, the vibration numbers of which are in the ratio of 2 : 1, say 200 and 100 respectively. They will generate a Differential Tone, the vibration number of which will be $200 - 100 = 100$, which Differential Tone will therefore coalesce and be indistinguishable from the lower of the two Simple Tones. This identity in pitch, of the Differential, and the lower of the Simple Tones will always occur, provided the ratio of the two tones is as 2 : 1; for

let $2n$ be the vibration number of the upper tone,
then n will be ,, ,, ,, lower ,,
consequently $2n - n = n$,, ,, ,, Differential ,,

If, however, the exact ratio be not preserved, the lower tone and the Differential will not coincide, and beats will be heard between them. For example, let the vibration numbers of the two tones be 200 and 99 respectively; then

Vib. No. of	upper tone	$= 200$
,, ,,	lower ,,	$= \ \ 99$
,, ,,	Diffl. ,,	$= 101$

and therefore $101 - 99 = 2$ beats per second will be heard.

We might therefore define an Octave between two Simple Tones, as that Interval at which the Differential generated by them coincides in pitch with the lower of the two tones; and we see that this perfect coincidence can only occur, when the ratio between the vibration numbers of the two tones is exactly $2 : 1$.

In the example given above, if we had taken 200 and 98 as the respective vibration numbers, that of the Differential would have been $200 - 98 = 102$, which would have given 4 beats per second with the lower tone; from which it is evident, that the more the interval is out of tune, the greater is the number of beats produced. Thus in order to tune two Simple Tones to an exact Octave, after tuning them approximately, one of them must be sharpened or flattened more and more, till the beats becoming less and less, finally vanish. This is an entirely mechanical operation and does not even need a musical ear. For suppose two forks give a false octave, producing beats, and it is required to tune the upper one to a true octave with the lower. Sharpen the former slightly and sound them again; if the beats are more rapid than before, then the higher fork was already too sharp and must be flattened gradually till the beats disappear; if on the other hand they are slower, the fork is too flat, and must be sharpened in a similar manner.

Fifth. Let $3n$ and $2n$ be the vibration numbers of two Simple Tones at this interval. Then

$3n - 2n = n$.... vib. no. of Differential of 1st order.
and $2n - n = n$.... ,, ,, ,, 2nd ,,

Thus a Fifth between Simple Tones is defined by the coincidence of Differentials of the 1st and 2nd order; and this coincidence can evidently only occur, when the ratio of the vibration numbers of the Simple Tones is as $3 : 2$. Differentials of the 2nd order are, however, generally weak, so that this interval between Simple Tones is by no means well defined.

If the ratio is not exactly that of 3 : 2, beats are generated. For instance, let the vibration numbers of the two tones be 300 and 201 respectively, then

$300 - 201 = 99$ .. Differential of 1st order.
$201 - 99 = 102$ .. ,, 2nd ,,

$102 - 99 = 3$ beats per second being produced. The more the tones are out of tune, the greater the rapidity of the beats; so that to tune the interval, one tone must be sharpened or flattened gradually, as the rapidity of the beats decreases, until they vanish altogether.

Fourth. Let $4n$ and $3n$ be the vibration numbers of two Simple Tones at this interval. Then

$4n - 3n = n$ .. vib. no. of Differential of 1st order
$3n - n = 2n$ .. ,, ,, ,, 2nd ,,
$\left.\begin{array}{l}4n - 2n = 2n\\ 2n - n = n\end{array}\right\}$ .. ,, ,, ,, 3rd ,,

A Fourth between Simple Tones, therefore, is only defined by the coincidence of Differentials of the 1st and 3rd, and of the 2nd and 3rd order. Inasmuch, however, as a 3rd Differential can only be heard under extremely favourable circumstances, this interval can scarcely be said to be defined at all. This is still more the case with the Thirds, the definition of which, in the case of Simple Tones, depends upon the existence of Differentials of the 4th order. Accordingly it is found, as stated before, that in the case of Simple Tones, intervals of any magnitude intermediate between a Minor Third and a Fourth, are usually of equal smoothness. For the same reason, it is impossible without extraneous aid to tune two Simple Tones to the exact interval of a Third, either Major or Minor; there is no check: they have no definition.

If, however, more than two Simple tones be employed, it becomes easy to tune these intervals. Indeed, it is better to tune the Fifth also by the aid of a third Tone; for, as we have seen, the interval of the Fifth alone, is only guarded by a Differential of the 2nd order; while if the Octave of the lower tone be present, a Differential of the 1st order becomes available. Suppose for example the vibration numbers of three Simple Tones be 200, 301, and 400 respectively, the 5th, 301, being mistuned, then

$301 - 200 = 101$ .. Differential of 1st order,
$400 - 301 = 99$ .. ,, ,, 1st ,,

$101 - 99 = 2$ beats per second being thus produced. Thus by flattening the middle note, till these beats vanish, we may obtain a perfect fifth.

Similarly, to rectify a mistuned Fourth, 301 and 400, for example, we may take a tone, 200, an octave below the higher one, and proceed as in the above case.

Again, in the case of a false Major Third, say 400 and 501, tune a Simple Tone 600 a perfect Fifth from the lower tone. Then

$501 - 400 = 101$ .. Differential of 1st order,
$600 - 501 = 99$ .. ,, ,, 1st ,,

$101 - 99 = 2$ beats being heard between Differentials of 1st order. Tune as before till the beats disappear.

Similarly in the case of the mistuned Minor Third, 600 and 501, take a third tone 400, a true Fifth below the higher tone, and proceed as above.

We come now to the definition of Intervals between Compound Tones, and in the first place we shall assume the Compound Tones in question to be such as are produced by the Human Voice, Harmonium, Piano, and stringed instruments in general; that is to say, we shall suppose them to consist of, at least, the first six partials.

Octave. Let the vibration numbers of the fundamentals of two Compound Tones, at the interval of an octave, be n and $2n$ respectively. Then the 2nd partial of the former will be $2n$ which will thus coincide with the other fundamental; or, in musical language

2nd partial.. d' ——— d' ..Fundamental.

Fundamental.. d

If the ratio of the vibration numbers be not exactly as 2 : 1, beats will be heard between the 2nd partial of one tone and the fundamental of the other. Suppose, for example, that the vibration numbers are 200 and 99. Then

2nd partial..198 ——— 200..Fundamental

Fundamental..99

and thus $200 - 198 = 2$ beats per second will be heard.

Inasmuch as these Fundamentals are Simple Tones, all that has been said above about the latter apply to the former; moreover, it will be noted, that in the case just taken, the number of beats due to the 2nd partial, viz. 2, is the same as that due to the Combination Tone of the 1st order (see page 174), and it is evident that this must always be the case.

An Octave between Compound Tones, therefore is defined,

1st, by the coincidence of the Differential Tone, generated between their two Fundamentals, with the lower of the Fundamentals; and

2nd, by the coincidence of one of the Fundamental Tones with the 2nd partial of the other.

These coincidences it is plain can only occur when the vibration numbers of the Fundamentals are in the exact ratio of 2 : 1. Consequently, this explains why this exact ratio is necessary to the perfection of this interval.

To tune the Octave is thus a very easy matter: the mere mechanical process, of altering the pitch of one tone, till all beats vanish. As this interval is so well defined, great accuracy in its tuning is necessary, the slightest error becoming evident to the ear in the form of beats.

Fifth. Let the vibration numbers of two Compound Tones at this interval be $3n$ and $2n$ respectively. Then

3rd partial.... $6n$ —— $6n$ .... 2nd partial

2nd ,, ... $4n$

$3n$ .... Fundamental

Fundamental.... $2n$

the 2nd partial of the former will exactly coincide in pitch with the 3rd of the latter; or musically

3rd partial....s————s....2nd partial

2nd ,, ...d

s_{I}....Fundamental

Fundamental....d_{I}

A Fifth between Compound Tones, therefore, though also guarded by Differentials, is chiefly defined by the coincidence of the 3rd partial of the lower, with the 2nd of the upper tone, and this coincidence can evidently only happen, when the vibration numbers of the Fundamentals are in the ratio of 2 : 3. If the vibration numbers vary from this ratio, beats will be heard between these partials. For example, let the vibration numbers be 201 and 300 respectively, then

3rd partial.... 603————600 ....2nd partial

2nd ,, ...402

300....Fundamental

Fundamental....201

and 603 — 600 = 3 beats per second will be heard. To tune a false Fifth, therefore, one of the tones must be altered, till these beats vanish.

Inasmuch as the definition of a Fifth depends upon the coincidence of 2nd and 3rd partials, while the definition of an Octave depends upon that of 1st and 2nd partials, we see that beats from a mistuned Fifth will not usually be so powerful as those from a mistuned Octave; that is to say, the same rigorous exactitude in tuning, which the octave demands, is not so essential in the case of the Fifth. As an illustration of this fact, it may be mentioned, that while the Octave is preserved intact, in all systems of temperament, the Fifth is always more or less tampered with.

Fourth. Let the vibration numbers of two Compound Tones at this interval be $3n$ and $4n$ respectively. Then, the 4th partial of the former will exactly coincide with the 3rd partial of the latter, thus,

4th partial.... $12n$	$12n$ ....3rd partial
3rd ,, ...$9n$	
	$8n$....2nd ,,
2nd ,, ...$6n$	
	$4n$....Fundamental
Fundamental...$3n$	

or in musical language, calling the Fundamentals $\mathbf{d}_{|}$ and $\mathbf{f}_{|}$

4th partial....d'	d'....3rd partial
3rd ,, ...s	
	f ....2nd ,,
2nd ,, ...d	
	$\mathbf{f}_{|}$....Fundamental
Fundamental....$\mathbf{d}_{|}$	

Thus a Fourth, between Compound Tones, is defined by the coincidence of 3rd and 4th partials, and for exact coincidence, it is obvious that the vibration numbers of the Fundamentals must be in the ratio 4 : 3. If they are not exactly in this ratio the inaccuracy will manifest itself in the form of beats. Let them be 400 and 301 for example: then

4th partial....	1204 ——	1200	...3rd partial
3rd ,, ...	903		
		800	....2nd ,,
2nd ,, ...	602		
		400	....Fundamental
Fundamental....	301		

4 beats per second will be produced.

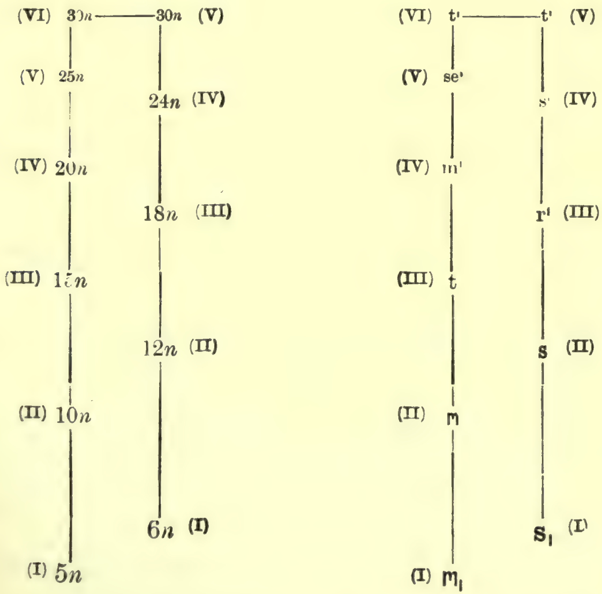
A Fourth is not so well defined as a Fifth, for not only are the coincident partials of a higher order, and therefore not so prominent in the former case, but also the dissonance between the 3rd and 2nd partials (**s** and **f** in the above) masks, to a certain extent, the beats between the 3rd and 4th partials of this interval, when not exactly in tune.

Major Third. Let $\mathrm{d}_{|} = 4n$ and $\mathrm{m}_{|} = 5n$ be the vibration numbers of two Compound Tones at this interval; then,

(V)	$20n$ ——	$20n$	(IV)		(V)	m' ——	m'	(IV)
(IV)	$16n$				(IV)	d'		
		$15n$	(III)	or			t	(III)
(III)	$12n$				(III)	s		
		$10n$	(II)				m	(II)
(II)	$8n$				(II)	d		
		$5n$	(I)				$\mathrm{m}_{	}$	(I)
(I)	$4n$				(I)	$\mathrm{d}_{	}$		

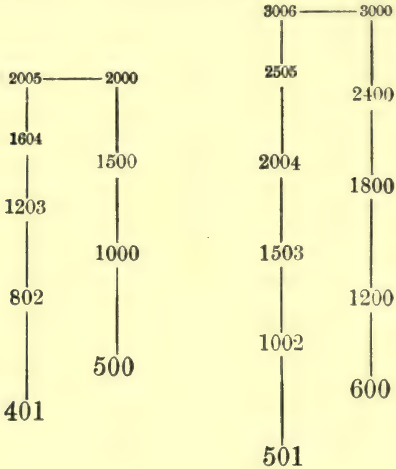
the 5th partial of the lower tone will exactly coincide with the 4th of the higher one. Thus a Major Third is even more ill-defined than a Fourth, the coincidence being between higher, and therefore usually weaker partials, and being masked more or less by the harsh dissonance of a semitone between the more powerful 3rd and 4th partials ($\mathbf{d}^{|}$ and $\mathbf{t}$ above).

Minor Third. Let $\mathbf{s}_{|} = 6n$ and $\mathbf{m}_{|} = 5n$ be the vibration numbers of two Compound Tones at this interval: then,



the 6th partial of the lower tone will coincide with the 5th of the higher. The Minor Third is still less defined, therefore, than the Major Third; the coincident partials being of a higher order, and obscured not only by the semitone dissonance between the 4th and 5th partials ($\mathbf{s}^{|}$ and $\mathbf{se}^{|}$ above) but by the tone dissonance between the 3rd and 4th ($\mathbf{r}^{|}$ and $\mathbf{m}^{|}$).

For a given departure from the exact ratios, the beats are more rapid in the case of the Thirds, than in the preceding intervals; for example, let 401 and 500, and 501 and 600 be the vibration numbers of the fundamentals of a Major and Minor Third respectively: then,

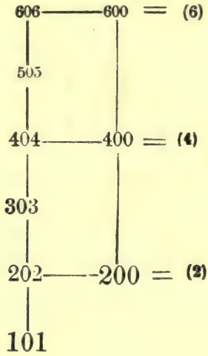


in the former case 5 beats and in the latter 6 beats per second will be produced.

As the intonation of the Thirds is guarded by such high, and therefore weak partials, a slight error in their tuning is much less evident, than in the case of the Fifth. Thus Thirds tuned in equal temperament are, as we shall see later on, mistuned to an extent, which if adopted with the Fifth, would render this latter interval unbearable.

Major and Minor Sixths. By pursuing the method adopted above, the student will find that the former of these two intervals between Compound Tones, is defined by the coincidence of the 3rd and 5th, and the latter by the coincidence of 5th and 8th partials. Inasmuch as the 8th partial is generally exceedingly weak, the Minor Sixth can scarcely be said to be defined at all.

In all the above intervals, we have only considered the lowest pair of coincident partials, as these are by far the most important; but it must not be forgotten, especially in the case of the Octave, and Fifth, that there are coincident pairs above those given. If the interval be not quite true, not only will beats be produced by this lowest pair, but by the higher also, and at a more rapid rate. Thus, let $\mathbf{d}_1 = 101$ and $\mathbf{d} = 200$: then,

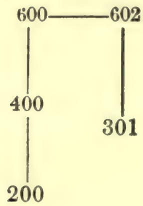


2 beats per second will be heard from the pair marked (2), 4 beats per second from that marked (4), and 6 from that marked (6).

In tuning Fifths, Thirds, &c., between Compound Tones with perfect exactness, a resonator tuned to the pitch of the coincident partials will be found of great service; for these partials being thus reinforced, it will be easy to discriminate any beats between them, from the beats of other partials; and furthermore the disturbing effect of any dissonating partials which may be present, will be much lessened.

It will be seen from the above, that the particular partials which coincide in any interval are given by the figures which denote its vibration ratio. Thus, the vibration ratio of the octave is 2 : 1, and the coincident partials are the 2nd and 1st; the vibration ratio of of the Fifth is 3 : 2, and the coincident partials are the 3rd and 2nd, and so on.

Furthermore, the preceding illustrations show, that in any particular interval, if the lower of the two tones is one vibration too sharp or too flat, the number of beats produced by the lowest pair of coincident partials is the same as the greater of the two numbers which denote its vibration ratio. Thus, in the case of the Major Third taken above,—401 and 500,—we found the number of beats per second to be 5, and that is the greater of the two numbers 5 : 4 which give its vibration ratio. Similarly, if the higher of the two tones of an interval be one vibration too sharp or too flat, the number of beats per second will be the smaller of the two numbers which denote its vibration ratio. For example, let the vibration numbers of a mistuned Fifth be 200 and 301 : Then



the number of beats per second will be $602 - 600 = 2$, which is the smaller of the two numbers, in the ratio, 3 : 2.

When the two Compound Tones forming an interval do not possess all the first six partial Tones, the above results require to be modified.

Thus for example, in wide open organ pipes, the tones of which consist of only the first two partials, the Octave is the only Interval which is defined by the coincidence of partials; the other Intervals being guarded merely by Differentials.

Again, in stopped organ pipes the tones of which only consist of the 1st and 3rd partials, the Twelfth is the only interval defined by the coincidence of the partials; the other intervals, even the Octave, being guarded by Differentials only.

In such cases as these, however, the definition is better than it would be if the tones were simple, more Differentials and those of a higher order being produced. For example, take the mistuned Major Third d = 400 and m = 501, and suppose each of these tones to consist of the first three partials only. Then the 1st horizontal line of the following table shows the partials of the one tone, and the 1st vertical line those of the other, the differentials of the 1st order being at the intersections.

	400	800	1200
501	101	299	699
1002	602	202	198
1503	1103	703	303

The following beats will be generated between these Differentials,

$$\left.\begin{array}{l} 202 - 198 = 4 \\ 303 - 299 = 4 \\ 703 - 699 = 4 \end{array}\right\} \text{ beats per second.}$$

The Major Third between tones consisting of the first three partials is guarded therefore by three sets of Differential Tones of the 1st order.

Summary of Definition of Intervals.

Simple Tones.

Octave.	1st Differential in unison with lower tone.				
Fifth.	1st	,,	,,	,,	2nd Differential.
Fourth.	1st	,,	,,	,,	3rd ,,

Any departure from true intonation produces beats between these unisons.

Other intervals practically undefined.

Ordinary Compound Tones.

The *Octave, Fifth and Fourth* defined as above, but also and chiefly as follows,

Octave.	2nd partial of lower tone unisonant with 1st partial of higher.							
Fifth.	3rd	,,	,,	,,	,,	,, 2nd	,,	,,
Fourth.	4th	,,	,,	,,	,,	,, 3rd	,,	,,
Major Third.	5th	,,	,,	,,	,,	,, 4th	,,	,,
Minor Third.	6th	,,	,,	,,	,,	,, 5th	,,	,,

and generally, in any interval the unisonant or defining partials are given by the numbers which denote its vibration ratio.

If the *lower* of the two tones of any interval be out of tune by 1 vibration per second, the number of beats generated (by lowest pair of defining partials) is the same as the *greater* of the two numbers which denote its vibration ratio; if the *higher* tone be out of tune by the same amount, the number of beats is the *smaller* of these two numbers.

CHAPTER XVI.

On the Relative Harmoniousness of the Consonant Intervals.

We have now to examine into the causes of the relative smoothness of those intervals which are usually called consonant.

With regard to perfectly simple tones, there is, as we have already seen, no element of roughness in any of these intervals, except in the case of the Thirds, and in these only when very low in pitch; consequently, there is found to be little or no difference in smoothness, between any of these intervals, when strictly Simple Tones are employed, and when the tones in question are in perfect tune.

With Compound Tones, however, the case is very different: not only do these intervals vary in smoothness—in harmoniousness—one with another, but the smoothness of any one particular interval varies according to the constitution, that is the quality, of its Compound Tones.

In the first instance, we shall consider these intervals as formed between Compound Tones, each consisting of the first six partials; and as before, we shall suppose, as is generally the case, that the intensity of these partials rapidly diminishes as we ascend in the series. In fig. 80 we have the ordinary Consonant Intervals, together with a few others, drawn out so as to show the first five or six partials of each tone. To facilitate comparison, the lower of the two tones in each interval, is supposed to be of the same pitch throughout, so that tones on the same horizontal lines are of the same pitch. The symbols for the partials diminish in size, as they rise above the fundamental, in order to represent roughly their diminution in intensity. As before, partials forming a tone dissonance are connected by a single line, those that dissonate at a semitone are joined by a double one. In comparing the intervals of the figure, it must be borne in mind, not only that the beats of the semitone are much worse than those of the tone, but also that these vary in themselves—the beats of the $\frac{9}{8}$ tone ($\left\{\begin{matrix}\mathbf{r}\\\mathbf{d}\end{matrix}\right.$ for example) not being so harsh as those of the $\frac{10}{9}$ tone ($\left\{\begin{matrix}\mathbf{m}\\\mathbf{r}\end{matrix}\right.$), nor those of the $\frac{16}{15}$ semitone ($\left\{\begin{matrix}\mathbf{d'}\\\mathbf{t}\end{matrix}\right.$), usually so discordant, as those of the $\frac{25}{24}$ semitone ($\left\{\begin{matrix}\mathbf{se}\\\mathbf{s}\end{matrix}\right.$). The small letters or asterisks in curved brackets show the positions of the Summation Tones generated by the fundamentals.

The facts thus summarized in fig. 80, will be found, on careful examination, to throw light on several fundamental phenomena in harmony relating to these intervals.

In the first place, it will be at once seen that with regard to Compound Tones such as those depicted, the Octave is the only perfectly consonant interval, that is, the only one absolutely free from roughness. Moreover, the student will readily perceive, that no roughness, except such as may be inherent in the tones themselves, can ever occur between two Compound Tones, at this interval, no matter what their constitution may be; for the higher of the two tones only adds to the lower one, elements which are already present.

The fact that the Octave is the only Interval devoid of all roughness, explains why this interval is the only one that can be used in all regions of the musical scale, on all instruments. Again the fact, that the Compound upper tone of the Octave, adds nothing new, but simply reinforces tones already present in the Compound lower tone, explains the similarity in effect of the two tones forming an Octave. We can thus understand, how it is that a company of men and women totally unskilled in music, and utterly unable to sing in Thirds, &c., yet experience no difficulty in singing together a tune in Octaves, and indeed when doing so usually consider themselves to be singing tones of the same pitch; in fact, such singing is called, even by musicians, unison singing.

Again, we see why a part in music for the Pianoforte, Harmonium, &c., may be doubled with impunity; for such addition adds nothing absolutely new; it simply reinforces the upper partials of tones already present, thus producing a brighter effect.

The Fifth as constituted in fig. 80 is not always a perfectly Consonant Interval, for as the figure shows the 3rd partial of the upper compound tone, dissonates with both the 4th and 5th of the lower one. The degree of roughness thus produced, will depend upon the intensity of these partials, and inasmuch as they are usually faint, the roughness will be but slight. Other things being

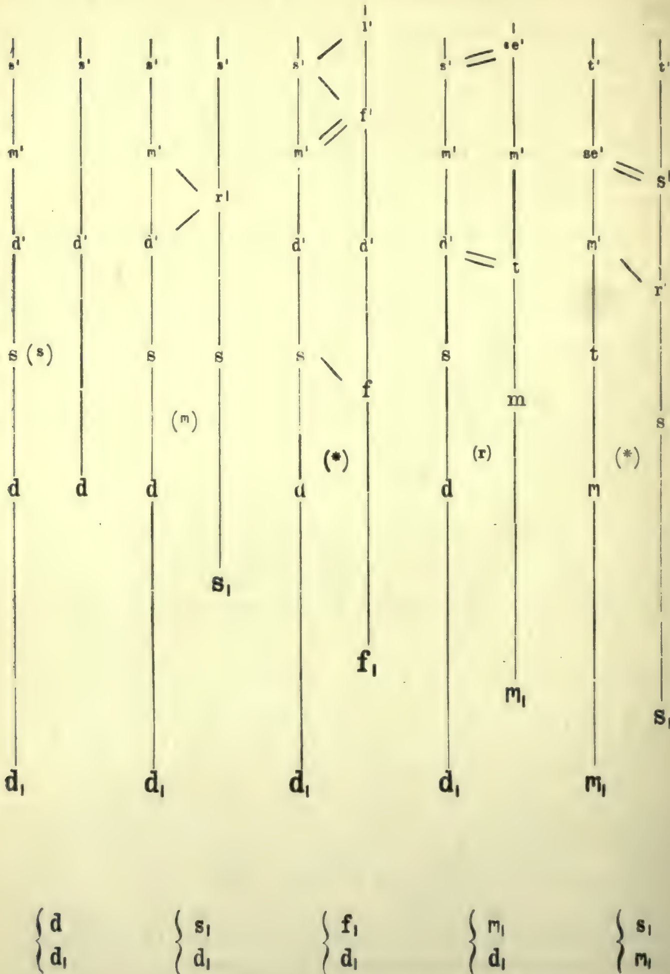

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Fig. 80.

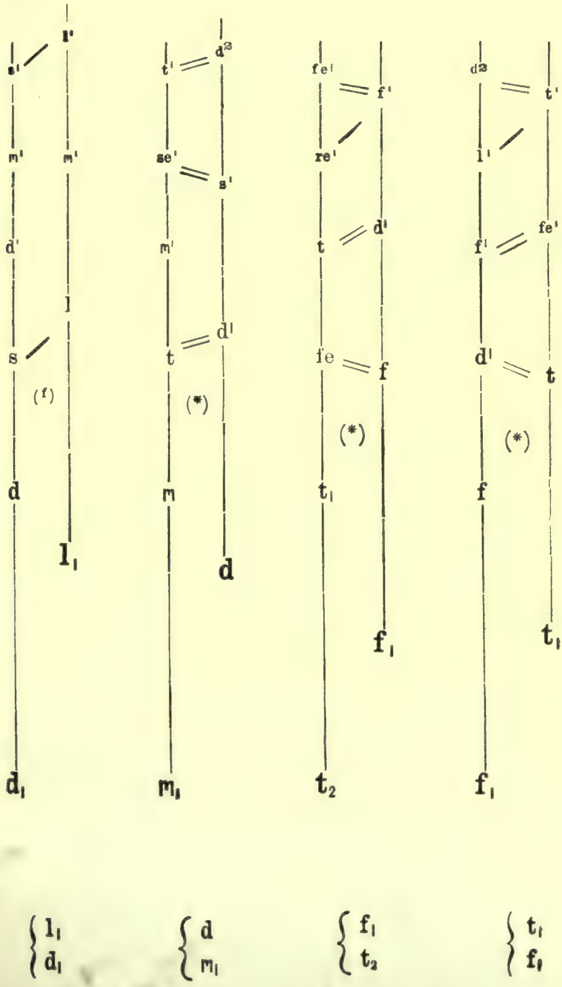


FIG. 80.

equal, the roughness of this interval will depend upon its position in the musical scale; such roughness becoming greater as we descend, and less as we ascend. Two reasons may be assigned for this; in the first place, the upper partials of low tones are usually stronger than those of higher ones, and consequently, when they beat with one another the beats are more intense, thus producing a harsher effect; secondly, partials that beat with one another in the lower part of the musical scale may be beyond beating distance in the upper part. To illustrate this fact, which of course applies to other intervals, we will take two or three cases of Fifths in different parts of the musical scale.

First, take $C = \mathbf{d} = 256$, then $\mathbf{s} = 384$.

$$\begin{array}{lr} \text{3rd partial of } \mathbf{s} = 384 \times 3 = & 1152 \\ \text{4th } \quad ,, \quad \mathbf{d} = 256 \times 4 = & 1024 \\ \hline \text{number of beats per second} = & 128 \end{array}$$

Now from the table on page 159, we know that 128 beats per second, in the neighbourhood of $C^2 = 1,024$, is only just within the beating distance; consequently we may conclude that fifths above middle C having the constitution assumed above, are devoid of all roughness whatever.

Next take $C_2 = \mathbf{d}_2 = 64$, then $\mathbf{s}_2 = 96$.

$$\begin{array}{lr} \text{3rd partial of } \mathbf{s}_2 = 96 \times 3 = & 288 \\ \text{4th } \quad ,, \quad \mathbf{d}_2 = 64 \times 4 = & 256 \\ \hline \text{number of beats per second} = & 32 \end{array}$$

From the table of page 159, we see that 32 beats per second in the region of $C = 256$ form a somewhat harsh dissonance. In fact, when C_2 and G_2 are strongly sounded on a harmonium, the harsh effect produced is due to the dissonating partials C and D, and consequently this harsh effect is about the same as that obtained by softly sounding the C and D digitals together,—a matter which can be easily put to the proof.

The above, therefore, explains the fact, that while an Octave may be played anywhere in the Musical Scale, a Fifth cannot be well used below a certain limit. On the other hand, we see that speaking generally, a Fifth is a perfectly consonant interval, when taken above middle C; we might therefore term this, the limit of a perfectly consonant Fifth on the Harmonium, Pianoforte, and stringed instruments in general.

A glance at fig. 80, shows that the Fourth is not so perfect an interval as the Fifth. Its roughness arises chiefly from the beats generated between the usually powerful 2nd partial of the upper tone, and the almost equally loud 3rd partial of the lower one. To this may be added the much softer semitone and tone dissonances between 4th, 5th, and 6th partials; and a still slighter disturbing element may be sometimes present in the Summation Tone midway between the 2nd partials. It may be noted also that the 2nd and 3rd pairs of dissonating partials, the 4th and 5th pairs, and also the Summation Tone, give rise to precisely the same number of beats.

As in the case of the Fifth, not only will the roughness of this interval vary with the varying intensities of dissonating partials, but other things being equal, with its position in the Musical Scale.

$$
\begin{array}{l}
\text{For example take } G = \mathbf{d} = 384, \text{ then } \mathbf{f} = 512 \\
\qquad \text{3rd partial of } \mathbf{d} = 384 \times 3 = 1152 \\
\qquad \text{2nd} \quad ,, \qquad \mathbf{f} = 512 \times 2 = 1024 \\
\hline
\qquad \text{number of beats per second} = \ \ 128
\end{array}
$$

which (see table, page 159) is only just within beating distance. It may be noticed that this number is the same that we obtained in the case of the Fifth $\left\{\begin{smallmatrix}G\\ \mathbf{C}\end{smallmatrix}\right.$, showing that in order to obtain a Fourth of approximately equal smoothness with a Fifth, we must take the former a Fifth higher in pitch. Thus using the term in the same sense as before, we might call this the lower limit of a perfectly consonant Fourth.

Coming now to the Thirds, we find in both Major and Minor, that the Third partial of the Upper Tone dissonates with the 4th partial of the lower one; but while in the latter they form only a tone dissonance, in the former they dissonate at the much more unpleasant interval of a semitone. On the other hand, while softer 4th and 5th partials respectively of the Minor Third beat at a semitone distance, the corresponding partials of the Major Third do not beat at all. Further, the Summation tones when present will add to the roughness; that of the Minor Third being slightly more detrimental than that of the Major.

The Thirds, in respect to their harmoniousness vary very greatly according to their position in the Musical Scale. They cannot be used very low in pitch, even when they are formed between Simple Tones: for as we have already seen the Thirds $\left\{\begin{smallmatrix}E_2\\ C_2\end{smallmatrix}\right.$ and $\left\{\begin{smallmatrix}E\flat_1\\ C_1\end{smallmatrix}\right.$ between Simple Tones are at the beating distance, that is, C_2 and C_1

are the limits respectively at, and below which, a Major and a Minor Third between Simple Tones, become dissonant. Thirds at or below these limits, between Compound Tones, contain of course these same elements of roughness, between their fundamentals; to which however must be added, the further roughnesses due to their beating overtones. Thirds, above these limits, must owe their roughness chiefly to beating overtones.

From fig. 80, we see that the smoothness of a Major or Minor Third between Compound Tones, above the limit just referred to, depends chiefly upon the loudness of the beats between 3rd, 4th, and 5th partials. Now observation shows that in the case of the Voice, Harmonium, and Piano, these partials generally become weak or even altogether absent above middle C; consequently, Thirds above this region, on these instruments will be as a rule sufficiently smooth. As we descend, however, from this region, Thirds rapidly deteriorate, for in the first place these partials begin to assert themselves, and secondly, the fundamentals are approximating to the beating distance.

To take an example:

$$\begin{array}{lrl} & \text{Let } C_{\text{I}} = 128, & \text{then } E_{\text{I}} = 160 \\ \text{4th partial of} & C_{\text{I}} = & 128 \times 4 = 512 \\ \text{3rd} \quad ,, & E_{\text{I}} = & 160 \times 3 = 480 \\ \hline \text{number of beats per second} & & = \ 32 \end{array}$$

and 32 beats per second, in the region of $C^{\text{I}} = 512$ are very harsh if at all prominent.

Again,

$$\begin{array}{lrl} & \text{Let } C_{\text{I}} = 128, & \text{then } E\flat_{\text{I}} = 153\frac{3}{5} \\ \text{4th partial of} & C_{\text{I}} = & 128 \times 4 = 512 \\ \text{3rd} \quad ,, & E\flat_{\text{I}} = & 153\frac{3}{5} \times 3 = 460\frac{4}{5} \\ \hline \text{number of beats per second} & & = \ 51\frac{1}{5} \end{array}$$

which is within beating distance in the region of $C^{\text{I}} = 512$. To this must be added, first, the roughness due to the $153\frac{3}{5} - 128 = 25\frac{3}{5}$, beats per second between the fundamentals, which are just about the beating distance, secondly, that due to the possible Summation Tone, and thirdly, that arising from the dissonant 5th and 4th partials.

It should be observed, that, though such 3rd and 4th partials may be absent or weak in tones which are produced softly, they may become very prominent in those tones when sung or played loudly; consequently a Third which may be perfectly smooth and harmonious when softly played or sung, may become rough and unpleasant when more loudly produced: a remark which evidently applies to other intervals also.

The foregoing explains, why Thirds were not admitted to the rank of consonances, until comparatively recent times. For the compass of men's voices (in respect to which, the music among classical nations was chiefly developed) lies chiefly below middle C, and as we have just seen, Thirds in the lower parts of that compass are actually dissonant.

We have, in the above, also, the explanation of the rule in harmony which forbids close intervals between the tenor and the bass, when these parts are low in pitch.

To sum up the comparative smoothness of the Thirds: we find that these intervals may be almost or quite devoid of roughness when somewhat high in pitch, and may even excel the Fourth in smoothness under these circumstances, but that they rapidly deteriorate, as we descend below middle C.

For Compound Tones of such constitution as depicted in fig. 80, the Major Sixth seems decidedly equal, if not slightly superior, to the Fourth. As in the case of the latter interval, the 2nd partial of its upper tone dissonates with the 3rd partial of the lower, at the interval of a tone, but the roughness due to dissonances between the 4th and 5th partials in the latter interval is wanting in the former. As a set off to this advantage, however, we see that the Summation Tone in the Major Sixth when present, produces a tone dissonance with the 3rd partial of the lower tone.

On the other hand, the Minor Sixth is the worst interval we have yet studied. Its chief roughness is due to the semitone dissonance between the 2nd partial of the upper and the 3rd partial of the lower tone, which are usually pretty loud. A subsidiary roughness is seen above between the 3rd, 4th, 5th and 6th partials.

As an example of the Major Sixth,

take $\mathbf{d} = 384$, then $\mathbf{l} = 640$

2nd partial of $\mathbf{l} = 640 \times 2 = 1280$

3rd ,, $\mathbf{d} = 384 \times 3 = 1152$

number of beats per second $= 128$

which we see from the table on page 159 is on the verge of the beating distance. G = 384 may be taken therefore, as the limit above which a Major Sixth is a perfectly smooth interval, and below which it gradually deteriorates.

On the other hand the Minor Sixth at this pitch has still an element of roughness, for

$$\begin{array}{llcl} & \text{let } \mathsf{m} = 384, \text{ then } \mathsf{d}^{\prime} = 614\tfrac{2}{5} & & \\ \text{2nd partial of} & \mathsf{d}^{\prime} = 614\tfrac{2}{5} & \times\, 2 = & 1228\tfrac{4}{5} \\ \text{3rd} \quad ,, & \mathsf{m} = 384 & \times\, 3 = & 1152 \\ \hline & \text{number of beats per second} = & & 76\tfrac{4}{5} \end{array}$$

which forms a harsh dissonance in this region.

On comparing the relative smoothness of the Fourth, Major Sixth, and Major Third, no valid reason appears for the precedence, which is usually granted to the first-named interval over the other two; and in fact no such precedence can be assigned to the Fourth, if the three intervals be judged by the ear alone, under similar circumstances. As Helmholtz remarks "the precedence given to the Fourth over the Major Sixth and Third, is due rather to its being the inversion of the Fifth, than to its own inherent harmoniousness."

In the Diminished Fifth and Augmented Fourth or Tritone the 2nd, 3rd, and 4th partials of the upper tone dissonate at the interval of a semitone with the 3rd, 4th, and 6th partials of the lower tone; after mentioning which elements of roughness, it is scarcely worth while to point out the tone dissonance between the 4th and 5th partials.

Although the above results apply fairly well to all instruments, the tones of which consist of the 1st six partials, to such instruments for example, as the pianoforte, harmonium, open pipes, of organs, and the human voice; yet it will not do to apply them, in a hard and fast manner, to any instrument whatever. The low tones of the harmonium, for example, especially if the instrument be loudly played, contain more than the 1st six partials, while those of open organ pipes, gently blown, often consist of fewer. Again, the tones of the human voice vary wonderfully in their constitution, not only in different voices, but also, and chiefly, according to the particular vowel sound produced. The influence which the vowel sounds have in modifying the roughness or smoothness of an interval, can best be realized by making a few experiments with men's voices. Let such intervals as the Major and Minor Thirds be

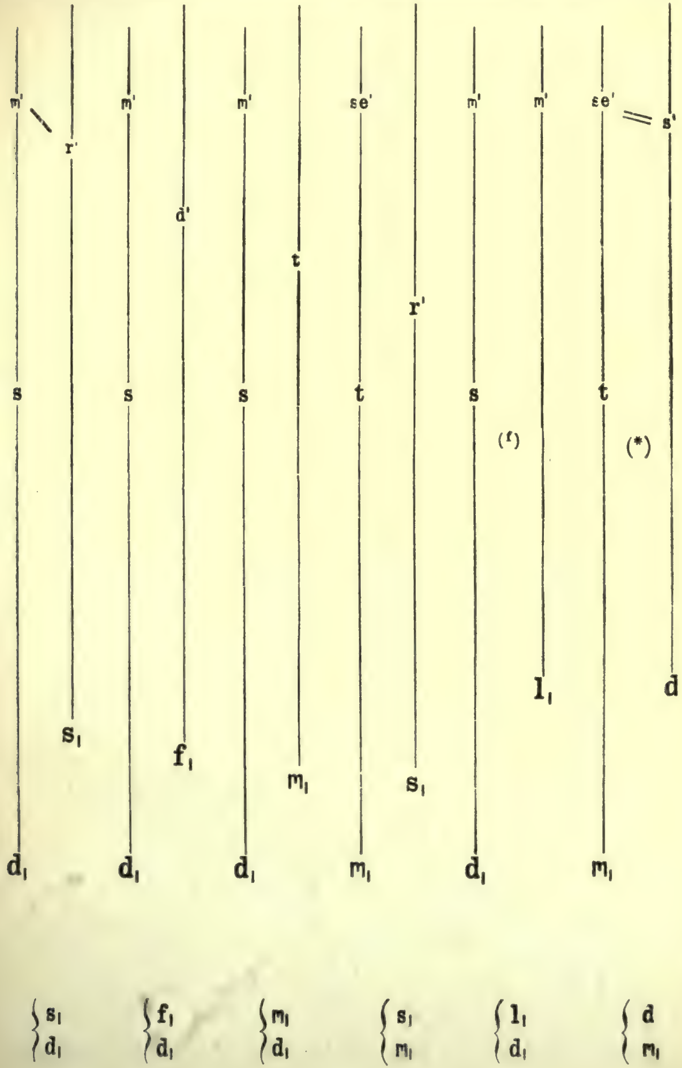


Fig. 81.

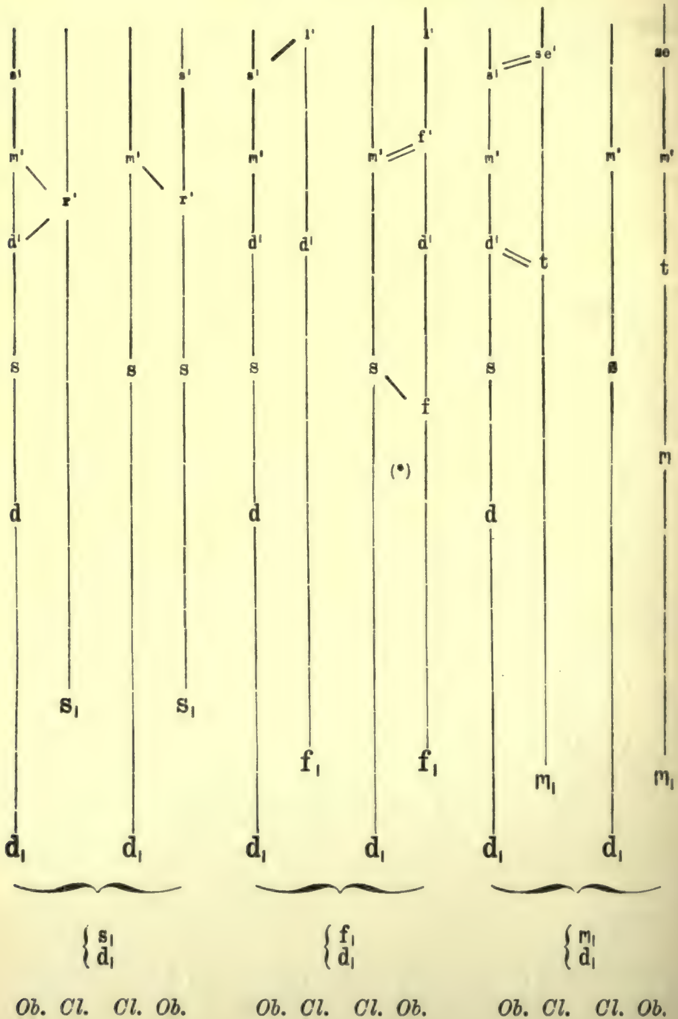


FIG. 82.

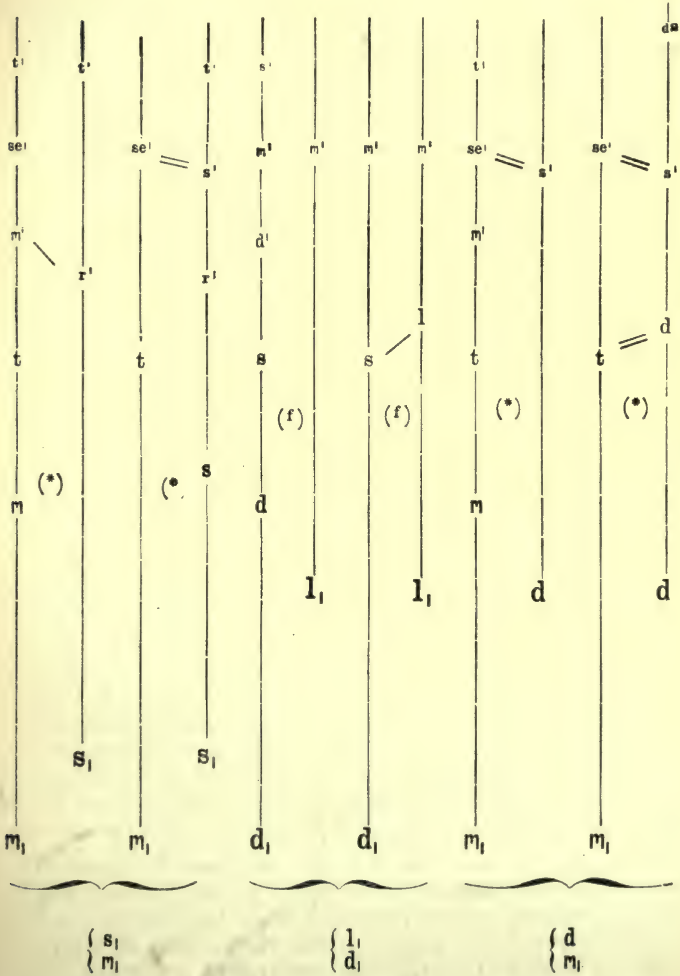


FIG. 82.

sounded by voices at different pitches below middle C, first on such vowel sounds as "a" in father, "i" in pine, and afterwards on "oo" in cool. The diminution in roughness in the latter case is very striking. The chief cause of the charm of soft singing, doubtless lies in the fact, that the upper dissonating partials of Thirds, Sixths, &c., become so faint, as to be practically nonexistent.

Intervals between Compound Tones consisting of the odd partials only,—such tones, for example, as are produced by the narrow stopped pipes of the organ and by the clarionet—may be more briefly noticed. Fig. 81 shows the ordinary consonant intervals between such tones, fully drawn out on the plan of fig. 80. The first thing that strikes us on looking at the figure, is the improvement noticeable in each interval; most of the dissonances of fig. 80 having vanished. The reader will be surprised, doubtless, by the apparent inferiority of the Fifth to most of the other intervals—to the Thirds—for example; but we must point out that it is for the most part only apparent. For we have already seen, that the Fifth is a perfectly smooth interval above middle C; consequently the Thirds can only be superior to the Fifth below that limit: and we have shown above that the Thirds rapidly deteriorate as they sink in pitch from that point, in consequence of their fundamentals approaching the Beating Distance.

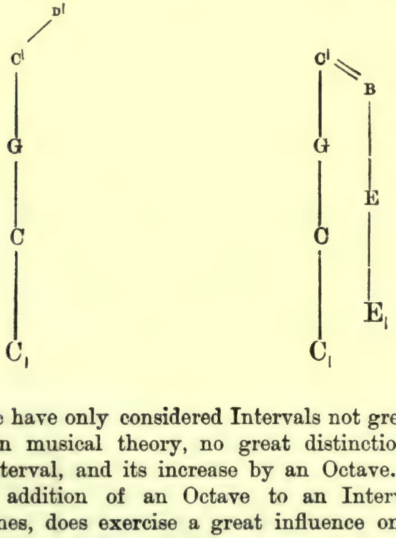
Another interesting case is that of Intervals between Compound Tones, one of which consists of only odd, and the other of the full scale of partials; such intervals as would be produced, for example, by a Clarinet sounding one tone and a Oboe the other. There will be two cases according as the lower tone is sounded on the former, or the latter instrument. Fig. 82 shows the ordinary consonant intervals, drawn out after the manner of the two preceding figures. Each interval is given twice: in those marked Ob Cl the lower tone of the interval is supposed to be sounded by the Oboe, while in those marked Cl Ob the Clarinet produces the lower tone.

It is evident at once, that it is not a matter of indifference, to which instrument the lower tone is assigned. The Fifth and Major Third are decidedly better when the lower tone is given to the the Clarinet; while the Fourth, Major Sixth, and Minor Sixth are smoother, when the Oboe takes the lower tone.

From the foregoing, it is quite clear that no hard and fast line can be drawn between Consonance and Dissonance; for as we have seen, every interval, between ordinarily constituted Compound

Tones, except the Octave, becomes a dissonance when taken sufficiently low in pitch. Furthermore, when the intervals of fig. 80 are taken in the same region of the musical scale, there is an uniform gradation of roughness, or diminution of smoothness in passing from the Perfect Fifth on the left, to the diminished Fifth on the right, which is usually looked upon as a dissonance. Again, with similar Compound Tones, and in the same region of pitch, some so-called dissonances are not inferior to intervals universally termed consonant. Compare for example,

$\left\{\begin{matrix} D^{1} \\ C_{1} \end{matrix}\right.$ & $\left\{\begin{matrix} E_{1} \\ C_{1} \end{matrix}\right.$



Hitherto, we have only considered Intervals not greater than an Octave, and in musical theory, no great distinction is drawn, between an interval, and its increase by an Octave. In reality, however, the addition of an Octave to an Interval, between Compound Tones, does exercise a great influence on its relative smoothness. Fig. 83 shows, on the same plan as before the Twelfth, Eleventh, Tenths, and Thirteenths.

The first point to be noted about these intervals is that the addition of an Octave to a Fifth, makes the Interval a perfect one; the addition of the Compound Tone (**s**) to the Compound Tone ($\mathbf{d}_1$) supplying no new partial tone to the latter. A Twelfth is therefore decidedly superior to a Fifth. On the other hand by comparing the Eleventh $\left\{\begin{matrix} \mathbf{f} \\ \mathbf{d}_1 \end{matrix}\right.$ of fig. 83 with the Fourth $\left\{\begin{matrix} \mathbf{f}_1 \\ \mathbf{d}_1 \end{matrix}\right.$ of fig. 80 it will be seen that the former is the worse of the two: for though the

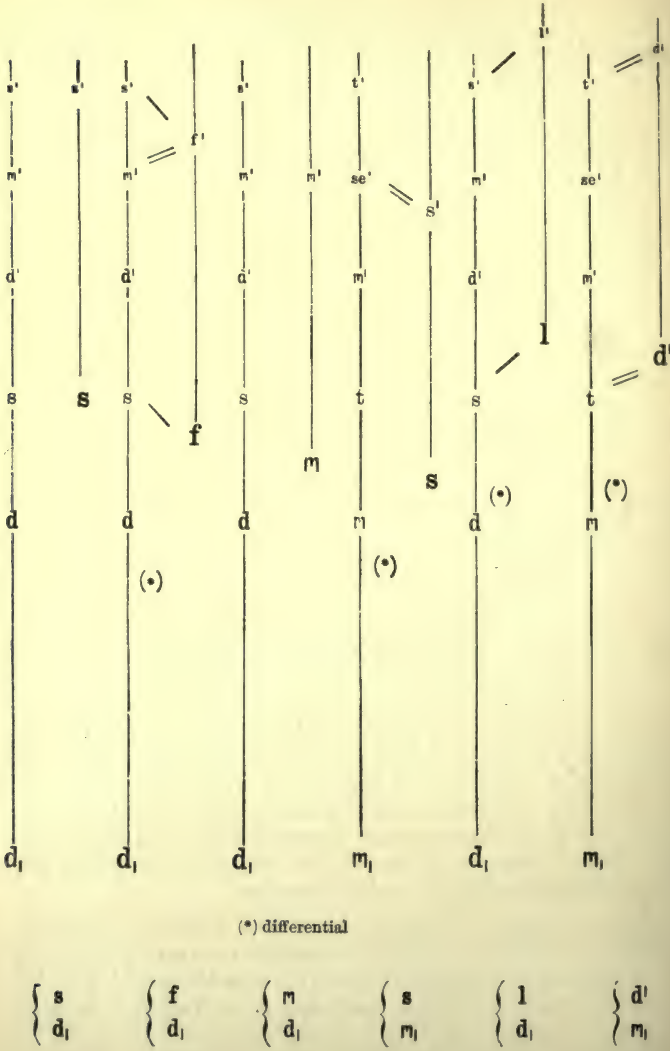


FIG. 83.

dissonances are at the same interval in each; in the latter, it is the dissonance of a 3rd partial against a 2nd, while in the former it is the dissonance of a 3rd partial against a Fundamental. Similarly the Major and Minor Thirteenths are inferior to the Major and Minor Sixths.

The Major Tenth however is greatly superior to the Major Third, the 3rd and 4th partial dissonance $\left\{ {\mathbf{d}' \atop \mathbf{t}} \right.$ of the latter, being absent in the former. With regard to the Minor Tenth and Minor Third, although in the former, the 3rd and 4th partial dissonance $\left\{ {\mathbf{m} \atop \mathbf{r}} \right.$ of the latter has disappeared, yet the semitone dissonance $\left\{ {\mathbf{se} \atop \mathbf{s}} \right.$ being now between 5th and 2nd partials, will be much more prominent in the former than in the latter, where it only occurs between 5th and 4th partials. Under these circumstances it is difficult to say which is the better interval of the two. Helmholtz holds that the Minor Third is the superior, and so obtains the following symmetrical rule to meet all cases:—

"Those intervals, in which the smaller of the two numbers expressing the ratios of the vibration numbers is odd, are made worse by having the upper tone raised an Octave;" while

"Those intervals, in which the smaller of the two numbers expressing the ratios of the vibration numbers is even, are improved by having the upper tone raised an Octave."

We conclude the present Chapter, by giving in fig. 84, Helmholtz's, graphic representation of the relative harmoniousness of musical intervals.

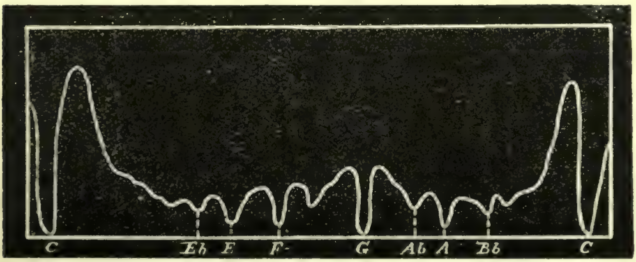


Fig. 84.

In this figure, the intervals are represented by the horizontal distances CE♭, CE, &c., measured from the point C; while the roughness of the intervals is shown by the vertical distances of the curved line from the corresponding points E2, E, &c., on the horizontal line. For example; the roughness of the interval $\left\{\begin{smallmatrix}E2\\C\end{smallmatrix}\right.$ is represented by the length of the vertical line over the point E2; the roughness of the interval $\left\{\begin{smallmatrix}E\\C\end{smallmatrix}\right.$, by the short vertical line over the point E, and so on. Thus, if we liken the curve to the outline of a mountain chain, the dissonances are represented by peaks, while the consonances correspond to passes.

According to this figure, the consonances in the order of their relative harmoniousness, are,

Octave,
Fifth,
$\left\{\begin{matrix}\text{Fourth,}\\ \text{Major Sixth,}\\ \text{Major Third,}\end{matrix}\right.$
Minor Third,
Minor Sixth.

In making use of the figure, however, the student must continually bear in mind, the assumptions on which it was calculated; viz., that the roughness vanishes when there are no beats; that it increases from this to a maximum for 33 beats per second; that it diminishes from this point as the number of beats per second increases; and lastly, that the intensity of the partial tones diminishes inversely as the square of their order. The conclusions expressed in the diagram, are therefore only true in those cases in which these assumptions are true, or approximately true.

Summary.

All the consonant intervals between *Simple Tones* are equally smooth or harmonious.

Intervals, whether between Simple or Compound Tones, having the following vibration ratios,

$$\frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \text{\&c.},$$

are *perfect* in their smoothness; they have no elements of roughness whatever.

The consonant intervals less than an Octave vary in smoothness according to the constitution or quality of their constituent Compound Tones, and according to their position in the scale of pitch. Helmholtz's arrangement for average qualities of tone is,

(1) Fifth,
(2) Fourth, Major Third, Major Sixth,
(3) Minor Third,
(4) Minor Sixth.

For other consonant intervals greater than an Octave, Helmholtz's rule applies, viz :—

Those intervals in which the smaller of the two numbers expressing the ratios of the vibration numbers, is *even*, are *improved* by having the upper tone raised an Octave, and *vice versa ;* thus,

Fifth	$\frac{3}{2}$	improved by becoming	Twelfth.
Major Third	$\frac{5}{4}$		Major Tenth.
Fourth	$\frac{4}{3}$	made worse by becoming	Eleventh.
Minor Third	$\frac{6}{5}$		Minor Tenth.
Major Sixth	$\frac{5}{3}$		Major Thirteenth.
Minor ,,	$\frac{8}{5}$		Minor ,,

CHAPTER XVII.

CHORDS.

WE have already seen that the Consonant intervals, within the Octave, are the Minor and Major Thirds, the Fourth, the Fifth, and the Minor and Major Sixths. If any two of these intervals be united, by placing one above the other, the interval thus formed between the two extreme tones, may or may not be consonant. In the former case the combination is termed a Consonant Triad.

In order to obtain all the Consonant Triads within the compass of an Octave, it is therefore only necessary to combine the above intervals two and two, and select those combinations, whose extreme tones form a consonant interval. The following table shows all the combinations of the above intervals, taken two at a time, whose extreme tones are at a smaller interval than an Octave.

(1)	Minor Third	+ Minor Third,	$\frac{6}{5} \times \frac{6}{5} = \frac{36}{25}$	
(2)	,,	+ Major ,,	$\frac{6}{5} \times \frac{5}{4} = \frac{3}{2}$	, Fifth
(3)	,,	+ Fourth,	$\frac{6}{5} \times \frac{4}{3} = \frac{8}{5}$	, Minor Sixth
(4)	,,	+ Fifth,	$\frac{6}{5} \times \frac{3}{2} = \frac{9}{5}$	
(5)	,,	+ Minor Sixth,	$\frac{6}{5} \times \frac{8}{5} = \frac{48}{25}$	
(6)	Major Third	+ Major Third,	$\frac{5}{4} \times \frac{5}{4} = \frac{25}{16}$	
(7)	,,	+ Fourth,	$\frac{5}{4} \times \frac{4}{3} = \frac{5}{3}$	, Major Sixth
(8)	,,	+ Fifth,	$\frac{5}{4} \times \frac{3}{2} = \frac{15}{8}$	
(9)	Fourth	+ Fourth,	$\frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$	

The only combinations in the above, the extreme tones of which form a consonant interval, are Nos. 2, 3, and 7. But each of these is capable of forming two Consonant Triads, according as the smaller of the constituent intervals is below or above. Consequently we find that there are altogether six Consonant Triads: viz.,

From (2) $\left\{\begin{array}{l}\text{Minor Third}\\ \text{Major ,,}\end{array}\right.$ as $\left\{\begin{array}{l}s\\ m\\ d\end{array}\right.$ 

$\left\{\begin{array}{l}\text{Major Third}\\ \text{Minor ,,}\end{array}\right.$ as $\left\{\begin{array}{l}m\\ d\\ l_1\end{array}\right.$ 

From (3) $\left\{\begin{array}{l}\text{Minor Third}\\ \text{Fourth}\end{array}\right.$ as $\left\{\begin{array}{l}d\\ l_1\\ m_1\end{array}\right.$ 

$\left\{\begin{array}{l}\text{Fourth}\\ \text{Minor Third}\end{array}\right.$ as $\left\{\begin{array}{l}d'\\ s\\ m\end{array}\right.$ 

From (7) $\left\{\begin{array}{l}\text{Major Third}\\ \text{Fourth}\end{array}\right.$ as $\left\{\begin{array}{l}m\\ d\\ s_1\end{array}\right.$ 

$\left\{\begin{array}{l}\text{Fourth}\\ \text{Major Third}\end{array}\right.$ as $\left\{\begin{array}{l}l\\ m\\ d\end{array}\right.$ 

If the lowest tone (d) of the 1st Triad be raised an Octave, we obtain the 4th Triad above $\left\{\begin{array}{l}d'\\ s\\ m\end{array}\right.$; while if the highest tone (s) of this same Triad be lowered an Octave we get the 5th Triad $\left\{\begin{array}{l}m\\ d\\ s_1\end{array}\right.$. Hence the 4th and 5th Triads are usually considered, to be derived from the 1st, and are called, respectively, its First and Second Inversions, or more briefly, its "*b*" and "*c*" positions.

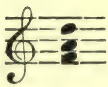
Again, if the lowest tone (l_1) of the 2nd Triad be raised an Octave we obtain the 6th Triad $\left\{\begin{array}{l}l\\ m\\ d\end{array}\right.$; while the lowering of the highest tone (m) through the same interval, produces the 3rd Triad $\left\{\begin{array}{l}d\\ l_1\\ m_1\end{array}\right.$. Hence, as before, the 6th and 3rd Triads are considered to be derived from the 2nd and are called its First and Second Inversions, or its "*b*" and "*c*" positions respectively.

The 1st Triad $\left\{\begin{matrix}s\\m\\d\end{matrix}\right.$, which has the Major Third below and the Minor Third above, is called a Major Triad, while the second $\left\{\begin{matrix}m\\d\\l_{,}\end{matrix}\right.$ which has the Minor Third below and the Major above, is termed a Minor Triad. The Six Consonant Triads may therefore be arranged as follows:

MAJOR TRIADS.

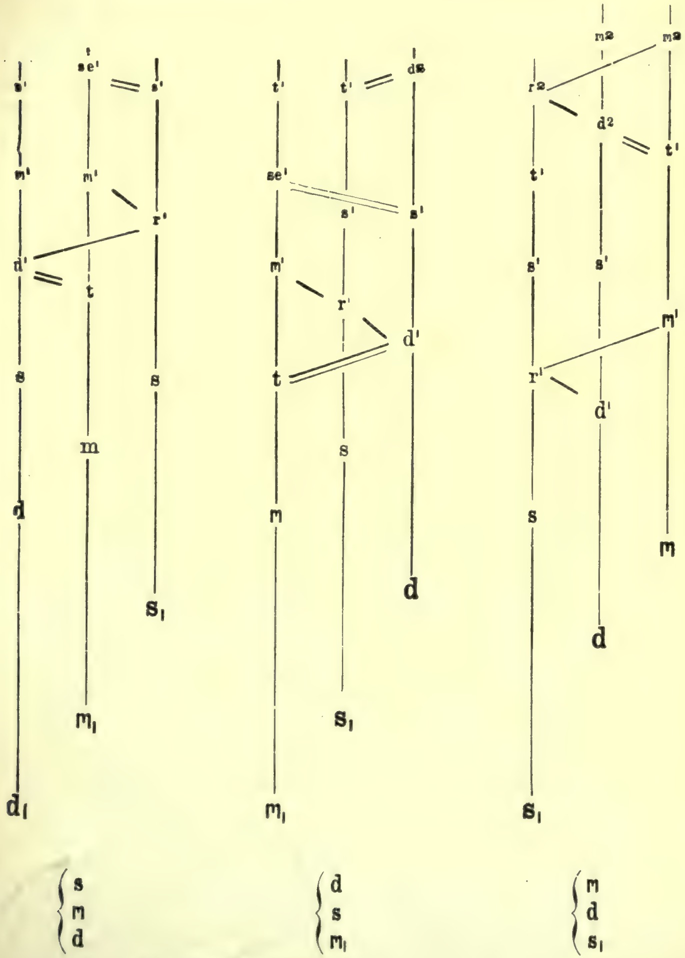
Normal	First Inversion	Second Inversion
$\left\{\begin{matrix}\text{Minor Third}\\\text{Major Third}\end{matrix}\right.$	$\left\{\begin{matrix}\text{Fourth}\\\text{Minor Third}\end{matrix}\right.$	$\left\{\begin{matrix}\text{Major Third}\\\text{Fourth}\end{matrix}\right.$
$\left\{\begin{matrix}s\\m\\d\end{matrix}\right.$ 	$\left\{\begin{matrix}d^{'}\\s\\m\end{matrix}\right.$ 	$\left\{\begin{matrix}m\\d\\s_{,}\end{matrix}\right.$ 
D*a*	D*b*	D*c*

MINOR TRIADS.

$\left\{\begin{matrix}\text{Major Third}\\\text{Minor Third}\end{matrix}\right.$	$\left\{\begin{matrix}\text{Fourth}\\\text{Major Third}\end{matrix}\right.$	$\left\{\begin{matrix}\text{Minor Third}\\\text{Fourth}\end{matrix}\right.$
$\left\{\begin{matrix}m\\d\\l_{,}\end{matrix}\right.$ 	$\left\{\begin{matrix}l\\m\\d\end{matrix}\right.$ 	$\left\{\begin{matrix}d\\l_{,}\\m_{,}\end{matrix}\right.$ 
L*a*	L*b*	L*c*

We shall first consider the Major Triads. An idea of the relative harmoniousness of these Triads may be obtained by the aid of fig. 85, in which these Triads are fully drawn out, on the same plan as in fig. 80, for Compound Tones containing the first six partials. In each Triad, the first six partials of the lowest tone, the first five of the middle tone, and the first four of the highest tone are given. The partials, that dissonate at the interval of a tone are connected, as before, by a single line, those dissonating at a semitone distance, by a double line. Whenever any partial dissonates with two partials of the same pitch, the first partial is connected with that partial of the other two which is of a lower order, that is, with the one which is presumably the louder of the two.

In order to facilitate the comparison of the Triads, an analysis of fig. 85 is given in the following table, in which the first horizontal line gives the names of the Triads; the next three lines show the



No. 1. No. 2. No. 3.

Fig. 85.

corresponding partials which beat at a semitone distance; and the remaining lines, those that beat at a tone distance. It has not been thought necessary to discriminate between the 16 : 15 and the 25 : 24 beats, nor between the 9 : 8 and 10 : 9.

	D*c*	D*a*	D*b*
Partials which beat at the interval of a *semitone.*	3 , 4	3 , 4 4 , 5	2 , 3 3 , 5 4 , 5
Partials which beat at the interval of a *tone.*	2 , 3 2 , 3 4 , 6 4 , 6	3 , 4 3 , 4	2 , 3 3 , 4

Comparing, in the first place, D*c* with D*a*: after eliminating the semitone dissonance between the 3rd and 4th partial, which is common to both, there remains in the latter, a semitone dissonance between a 4th and 5th partial, which is absent in the former. On the other hand, among the tone dissonances, D*c* has two, between the 2nd and 3rd partials, as against the two between 3rd and 4th partials in D*a*: the former being of course the more prominent. The two dissonances of 4th against 6th partials are so slight, that they may be disregarded. It is difficult, on the whole, to decide between these two Triads; there is probably not much difference between them. Helmholtz gives the preference to D*c*, which we have therefore put first in the table.

If there be any doubt concerning the relative harmoniousness of D*c* and D*a*, there can be none with respect to D*b*. Comparing it with D*a*: after discarding the semitone dissonance of a 4th against a 5th partial, which is common to both, there remains in D*b* the two semitone dissonances of a 2nd against a 3rd and a 3rd against a 5th partial, while in D*a* there is only that of a 3rd against a 4th. Again, among the tone dissonances, after throwing out that of a 3rd against a 4th partial from both, there remains in D*b* the dissonance of a 2nd against a 3rd, while in D*a* there is the less prominent dissonance of a 3rd against a 4th. Thus D*b* is decidedly the least harmonious of the Major Triads.

Coming now to the Minor Triads: fig. 86 shows them drawn out on the same plan as in fig. 85, and the following table contains an

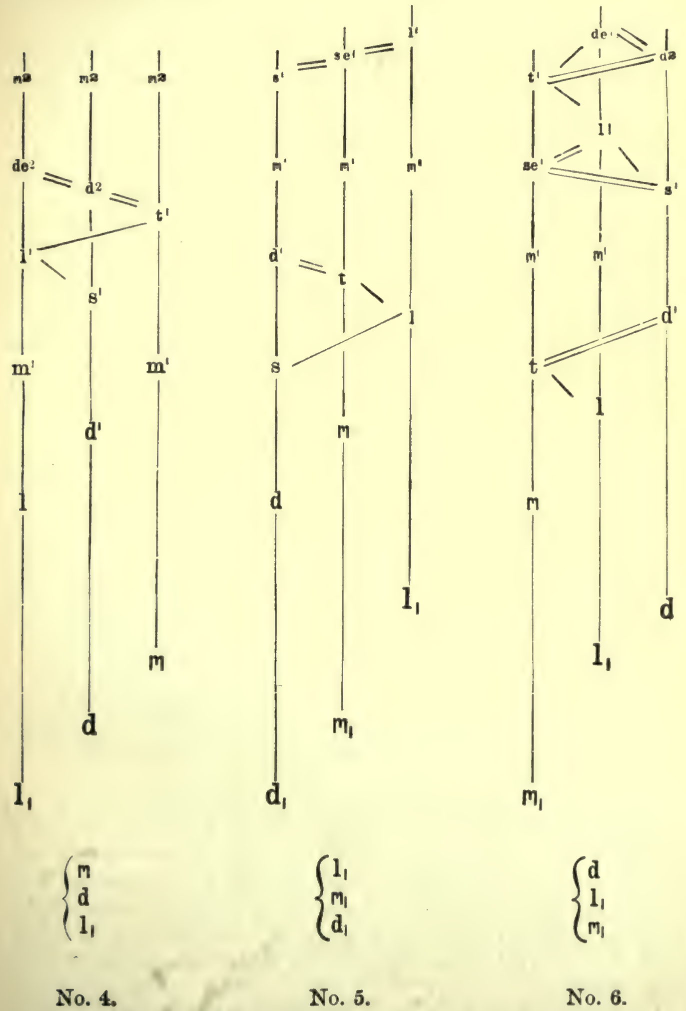


FIG. 86.

analysis of the results, arranged in the same way as in the case of the Major Triads.

	L*a*	L*b*	L*c*
Partials which beat at the interval of a *semitone.*	3 , 4 4 , 5	3 , 4 4 , 5 5 , 6	2 , 3 3 , 5 4 , 5 4 , 5 4 , 6
Partials which beat at the interval of a *tone.*	3 , 4 3 , 4 3 , 5	2 , 3 2 , 3 4 , 6	2 , 3 3 , 4 4 . 6 5 , 6

In comparing L*a* and L*b*, we may first eliminate the semitone dissonances of a 3rd against a 4th, and 4th against 5th partials, which are common to both; the slight semitone dissonance of a 5th against a 6th still remaining in the case of the latter. Further the two tone dissonances of 2nd against 3rd partials in L*b* will be more prominent than the two between 3rd and 4th partials in L*a*. For these reasons L*a* seems slightly more harmonious than L*b*, and has accordingly been placed first in the Table. It is only right to state, however, that Helmholtz places L*b* before L*a*.

About L*c* there can be no doubt: it has no fewer than five semitone dissonances, including the prominent one of a 2nd against a 3rd partial; it is decidedly the least harmonious of the six Triads.

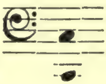
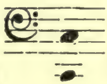
It must be recollected that the above results refer only to the isolated triads; in order to test these conclusions, each chord must be struck separately, unconnected with any others, and judged entirely by its own inherent harmoniousness. Furthermore, they must not be taken too low in the scale, or beats between the fundamentals may occur; and lastly, it must be borne in mind, that the intervals are supposed in the above, to be in just intonation.

On comparing D*a* with L*a* in the above tables, it will be found that they appear very nearly on an equality with regard to their harmoniousness. The same result is obtained on comparing their constituent intervals; each consisting of a Major and a Minor

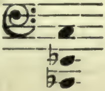
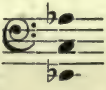
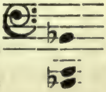
Third, and a Fifth. Thus we should expect a Minor Triad to sound as well as a Major Triad. This, however, as every one knows, is not the case. The cause of this must be looked for in the Differential Tones.

The Differential Tones of these Triads, can be found by ascertaining the Differentials generated by their constituent tones. Thus in the Triad $\left\{\begin{matrix}s\\m\\d\end{matrix}\right.$ we find from the table on page 130 Chap. XII, that $\left\{\begin{matrix}m\\d\end{matrix}\right.$ generates a Differential d_2; $\left\{\begin{matrix}s\\m\end{matrix}\right.$ generates d_2, and $\left\{\begin{matrix}s\\d\end{matrix}\right.$ produces d_1. Proceeding in this way, we shall find the Differential Tones of the Major and Minor Triads are as follows:

DIFFERENTIAL TONES OF THE MAJOR TRIADS.

	D*a*.	D*b*.	D*c*.
Triads,	$\left\{\begin{matrix}s\\m\\d\end{matrix}\right.$ 	$\left\{\begin{matrix}d^1\\s\\m\end{matrix}\right.$ 	$\left\{\begin{matrix}m\\d\\s_1\end{matrix}\right.$ 
Differential Tones,	$\left\{\begin{matrix}d_1\\d_2\end{matrix}\right.$ 	$\left\{\begin{matrix}s_1\\d_1\\d_2\end{matrix}\right.$ 	$\left\{\begin{matrix}d_1\\d_2\end{matrix}\right.$ 

DIFFERENTIAL TONES OF THE MINOR TRIADS.

	L*a*.	L*b*.	L*c*.
Triads,	$\left\{\begin{matrix}m\\d\\l_1\end{matrix}\right.$ 	$\left\{\begin{matrix}l\\m\\d\end{matrix}\right.$ 	$\left\{\begin{matrix}d\\l_1\\m_1\end{matrix}\right.$ 
Differential Tones,	$\left\{\begin{matrix}l_2\\d_2\\f_3\end{matrix}\right.$ 	$\left\{\begin{matrix}f_1\\l_2\\d_2\end{matrix}\right.$ 	$\left\{\begin{matrix}s_2\\l_3\\f_3\end{matrix}\right.$ 

We see from the above tables that the Differentials of the Major Triads are not only harmless to their respective chords, but actually improve them, supplying as it were a natural and true bass. On the other hand, in the Minor Triads, we find Differential Tones (**f** and **s** above) that are entirely foreign to the chords. They are not indeed close enough to beat, nor are they sufficiently distinct to destroy the harmony, but "they are enough to give a mysterious, obscure effect to the musical character and meaning of these chords, an effect for which the hearer is unable to account, because the weak differential tones on which it depends are concealed by other louder tones, and are audible only to a practised ear. Hence minor chords are especially adapted to express mysterious obscurity or harshness."

It must be remembered, that on tempered instruments, the differentials will not be exactly of the pitch given above, consequently those of the Major Triads will not fit in so well to the chords. Hence the superiority of Major to Minor chords, though still perceptible on tempered instruments, is not so marked as when the intervals are justly intoned.

It was shown in the last Chapter, that either tone of a Consonant Interval may be raised or lowered by an Octave, not indeed without somewhat altering the degree of its harmoniousness, but without losing its consonant character. By thus raising or lowering one or more of their tones, the Consonant Triads may be obtained in a great variety of distributions. We shall proceed to ascertain theoretically, the more harmonious of these distributions, in which the extreme tones of the Triad are within the compass of two Octaves.

In order to ascertain the more harmonious of these distributions of the six fundamental Triads, we shall in the first place have to bear in mind the rules, concerning the enlargement of an Interval by an Octave, which we obtained in the last Chapter (see page 203); and in the second place, we shall have to note the effect of Differential Tones. These two considerations will be sufficient to guide us in this enquiry.

With regard to the first, it will be convenient to briefly recapitulate the essential part of the results on page 203, viz.—

Minor Tenths	*are inferior to*		*Minor Thirds.*
Elevenths	,,	,,	*Fourths.*
Thirteenths	,,	,,	*Sixths.*

but the Fifth and Major Third are improved by being enlarged by an Octave.

As to the second consideration just referred to, the Differentials of the Consonant intervals, within the Octave, have been already given in Chapter XII. It will be convenient, however, to give them here again, together with those of all the other Consonant intervals, within the Compass of two Octaves,

TABLE I.

	Octave	Fifth	Twelfth	Fourth	Major 3rd
Interval	$\left\{\begin{array}{l} d^{1} \\ d \end{array}\right.$	$\left\{\begin{array}{l} s \\ d \end{array}\right.$ or $\left\{\begin{array}{l} m^{1} \\ l \end{array}\right.$	$\left\{\begin{array}{l} s^{1} \\ d \end{array}\right.$ or $\left\{\begin{array}{l} m^{1} \\ l_{1} \end{array}\right.$	$\left\{\begin{array}{l} d^{1} \\ s \end{array}\right.$ or $\left\{\begin{array}{l} l \\ m \end{array}\right.$	$\left\{\begin{array}{l} m \\ d \end{array}\right.$
Differential	d	d_{1} or l_{1}	d^{1} or l	d_{1} or l_{2}	d_{2}

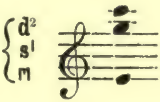
TABLE II.

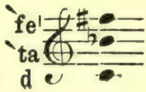
	Eleventh	Minor 3rd	Major 10th	Major 6th	Minor 6th
Intervals	$\left\{\begin{array}{l} d^{1} \\ s_{1} \end{array}\right.$ or $\left\{\begin{array}{l} l^{1} \\ m \end{array}\right.$	$\left\{\begin{array}{l} s \\ m \end{array}\right.$ or $\left\{\begin{array}{l} d^{1} \\ l \end{array}\right.$	$\left\{\begin{array}{l} m^{1} \\ d \end{array}\right.$	$\left\{\begin{array}{l} l \\ d \end{array}\right.$ or $\left\{\begin{array}{l} m^{1} \\ s \end{array}\right.$	$\left\{\begin{array}{l} d^{1} \\ m \end{array}\right.$
Differentials	m or de^{1}	d_{2} or f_{2}	s	f_{1} or d	s_{1}

TABLE III.

	Minor 10th	Major 13th	Minor 13th
Intervals	$\left\{\begin{array}{l} s^{1} \\ m \end{array}\right.$ or $\left\{\begin{array}{l} d^{1} \\ l_{1} \end{array}\right.$	$\left\{\begin{array}{l} l^{1} \\ d \end{array}\right.$ or $\left\{\begin{array}{l} m^{1} \\ s_{1} \end{array}\right.$	$\left\{\begin{array}{l} d^{2} \\ m \end{array}\right.$
Differentials (approximately)	$\grave{\text{ta}}$ or $\grave{\text{ma}}$	$\grave{\text{ma}}^{1}$ or $\grave{\text{ta}}$	$\grave{\text{fe}}^{1}$

Considering in the first place the distributions of the Major Triads, it is obvious that no such Triad can be injured by differential tones if the intervals of which it consists occur in Tables I or II above. For the Differential of each Interval in Table I only duplicates one or other of the Constituent Tones of such interval; while the Differential Tone of each interval in Table II will either coincide with, or duplicate the tone that must be added to that interval, to make it a Major Triad. On the other hand, those Triads which contain either of the intervals in Table III, must be disturbed more or less by their differentials: for in the first place, the differentials are foreign to the scale, and will consequently sound strange and disturbing; and secondly, they may produce audible beats with the third tone of the Triad or one of its overtones, as for example,



Differentials 

in which the **s**^{1} will dissonate against the **fe**^{1}.

Both the rules concerning the widening of the Consonant Intervals, and the Differentials generated by such Intervals, therefore, teach the same fact, viz.: that in selecting the most harmonious distributions of the Major Triads, the following intervals must be avoided.

The Minor Tenth.

The Thirteenths.

On examining all the possible distributions of the Major Triad, within a compass of two octaves, and rejecting those that contain either a Minor Tenth or a Thirteenth, the following Triads appear the more harmonious, the differentials being shown below :—

THE MOST PERFECT DISTRIBUTIONS OF THE MAJOR TRIADS.

	1	2	3	4	5	6
Triads.	m^1 s d	m^1 d^1 s	s^1 m^1 d	s^1 m^1 d^1	d^1 m s_1	d^1 s m
Differentials	s d d_1	d d_1	d^1 s d_1	d d_1	m s_1 d_1	s_1 d_1 d_2

It is interesting to observe how closely the above Triads, taken in conjunction with their Differential Tones, approximate to an ordinary Compound Tone.

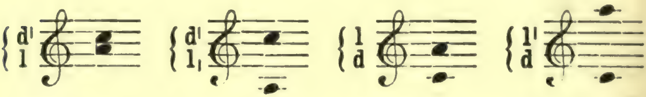
The other Distributions of the Major Triads are those that contain the intervals forbidden in the above. They all generate unsuitable Differentials, which without making them dissonant, cause them to be slightly rougher than those just considered. The following Table contains these Triads, together with the Differentials they generate.

THE LESS PERFECT DISTRIBUTIONS OF THE MAJOR TRIADS.

	7	8	9	10	11	12
Triads.	m^1 d s_1	m^1 d^1 s_1	s^1 m d	s^1 d^1 m	d^2 s m	d^2 s^1 m
Differentials	ta s d_2	ta m d_1	d^1 ta d_2	ta d s_1	fe^1 m^1 d_2	fe^1 ta d

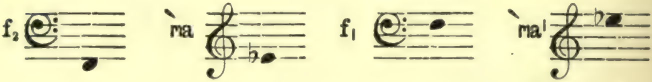
In the last two, the Differentials actually produce beats, causing them to be much the least pleasing; in fact, they are rougher than the better distributions of the Minor Triad.

We have now to ascertain the more advantageous positions of the Minor Triads. Taking { m' d' l  as the type of a Minor Triad, and keeping within the compass of two Octaves, the 3rd and root of the Triad must have one or other of the following positions



Minor 3rd. Minor 10th. Major 6th. Major 13th.

But from Table II and III, page 213, we see that these intervals generate respectively the following Differentials—



the second and fourth of which are foreign to the scale, while the first and third do not belong to the Minor Triad in question. It follows therefore, that every Minor Triad must generate at least one disturbing Differential tone.

Further, in order that there may be only one such Differential, the intervals which "m" makes with both the "l" and the "d" in the above four intervals, must be selected from those in Table I, page 213; for if it form with the "l" or "d" any of the intervals of Tables II and III, page 213, other differentials not belonging to the Minor chord will be introduced. On examination it will be found, that the following are the only three distributions of the Minor Triad that answer this test, that is, which have only one disturbing Differential Tone.

THE MORE PERFECT DISTRIBUTIONS OF THE MINOR TRIADS.

	1	2	3
Triads.	l^{1} m^{1} d^{1}	m^{1} d^{1} l	m^{1} d^{1} l_{1}
Differentials	f l_{1} d_{1}	l_{1} d_{1} f_{2}	l \`ma d_{1}

The other distributions which do not sound so well are—

THE LESS PERFECT DISTRIBUTIONS OF THE MINOR TRIAD.

	4	5	6	7	8	9	10	11	12
Triads.	l^{1} m d	l^{1} m^{1} d	d^{1} l m	d^{1} m l_{1}	m^{1} l d	m^{1} d l_{1}	d^{1} l_{1} m_{1}	d^{1} l m_{1}	l d m_{1}
Differentials.	\`ma^{1} de^{1} d_{2}	\`ma^{1} s l_{1}	s_{1} l_{2} f_{2}	\`ma s_{1} l_{2}	s l_{1} f_{1}	l s f_{3}	\`fe \`ma l_{3}	\`fe de f_{2}	de f_{1} s_{2}

In testing the results of this theoretical investigation, the student must be again reminded, that all the above intervals are supposed to be in just intonation. On a tempered instrument, the Combination Tones will be very different from those given above.

CONSONANT TETRADS.

If one of the tones of a consonant Triad be duplicated by adding its Octave, a consonant Tetrad or chord of four tones is obtained. A great variety of chords may be thus formed, and the method of investigating them, which we adopted in the case of the six original Triads, that is, by setting out the partials of each tone, would be somewhat laborious. Nevertheless this method is occasionally convenient, especially when two or three chords only have to be compared with one another. As an example, we will compare the following two Tetrads, the fundamentals being supposed to be of the same pitch, say D.

I I II II

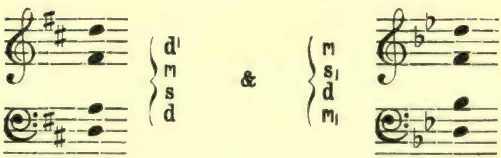
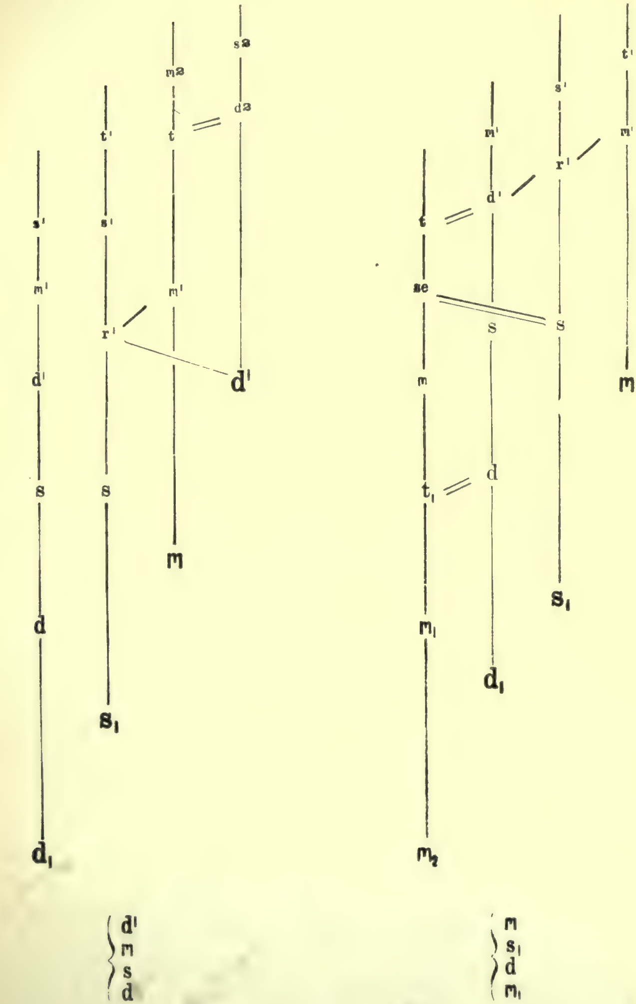


Fig 87 shows the full partials of each tone, up to the 6th in the Bass, 5th in the Tenor, 4th in the Contralto, and 3rd in the Soprano, tone dissonances being connected by a single, and semitone dissonances by a double line, as before; while below are given the Differentials generated by the four fundamentals of each chord. The following is a summary of the partial dissonances:

	I	II
Semitone	2 against 3	2 against 3
		2 ,, 5
		4 ,, 6
Tone	1 ,, 3	2 ,, 3
	2 ,, 3	3 ,, 4



d_2 d_1 s_1 m s (Differentials) **s_3 d_2 ta_2 d_1 s_1 t_1**

FIG. 87.

Thus, as far as the partials go, the balance is largely in favour of No 1. The differentials tell the same tale; for four of those in No. 1 are identical with partial tones, and the other rather improves the chord than otherwise. On the other hand the **ta_2** in No 2 is decidedly detrimental, for not only is it foreign to the chord, but, being at such a low pitch, it will produce audible beats with **$d_{|}$**.

In order to obtain some general results concerning these four-part chords, it will be better to apply the results that were arrived at in the case of the Triads.

Taking the Major Tetrads first, the following may be selected as a typical chord.

$$\left\{\begin{array}{l} d^{|} \\ s \\ m \\ d \end{array}\right.$$

The rule in the case of the Major Triads was, to avoid Thirteenths and Minor Tenths. In transposing the tones of the above chord, therefore, and keeping within the compass of two Octaves, the "**s**" must not be more than a Minor Third above the "**m**," otherwise a Minor Tenth will be formed; nor must the "**s**" be transposed more than a Sixth below the "**m**," or a Major Thirteenth will occur; lastly the "**d**" must not be more than a Minor Sixth above "**m**," if we wish to avoid making a Minor Thirteenth, but it may be placed as far below it as we please. These rules may be briefly enunciated in the words of Helmholtz:—"Those Major chords are most harmonious, in which the Root or the Fifth does not lie more than a Sixth above the Third, or the Fifth does not lie more than a Sixth below it."

It follows from this, that the "**m**" and "**s**" must not be duplicated by the Double Octave. There is another reason for this rule, however, namely that the differentials thus generated, will interfere with the other tones of the chord; thus—

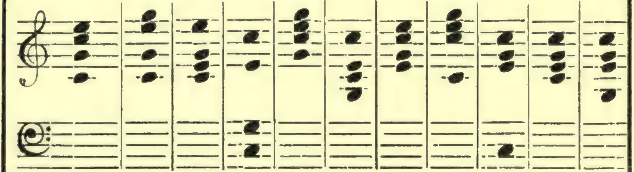
	$\left\{\begin{array}{l} m^{|} \\ m_{|} \end{array}\right.$	$\left\{\begin{array}{l} s^{|} \\ s_{|} \end{array}\right.$
Differentials	**t**	**$r^{|}$**

In the first case the "**t**" will beat with the "**$d^{|}$**" and the "r'" with both "d'" and "m'"; the first being the more objectionable.

The following are the Distributions of the Major Tetrads within the compass of two Octaves, which contain no Thirteenths or Minor Tenths, and which therefore have no disturbing Differential Tones.

THE MOST PERFECT DISTRIBUTIONS OF THE MAJOR TETRADS WITHIN THE COMPASS OF TWO OCTAVES.

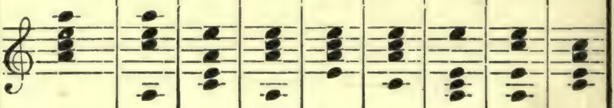
1	2	3	4	5	6	7	8	9	10	11
m'	s'	m'	d'	s'	d'	m'	s'	d'	d'	d'
d'	m'	s	m	m'	m	d'	m'	s	s	s
s	s	m	s_1	d'	d	s	d'	m	m	m
d	d	d	d_1	s	s_1	m	d	d_1	d	s_1



We see from this, that the tones of a Major chord in its First Inversion or "*b*" position must lie closely together as in 7; that the tones of a Major chord in the Second Inversion or "*c*" position must not have a greater compass than an Eleventh, as in 5, 6, and 11; but that to Major chords in their normal position more freedom may be allowed.

With regard to Minor Tetrads, we have already seen that they must have at least one false differential tone. The only Minor Tetrad with but one such Differential is No 1 in the Table below, which has the false differential f and its double octave f_2. The remaining Minor Tetrads may contain two, three, or even four disturbing differentials. The following Table contains all those within the compass of two Octaves, which generate only two false differentials; such differentials only being shown.

Best Distributions of Minor Tetrads.

	1	2	3	4	5	6	7	8	9
Tetrads.	l' m' d' l	l' m' d' l_1	m' l m d	m' d' l l_1	m' d' l m	m' d' l d	m' m d l_1	m' d' m l_1	d' l m d
									
Differentials.	f f_2	f m̀a	s f_1	m̀a f_2	s_1 f_2	s f_1 f_2	s f_3	m̀a s_1	s_1 f_1 f_2

From this Table it is evident, that a Minor chord in its Second Inversion or "*c*" position, must have its tones close together as in 5; that the tones of the First Inversion or "*b*" position must be within a Major Tenth, as in 3, 6, and 9.

We bring this discussion to a conclusion with an extract from the Chapter on Transposition of chords in Helmholtz' work, from which the present Chapter has been largely taken.

"In musical theory, as hitherto expounded, very little has been said of the influence of the Transposition of chords on harmonious effect. It is usual to give as a rule that close intervals must not be used in the bass, and that the intervals should be tolerably evenly distributed between the extreme tones. And even these rules do not appear as consequences of the theoretical views and laws usually given, according to which a consonant interval remains consonant in whatever part of the scale it is taken, and however it may be transposed or combined with others. They rather appear as practical exceptions from general rules. It was left to the musician himself to obtain some insight into the various effects of the various positions of chords, by mere use and experience. No rule could be given to guide him.

"The subject has been treated here at such length in order to show that a right view of the cause of consonance and dissonance leads to rules for relations which previous theories of harmony could not contain. The propositions we have enunciated agree, however, with the practice of the best composers, of those, I mean, who studied vocal music principally, before the great development of instrumental music necessitated the general introduction of tempered intonation, as anyone may easily convince himself by examining those compositions which aimed at producing an impression of perfect harmony. Mozart is certainly the composer who had the surest instinct for the delicacies of his art. Among his vocal compositions the *Ave verum corpus* is particularly celebrated for its wonderfully pure and smooth harmonies. On examining this little piece as one of the most suitable examples for our purpose we find in its first clause, which has an extremely soft and sweet effect, none but Major chords, and chords of the dominant Seventh. All these Major chords belong to those which we have noted as having the more perfect positions. Position 2 occurs most frequently, and then 8, 10, 1, and 9. It is not till we come to the final modulation of this first clause that we meet with two minor chords, and a major chord in an unfavourable position. It is very striking, by way of comparison, to find that the second clause of the same piece, which is more veiled, longing, and mystical, and laboriously modulates through bolder transitions and harsher dissonances, has many more minor chords, which, as well as the major chords scattered among them, are for the most part brought into unfavourable positions, until the final chord again restores perfect harmony.

"Precisely similar observations may be made on those choral pieces of Palestrina, and of his contemporaries and successors, which have simple harmonic construction without any involved polyphony. In transforming the Roman Church music, which was Palestrina's task, the principal weight was laid on harmonious effect, in contrast to the harsh and unintelligible polyphony of the older Dutch system, and Palestrina and his school have really solved the problem in the most perfect manner. Here also we find an almost uninterrupted flow of consonant chords, with dominant Sevenths, or dissonant passing notes, charily interspersed. Here also the consonant chords wholly, or almost wholly, consist of those major and minor chords which we have noted as being in the more perfect positions. But in the final cadence of a few clauses, on the contrary, in the midst of more powerful and more frequent dissonances, we find a predominance of the unfavourable positions of the major and minor chords. Thus that expression which modern music endeavours to attain by various discords and an abundant introduction of dominant Sevenths, was obtained in the school of Palestrina by the much more delicate shading of various transpositions of consonant chords. This explains the deep and tender expressiveness of the harmony of these compositions, which sound like the songs of angels with hearts affected but undarkened by human grief in their heavenly joy. Of course such pieces of music require fine ears both in singer and hearer, to let the delicate gradation of expression receive its due, now that modern music has accustomed us to modes of expression so much more violent and drastic."

"The great majority of Major Tetrads in Palestrina's 'Stabat Mater' are in the positions 1, 10, 8, 5, 3, 2, 4, 9, and of minor tetrads in the positions 9, 2, 4, 3, 5, 1. For the major chords one might almost think that some theoretical rule led him to avoid the bad intervals of the Minor Tenth and the Thirteenth. But this rule would have been entirely useless for minor chords. Since the existence of combinational tones was not then known, we can only conclude that his fine ear led him to this practice, and that the judgment of his ear exactly agreed with the rules deduced from our theory.

"These authorities may serve to lead musicians to allow the correctness of my arrangement of consonant chords in the order of their harmoniousness. But anyone can convince himself of their correctness on any justly intoned instrument. The present system of tempered intonation certainly obliterates somewhat of the more delicate distinctions, without, however, entirely destroying them."

Summary.

Triads in their closest distribution.

Major Triads are smoother or more harmonious than Minor Triads, because their differential tones form part of the chord, which is not the case with the Minor Triads.

A triad is not equally smooth in its three positions; arranged in the order of their smoothness, we have for

Major Triads,

1st	(*c*)	position or 2nd inversion,	as:	D*c*, S*c*, F*c*.
2nd	(*a*)	,, normal triad,	,,	D*a*, S*a*, F*a*.
3rd	(*b*)	,, 1st inversion,	,,	D*b*, S*b*, F*b*.

Minor Triads.

1st	(*b*) position	or	1st inversion	as L*b*, R*b*, M*b*.
2nd	(*a*)	,,	normal triad,	,, L*a*, R*a*, M*a*.
3rd	(*c*)	,,	2nd inversion,	,, L*c*, R*c*, M*c*.

Triads in various distributions.

The smoothest distributions of the Major Triads are those in which Thirteenths, and Minor Tenths are absent.

The smoothest distributions of the Minor Triads are the following, taking L as the type.

Tetrads.

Those Major Tetrads are most harmonious in which the Root or Fifth does not lie more than a Sixth above the Third; or the Fifth does not lie more than a Sixth below it.

To take the D chord for example, those distributions are smoothest in which the **d** does not lie more than a Sixth above the **m**; and in which the **s** does not lie more than a Sixth above or below the same note; therefore

> The Tones of Major Tetrads in the *b* position should lie as closely together as possible: and in the *c* position the extreme compass of the chord should not exceed an Eleventh.
>
> The tones of a Minor Tetrad in the *c* position should be as closely together as possible; and in the *b* position, the extreme compass of the chord should not exceed a Major Tenth.

In this Chapter, a chord has been scientifically studied as a thing *by itself;* in the art of music it is generally also considered in relation to what goes before and after; where the requirements of Art do not accord with those of Science in this respect, the latter must give way to the former.

CHAPTER XVIII.

Temperament.

In Chap. V, we saw how, by means of Helmholtz's Syren, it may be proved, that, for two tones to be at the interval of a Fifth, it is requisite that their vibration numbers be in the ratio of 3 : 2; and that for two tones to form the interval of a Major Third, their vibration numbers must be as 5 : 4.

In Chap. XV we have seen the reason for this; the vibration numbers must be exactly in these ratios, in order to avoid beats, between Overtones on the one hand, and Combination Tones on the other.

The vibration numbers of the tones of each of the Major Triads $\left\{\begin{matrix}\mathbf{s}\\ \mathbf{m}\\ \mathbf{d}\end{matrix}\right.$, $\left\{\begin{matrix}\mathbf{r}'\\ \mathbf{t}\\ \mathbf{s}\end{matrix}\right.$, $\left\{\begin{matrix}\mathbf{d}'\\ \mathbf{l}\\ \mathbf{f}\end{matrix}\right.$, being therefore in the ratios 6 : 5 : 4; the vibration numbers of all the tones of the diatonic scale can be readily calculated on any given basis, after the manner shown in Chap. V (which the student is recommended to read again, before proceeding with the present chapter). There, 288 having been chosen as the vibration number of **d**, the vibration numbers of the other notes were found to be as follows:

$$
\begin{array}{ll}
576 = \mathrm{d}^{\prime} & \\
 & \frac{16}{15} \\
540 = \mathrm{t} & \\
 & \frac{9}{8} \\
480 = \mathrm{l} & \\
 & \frac{10}{9} \\
432 = \mathrm{s} & \\
 & \frac{9}{8} \\
384 = \mathrm{f} & \\
 & \frac{16}{15} \\
360 = \mathrm{m} & \\
 & \frac{10}{9} \\
324 = \mathrm{r} & \\
 & \frac{9}{8} \\
288 = \mathrm{d} &
\end{array}
$$

The vibration ratios of the intervals between the successive notes were then calculated from the numbers thus obtained, and were found to be those given above.

The Fifths which can be formed from these tones are:

$$\left\{\begin{matrix}\mathrm{s}\\ \mathrm{d}\end{matrix}\right. \quad \left\{\begin{matrix}\mathrm{l}\\ \mathrm{r}\end{matrix}\right. \quad \left\{\begin{matrix}\mathrm{t}\\ \mathrm{m}\end{matrix}\right. \quad \left\{\begin{matrix}\mathrm{d}^{\prime}\\ \mathrm{f}\end{matrix}\right. \quad \left\{\begin{matrix}\mathrm{r}^{\prime}\\ \mathrm{s}\end{matrix}\right. \quad \left\{\begin{matrix}\mathrm{m}^{\prime}\\ \mathrm{l}\end{matrix}\right. \quad \left\{\begin{matrix}\mathrm{f}^{\prime}\\ \mathrm{t}\end{matrix}\right. .$$

If, from the vibration numbers above, the vibration ratio of each of these Fifths be calculated, it will be found that each of the following five intervals has the exact vibration ratio of 3 : 2; that is to say, the following are Perfect Fifths:

$$\left\{\begin{matrix}\mathrm{s}\\ \mathrm{d}\end{matrix}\right. \quad \left\{\begin{matrix}\mathrm{t}\\ \mathrm{m}\end{matrix}\right. \quad \left\{\begin{matrix}\mathrm{d}^{\prime}\\ \mathrm{f}\end{matrix}\right. \quad \left\{\begin{matrix}\mathrm{r}^{\prime}\\ \mathrm{s}\end{matrix}\right. \quad \left\{\begin{matrix}\mathrm{m}^{\prime}\\ \mathrm{l}\end{matrix}\right. .$$

Thus, for example,

$$\left\{\begin{matrix}\mathrm{t}\\ \mathrm{m}\end{matrix}\right. = \frac{540}{360} = \frac{3}{2}.$$

The remaining two are exceptions; for from the vibration numbers above we have

$$\left.\begin{matrix} f^{\prime} \\ t \end{matrix}\right\} = \frac{768}{540} = \frac{64}{45} \qquad \& \qquad \left.\begin{matrix} l \\ r \end{matrix}\right\} = \frac{480}{324} = \frac{40}{27}.$$

Again, the Minor Thirds that can be formed from the notes of the diatonic scale are:

$$\begin{Bmatrix} f \\ r \end{Bmatrix} \begin{Bmatrix} s \\ m \end{Bmatrix} \begin{Bmatrix} d^{\prime} \\ l \end{Bmatrix} \begin{Bmatrix} r^{\prime} \\ t \end{Bmatrix}.$$

If, as above, the vibration ratios of these intervals be calculated, it will be found that the following three are true Minor Thirds, that is, the vibration numbers of their constituent tones are in the exact ratio of 6 : 5:

$$\begin{Bmatrix} s \\ m \end{Bmatrix} \begin{Bmatrix} d^{\prime} \\ l \end{Bmatrix} \begin{Bmatrix} r^{\prime} \\ t \end{Bmatrix}.$$

The other interval is not a true Minor Third, for:

$$\left.\begin{matrix} f \\ r \end{matrix}\right\} = \frac{384}{324} = \frac{32}{27}.$$

Of these three exceptions, we may at once dismiss $\begin{Bmatrix} f^{\prime} \\ t \end{Bmatrix}$ as being a well recognized interval, the Diminished Fifth, less by a Semitone than the Perfect Fifth, for which it is not likely to be mistaken. But it is otherwise with the other two; they are very nearly a Perfect Fifth and a true Minor Third, respectively, for as we have just seen

$$\left.\begin{matrix} l \\ r \end{matrix}\right\} = \frac{120}{81} \text{ while } \frac{3}{2} = \frac{120}{80}$$

$$\text{and} \left.\begin{matrix} f \\ r \end{matrix}\right\} = \frac{32}{27} = \frac{96}{81} \text{ while } \frac{6}{5} = \frac{96}{80}$$

The imperfection of these intervals will be best seen by sketching the Compound Tones of $\begin{Bmatrix} l \\ r \end{Bmatrix}$ up to the beating partials, taking the vibration numbers given above; thus,—

$$\begin{array}{lcl} 972 = \acute{\mathrm{l}}^{\prime} & \text{———————} & \mathrm{l}^{\prime} = 960 \\ \quad\quad\;\;\; | & & \;\;| \\ 648 = \mathrm{r}^{\prime} & & \;\;| \\ \quad\quad\;\;\; | & & \mathrm{l} = 480 \\ 324 = \mathrm{r} & & \end{array}$$

We see from this, that with these vibration numbers, the mistuned Fifth $\left\{\begin{matrix}\mathrm{l}\\ \mathrm{r}\end{matrix}\right.$ produces 972 — 960 = 12 beats per second, between the 2nd and 3rd partials.

Similarly, with the mistuned Minor Third $\left\{\begin{matrix}\mathbf{f}\\ \mathrm{r}\end{matrix}\right.$; taking $\mathbf{r} = 324$, 24 beats per second would be heard.

It is obvious, therefore, that, although one might fail to perceive that the **r** of the scale is not in tune with **f** and **l** as long as we are concerned with *melody* only; yet as soon as these tones are sounded together, the discordance becomes very conspicuous. For true harmony, therefore, another tone is required in the scale, to form a Perfect Fifth with **l**. Calling this tone *rah* ($\grave{\mathbf{r}}$), we can deduce its vibration number in the scale above from the fact that

$$\left\{\begin{matrix}\mathrm{l}\\ \grave{\mathrm{r}}\end{matrix}\right. = \frac{3}{2}.$$

For $\mathbf{l} = 480$, therefore

$$3 \times \grave{\mathbf{r}} = 2 \times 480$$

$$\text{and thus } \grave{\mathbf{r}} = \frac{2 \times 480}{3} = 320$$

This tone not only forms a Perfect Fifth with **l**, but also a true Minor Third with **f**, for

$$\left\{\begin{matrix}\mathbf{f}\\ \grave{\mathrm{r}}\end{matrix}\right. = \frac{384}{320} = \frac{6}{5}$$

The relations of $\grave{\mathbf{r}}$ with the adjacent tones, which the student can readily calculate for himself, are as follows:

$$\begin{array}{ccc} & m & \\ \frac{9}{8} = & & = \frac{10}{9} \\ & r & \\ & \grave{r} & \\ \frac{10}{9} = & & = \frac{9}{8} \\ & d & \end{array}$$

The interval between $\left\{ \begin{array}{c} \grave{r} \\ r \end{array} \right.$ is termed a comma, its vibration ratio being $\frac{324}{320} = \frac{81}{80}$.

In harmony, $\grave{r}$ is required in the Minor Chord $\left\{ \begin{array}{c} l \\ f \\ \grave{r} \end{array} \right.$ and its inversions, while **r** is wanted in the Chord $\left\{ \begin{array}{c} r' \\ t \\ s \end{array} \right.$; but even in melody, $\grave{r}$ sounds better than **r** after the tones **f** and **l**, and similarly **r** better than $\grave{r}$ after **s** and **t**.

It will be seen, therefore, that in order to execute a piece of music which is entirely in one key, say C Major, and which has no Chromatics, we should require eight tones to the Octave, viz., the eight tones given in the middle column of fig. 88. If, however, the piece of music in question changes key, we shall require other tones. Suppose in the first place it passes into the First Sharp key. The **s** of the middle column, that is, the dominant of the original key, then becomes the **d**, or tonic of the First Sharp key, as shown in the right-hand column of fig. 88 Proceeding upwards from the tone **d**, we find that the $\grave{r}$ and **m** of this key will correspond to the **l** and **t** of the original key, but a new tone will be required for **r**. Going downwards the $l_{|}$, $s_{|}$, and $f_{|}$ of the new key correspond to the **m**, **r**, and **d** of the old, but a new tone will be necessary for $t_{|}$. Thus if the music passes into the First Sharp key, two more tones will be required.

For a change into the First Flat key, two new tones will also be wanted. For in this case, the **f** of the original key, becomes the **d** of the First Flat key, as shown in the left-hand column of fig. 88. Ascending from this **d**, the **r** and **m** of the new key will correspond exactly to the **s** and **l** of the original one, but new tones will be necessary for $\grave{r}$ and **f**. Descending, the **m**, $\grave{r}$, and **d** of the original key will serve for the $t_{|}$, $l_{|}$ and $s_{|}$ of the new one.

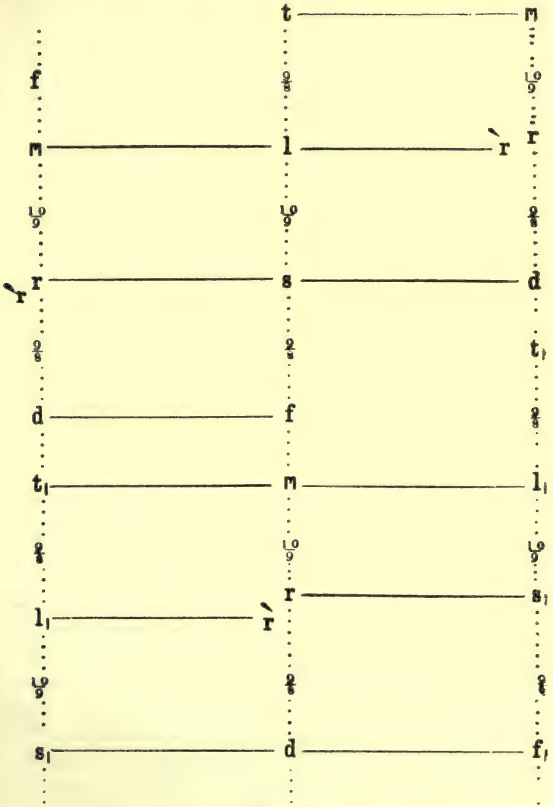


FIG. 88.

The changes of key in modern music, are, however, rarely confined to the above, and are often very extensive. A study of fig. 89 will show that in general every change of key of one remove either to the right or left of the central key, requires two new tones. Thus starting in C Major without Chromatics, 8 tones are required to the octave; on passing into G, two new tones are required; on further changing to D two more will be wanted; passing from this into A two more have to be brought forward, while if the music then enters the key of E, all the tones of this key will be of different pitches to those in the original key C. We have supposed here, that the music passes gradually through the keys of G, D and A; of course, if the change be a sudden one from C to E, the case would be somewhat different; The E of the central column would become the **d** of key E, the A would become its **f**, the B its **s**; only five new tones being therefore required for this key. It may be also noticed, that the E and B of the centre column are not of exactly the same pitch as the É and B́ of last column on the right, which are derived by transition through the intermediate keys; the latter being one comma higher than the former.

Similarly each transition to the left of the central key requires two new tones. Further, the È♭, À♭, and D̀♭ of the extreme left hand columns, are not of the same pitch as the E♭, A♭, D♭ of the central column, but are one comma flatter.

Thus to perform music, which modulates through the major keys of fig. 89, in the major mode only, requires a very large number of tones to the Octave. If to this, the minor mode be also added, a still larger number is necessary. Moreover, there are many more keys than those of fig. 89 used in modern music, so that the student will readily perceive that the number of tones to the Octave, thus required in modern music, is very large indeed.

All this presents no difficulty in the case of the voice, which is capable of producing tones of every possible gradation of pitch within its compass, and which, governed by the ear, readily forms the tones necessary to perfect harmony. Nor does it present any real difficulty in the violin class of instruments, which also may be made to emit tones of every gradation of pitch within their compass. The real difficulty is met with in such instruments as the Organ, Harmonium, Piano, &c., which have fixed tones, and consequently only possess a certain limited number of notes. On these instruments, which have but few notes to the Octave (generally only twelve), it is obviously impossible to execute music written in various keys and modes, in true intonation. The only thing that can be done is so to tune the fixed notes of the instrument, that the imperfections shall be as small as possible. The problem therefore is:—how so to tune an instrument, with but twelve tones to the Octave, as to be able to play in various keys and modes, with the smallest amount of imperfection. Any system of tuning by which this is brought about, is called a Temperament, and the false intervals thus obtained are termed tempered intervals.

D♭ A♭ E♭ B♭ F C G D Á É B́

r ....s ....d

t....m....l .. .r

t_1

r ....s ....d .. f

l ... r r....s ....d .... f

t_1....m....l....r

t_1....m....l_1

s ....d .... f

r....s.... d ....f

t_1....m....l_1.. .r r ....s_1

f t_1....m ....l_1....r

r.. .s_1....d ....f

m ....l_1....r

r ....s_1....d . ..f_1

t_1....m....l_1 ....r

r ....s_1....d ....f_1 t_1....m_1

r

r... s_1 ....d ....f_1

t_1....m_1.....l_1 ...r

r_1

d ....f_1 t_1....m_1....l_1....r_1

r_1....s_1... d... f_1

t_1....m_1.... l_1....r_1

FIG. 89.

There are many possible Temperaments, but only two are of any practical importance: Mean-Tone Temperament and Equal Temperament.

Mean-Tone Temperament.

This was the temperament used in tuning the Organ until about 1840. Its principle will be seen from the following considerations.

Starting with $C_{|}$ and tuning upwards four true Fifths consecutively we obtain the following notes:

$$\begin{array}{ll} \acute{E}^{|} & \\ & \Big\} = \frac{3}{2} \\ \acute{A} & \\ & \Big\} = \frac{3}{2} \\ D & \\ & \Big\} = \frac{3}{2} \\ G_{|} & \\ & \Big\} = \frac{3}{2} \\ C_{|} & \end{array}$$

but as the Fifth $\left\{ \begin{smallmatrix} \acute{A} \\ D \end{smallmatrix} \right.$ is a true one, while the $\left\{ \begin{smallmatrix} A \\ D \end{smallmatrix} \right.$ of the diatonic scale is, as we have already seen, smaller by a comma than a true Fifth, the $\acute{A}$ in the above will be a comma sharper than the A of the perfect scale. Consequently the $\acute{E}^{|}$ will also be sharper by a comma than the $E^{|}$ two octaves above the $E_{|}$ which makes a true Major Third with $C_{|}$.

Using the vibration ratios, we may put the same thing thus. The interval between $C_{|}$ and $\acute{E}^{|}$ in the above is the sum of four true Fifths, that is

$$\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{81}{16}$$

The interval between $C_{\prime}$ and $E^{\prime}$ is the sum of two Octaves and a Major Third, that is

$$\frac{2}{1} \times \frac{2}{1} \times \frac{5}{4} = \frac{20}{4} = \frac{80}{16}$$

Thus the interval between $E^{\prime}$ and $E^{\prime}$ is the difference between the above, viz:

$$\frac{81}{16} \div \frac{80}{16} = \frac{81}{16} \times \frac{16}{80} = \frac{81}{80}$$

that is, a comma.

Now, in the Mean-Tone Temperament, each of the four Fifths above is flattened a quarter of a comma, and consequently the $E^{\prime}$ thus obtained forms a perfectly true Major Third with the $C^{\prime}$. Thus, starting with C and tuning upwards two of these flattened Fifths, and a true Octave down, we obtain the notes C, G, and D; then again starting from this D, and tuning up two of these Fifths, and another Octave down, we get the additional notes A and E, all the Fifths being a quarter of a comma flat, but $\left\{\begin{matrix}E\\C\end{matrix}\right.$ being a true Major Third.

Now if we start from E, and repeat this process, that is, tune two of these flattened Fifths up, and an Octave down, and again two flat Fifths up, and an Octave down, we shall have obtained altogether the following notes:

$C^{\prime}$

B

A

G♯

G

F♯

E

D

C♯

C

the G♯ thus necessarily forming a true Major Third with E. Now in the above $\left\{\begin{smallmatrix}G\\C\end{smallmatrix}\right.$ is a true Fifth less a quarter of a comma; but $\left\{\begin{smallmatrix}E\\C\end{smallmatrix}\right.$ is a true Major Third; therefore $\left\{\begin{smallmatrix}G\\E\end{smallmatrix}\right.$ is a true Minor Third less a quarter of a comma. Again, $\left\{\begin{smallmatrix}B\\E\end{smallmatrix}\right.$ is a true Fifth less a quarter of a comma, but it has just been shown that $\left\{\begin{smallmatrix}G\\E\end{smallmatrix}\right.$ is a true Minor Third less a quarter of a comma; therefore $\left\{\begin{smallmatrix}B\\G\end{smallmatrix}\right.$ is a true Major Third. In a similar manner, it may be shown successively, that all the other Major Thirds in the above are true intervals, and that all the Minor Thirds are flatter by a quarter of a comma, than true Minor Thirds.

Now, starting from C, and tuning two Fifths, each flattened by a quarter of a comma, downwards, and an Octave up: again two flat Fifths down and an Octave up, the following additional tones, printed in italics below, are obtained:

C'

B

B♭

A

A♭ G♯

G

F♯

F

E

E♭

D

C♯

C

the A♭ thus necessarily forming a true Major Third with C.

We should have to obtain several more tones, in this way, to form a complete scale in this temperament, but, as we are supposing but 12 tones to the Octave, we must stop here; in fact, we have already exceeded that number, and must throw out either G♯ or A♭; we will suppose the former.

Now, in the above $\left\{\begin{smallmatrix}E\flat'\\A\flat\end{smallmatrix}\right.$ is a true Fifth less a quarter of a comma, and as we have just seen $\left\{\begin{smallmatrix}C'\\A\flat\end{smallmatrix}\right.$ is a true Major Third, therefore $\left\{\begin{smallmatrix}E\flat'\\C'\end{smallmatrix}\right.$

is a true Minor Third less a quarter of a comma. Again, $\{{G \atop C}$ is a true Fifth less a quarter of a comma, but we have just shown that $\{{E\flat \atop C}$ is a true Minor Third less the same amount, therefore $\{{G \atop E\flat}$ is a true Major Third. Proceeding upwards in this way it may be shown that all the Major Thirds in the above are true intervals except $\{{A\flat \atop E}$ $\{{F \atop C\sharp}$ $\{{B\flat \atop F\sharp}$ ${E\flat \atop B_1}$, but the Minor Thirds are flatter than the corresponding true intervals by a quarter of a comma. Further, since the Octaves and Major Thirds are true intervals, it follows that all the Minor Sixths (except four) must be true also. Again, since the Fifths and Minor Thirds are flatter than the corresponding true intervals by a quarter of a comma, it follows that the Fourths and Major Sixths must be sharper than the corresponding true intervals by the same amount.

We have seen that the D in the above is derived from C, by tuning upwards successively, two true Fifths, each less a quarter of a comma, and then an Octave down. Now the E, A, and B above are derived in exactly the same manner from the D, G, and A, respectively. Consequently the four intervals $\{{D \atop C}$ $\{{E \atop D}$ $\{{A \atop G}$ and $\{{B \atop A}$ are precisely similar. Moreover, it is easy to show that the interval $\{{G \atop F}$ is similar to these four; for since $\{{A \atop F}$ and $\{{B \atop G}$ have been shown to be true Major Thirds, they are equal to one another; take away the $\{{A \atop G}$ from each, and the remaining intervals $\{{G \atop F}$ and $\{{B \atop A}$ must be equal. Thus in Mean-Tone Temperament, there is no distinction between the Greater and Smaller step—between the $\frac{9}{8}$ and $\frac{10}{9}$ interval. The Major Third, $\{{\mathbf{m} \atop \mathbf{d}}$ for example, is composed of two precisely equal intervals, $\{{\mathbf{r} \atop \mathbf{d}}$ and $\{{\mathbf{m} \atop \mathbf{r}}$, the **r** being exactly midway between the two tones **m** and **d**. The vibration number of this **r** would thus be the geometrical *mean* of the vibration numbers of **d** and **m**. It is from this circumstance that the term *Mean-Tone* Temperament is derived.

In the above, we have seen that $\{{C' \atop E}$ is a true Minor Sixth and its vibration ratio is consequently $\frac{8}{5}$: we have also seen that $\{{G\sharp \atop E}$ is a true Major Third, and its vibration ratio is $\frac{5}{4}$: therefore the vibration ratio of $\{{C' \atop G\sharp}$ is

$$\frac{8}{5} \div \frac{5}{4} = \frac{8}{5} \times \frac{4}{5} = \frac{32}{25}$$

But $\{{C \atop A\flat}$, as we have seen, is a true Major Third and its vibration ratio is $\frac{5}{4}$. Thus the A♭ and G♯ above are tones of different pitch, and $\left\{ {A\flat \atop E} \right.$ cannot therefore be a true Major Third. Consequently in Mean-Tone Temperament if the number of tones to the Octave be restricted to 12, the Major Thirds cannot all be true.

With the scale constructed as above, the Major Thirds in the major keys of C, G, D, F, B♭ and E♭ are true, but the more remote keys will have one or more of their Major Thirds false; for example, the dominant chord of A would have to be played as $\left\{ \begin{matrix} B \\ A\flat \\ E \end{matrix} \right.$ and as we have just seen $\left\{ {A\flat \atop E} \right.$ is not a true Major Third.

Instruments of 12 tones to the Octave, tuned in Mean-Tone Temperament as above, can thus only be used in C and the more nearly related keys, viz., in E♭, B♭, F, G and D Major, and C, G, and D Minor: or if G♯ be retained instead of A♭, in B♭, F, C, G, D and A Major and G, D and A Minor. The other keys, which are more or less discordant, used to be termed "Wolves." Of course in these "wolves" not only will some of the Major Thirds be false, but some of the other intervals will differ from what they are in the better keys: for example, $\left\{ {E\flat' \atop A\flat} \right.$ is a true Fifth less a quarter of a comma, therefore if G♯ be retained in preference to A♭, the Fifth $\left\{ {E\flat' \atop G\sharp} \right.$ will no longer be equal to this amount.

Some old instruments, tuned in Mean-Tone Temperament were furnished with additional tones, such as G♯, D♯ and D♭, thus extending the number of keys that could be employed. The English Concertina, an instrument which is generally tuned on the Mean Tone system, is furnished with G♯ and D♯ as well as A♭ and E♭.

Equal Temperament.

In this system of tuning, which is the one now universally adopted for key-board instruments, the Octave is supposed to be divided into twelve exactly equal intervals, each of which is termed an equally tempered semitone. In consequence, however, of the extreme difficulty of thus tuning an instrument, these intervals are never exactly equal, and often very far from being so.

The vibration ratio of the equally tempered semitone is evidently

$$\sqrt[12]{\frac{2}{1}}$$

for twelve of these intervals *added* together form an Octave, that is, twelve of their vibration ratios *multiplied* together must amount to

§. In order to compare the other equally tempered intervals, with the corresponding true ones, we must go a little deeper into the subject.

Starting from any given tone, say C, let the other twelve letters in line I below, represent twelve other tones, obtained from it, by successively ascending twelve true Fifths; and let the letters after C in line II denote seven other tones derived from the same tone C by ascending seven Octaves:

$$\begin{array}{lccccccccccccccccccccccccc}
 & & \frac{3}{2} & & \frac{3}{2} & & \frac{3}{2} & & \frac{3}{2} & & \frac{3}{2} & & \frac{3}{2} & & \frac{3}{2} & & \frac{3}{2} & & \frac{3}{2} & & \frac{3}{2} & & \frac{3}{2} & & \frac{3}{2} & \\
\text{I.} & C & & G & & D & & A & & E & & B & & F\sharp & & C\sharp & & G\sharp & & D\sharp & & A\sharp & & E\sharp & & B\sharp
\end{array}$$

$$\begin{array}{lccccccccccccccc}
\text{II.} & C & & C & & C & & C & & C & & C & & C & & C \\
 & & \frac{2}{1} & & \frac{2}{1} & & \frac{2}{1} & & \frac{2}{1} & & \frac{2}{1} & & \frac{2}{1} & & \frac{2}{1} &
\end{array}$$

Inasmuch as the interval between any two successive tones in I is a just Fifth, the vibration ratio of which is $\frac{3}{2}$, it is obvious that the vibration ratio of the interval between the extreme tones of this line is $\frac{3}{2}$ multiplied by itself twelve times

$$= \left(\frac{3}{2}\right)^{12}$$

Similarly the vibration ratio of the interval between the extreme tones of line II

$$= \left(\frac{2}{1}\right)^{7}$$

Consequently the vibration ratio of the interval between the B♯ on the extreme right of line I and the C on the extreme right of II

$$= \left(\frac{3}{2}\right)^{12} \div \left(\frac{2}{1}\right)^{7}$$

$$= \left(\frac{3}{2}\right)^{12} \times \left(\frac{1}{2}\right)^{7}$$

$$= \frac{531441}{524288}.$$

This interval is called the Comma of Pythagoras, which may therefore be defined as the difference between twelve true Fifths and seven Octaves. This cumbrous vibration ratio cannot be expressed in any simpler way with perfect exactness, but it very nearly equals

$$\frac{74}{73}.$$

This fraction may be used instead of the one above, for all practical purposes, and we shall so employ it here. Roughly speaking, the comma of Pythagoras is about $\frac{1}{5}$ of a semitone, or $\frac{12}{11}$ of the comma of Didymus which has been often referred to above, and the vibration ratio of which is $\frac{81}{80}$.

We have just seen that the B♯ in line I above is sharper than the C in line II beneath it, by the small interval termed the comma of Pythagoras. If therefore each of the twelve true Fifths in I be diminished by $\frac{1}{12}$ of this interval, that is by $\frac{1}{11}$ of the ordinary comma (of Didymus), the B♯ will coincide with the C. Thus the tones of I may be written

III. C G D A E B F♯ C♯ G♯ D♯ A♯ F C

in which each successive Fifth, C—G, G—D, &c., is a true Fifth diminished by $\frac{1}{11}$ of the ordinary comma. Reducing these tones so as to bring them within the limit of an Octave they may be arranged in order of pitch thus:

IV. C C♯ D D♯ E F F♯ G G♯ A A♯ B C.

By observing the way in which these tones have been obtained above, it will be seen, that the successive intervals C—C♯, C♯—D, &c., are all equal to one another. For C♯ (see III above) was obtained from C, by ascending 7 of the flat Fifths just referred to; now if we ascend 5 of these same Fifths from C♯ we reach C, and ascending two more of these Fifths from C we reach D; therefore D is obtained by ascending 7 of these flat Fifths from C♯; consequently the interval C—C♯ must be equal to the interval C♯—D, and so on.

The tones represented in line IV above are therefore the tones of Equal Temperament. Thus the Fifths in this Temperament are $\frac{1}{11}$ of a comma, or about the $\frac{1}{60}$ part of a Semitone flatter than true Fifths; and as the Octaves are perfect, the Fourths must be sharper than true Fourths by the same amount.

The Major Third (C—E above, for example) was obtained by ascending four true Fifths each flattened by $\frac{1}{11}$ of a comma and dropping down two Octaves. Now the Major Third obtained by ascending four *true* Fifths and descending two Octaves, is, as we have already seen, sharper than a true Major Third, by one comma; therefore the Major Third of the Equal Temperament is sharper than a true Major Third, by a comma, less four times the amount by which each Fifth is flattened, that is, less $\frac{4}{11}$ of a comma; in other words, the Major Third of Equal Temperament is $\frac{7}{11}$ of a comma sharper than a true Major Third.

Again, the Equal Tempered Fifth is too flat by $\frac{1}{11}$ of a comma, and the Major Third too sharp by $\frac{7}{11}$ of a comma; therefore the difference between them, that is, the Minor Third is $\frac{8}{11}$ of a comma too flat. Further, as the Octaves are perfect, the Major Sixth must be $\frac{8}{11}$ of a comma too sharp, and the Minor Sixth $\frac{7}{11}$ of a comma too flat.

The following Table gives in a form convenient for reference the amount by which the Consonant intervals, both in Mean Tone and Equal Temperament, differ from the corresponding intervals in just Intonation.

INTERVAL.	MEAN TONE.	EQUAL.
Minor Thirds	$\frac{1}{4}$ comma flat	$\frac{8}{11}$ comma flat
Major Thirds	true	$\frac{7}{11}$,, sharp
Fourths	$\frac{1}{4}$ comma sharp	$\frac{1}{11}$,, ,,
Fifths	$\frac{1}{4}$,, flat	$\frac{1}{11}$,, flat
Minor Sixths	true	$\frac{7}{11}$,, ,,
Major Sixth	$\frac{1}{4}$ comma sharp	$\frac{8}{11}$,, sharp

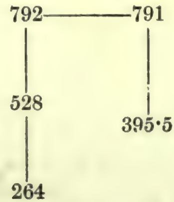
For further purposes of comparison, the following table is given, the 2nd column of which shows the vibration number of the tones of the diatonic scale on the basis C = 264; to which is added those of the E♭, a Minor Third above C, the A♭, a Minor Sixth above C, the B♭, a fourth above F, and the F♯ and C♯, a diatonic Semitone below G and D respectively: all in true intonation. The 3rd and 4th columns give the vibration numbers of the twelve tones, into which the Octave is divided in Mean-Tone and Equal Temperament respectively; the numbers being calculated as far as the first decimal place:

TONE.	JUST.	MEAN TONE.	EQUAL.
C¹	528	528	528
B	495	493·5	498·4 249.2
B♭	469·3	472·3	470·4
A	440	441·4	444
A♭	422·4	422·4	419·1
G	396	394·8	395·5
F♯	371·2	368·9	373·3
F	352	353·1	352·4
E	330	330	332·6
E♭	316·8	315·8	313·9
D	297	295·2	296·3
C♯	278·4	275·8	279·7
C	264	264	264

On comparing the Equal with the Mean Tone Temperament, we see that the former has its Fifth better in tune than the latter, but that it is inferior in all its other intervals, especially in the Major Thirds. On the other hand it must be recollected that in Mean Tone Temperament, it is only possible to play in a limited number of keys, while in Equal Temperament all keys are equally good or equally bad. If it be desired therefore to play in all keys, the Equal Temperament is decidedly the better; in fact, the only one possible under these circumstances.

As Equal Temperament is the one now universally employed on instruments with fixed keys, it will be of advantage to be able to compare its intervals with those of just Intonation, without the necessity of using the somewhat cumbrous vibration ratios. We can do this by employing a method devised by Mr. Ellis. Suppose a piano to be accurately tuned in Equal Temperament, the Octave being divided into twelve exactly equal parts. Further suppose each of these twelve equal semitones to be accurately divided into 100 equal parts; each of these minute intervals, Mr. Ellis has termed a Cent, so that there are 1,200 equal Cents in the Octave. Fig. 90 shows the magnitudes of the intervals in Equal and in True Intonation, expressed in these Cents.

The evils of Equal Temperament arise chiefly, of course, from the fact that overtones, which should be coincident, are not so, but produce audible beats. In addition to this, the Differentials, except in the case of the Octave, do not exactly correspond with any tones of the scale, and may generate beats with some adjacent tone, if this latter be sounding at the time. In the case of the Fifths and Fourths, these beats, being very slow, do not produce any very bad effects: for example, with the Fifth from C = 264 we have only one beat per second, thus:



Differential Tone = 395·5 — 264 = 131·5 $\quad C_{|} = 132$.

With the other intervals the case is different, more rapid beats being generated. The 2nd column of the following table shows the number of beats per second produced between partials which are

EQUAL.	CENTS.	TRUE.
Semitone	100	
	112	Diatonic Semitone : $\left\{\begin{smallmatrix} d \\ t \end{smallmatrix}\right.$
	182	Smaller Step : $\left\{\begin{smallmatrix} m \\ r \end{smallmatrix}\right.$
Tone	200	
	204	Greater Step : $\left\{\begin{smallmatrix} r \\ d \end{smallmatrix}\right.$
Minor Third	300	
	316	Minor Third
	386	Major Third
Major Third	400	
	498	Fourth
Fourth	500	
	590	Tritone : $\left\{\begin{smallmatrix} t \\ f \end{smallmatrix}\right.$
Tritone	600	
Fifth	700	
	702	Fifth
Minor Sixth	800	
	814	Minor Sixth
	884	Major Sixth
Major Sixth	900	
	996	Minor Seventh : $\left\{\begin{smallmatrix} f \\ s_1 \end{smallmatrix}\right.$
Minor Seventh	1000	
	1088	Major Seventh
Major Seventh	1100	
Octave	1200	Octave

FIG. 90

coincident in just Intonation when the interval given in the 1st column is played in Equal Temperament, the lower tone of each interval being C = 264.

Interval.	Beats.
Fifth	1
Fourth	1·2
Major Third	10·4
Major Sixth	12
Minor Third	14·5
Minor Sixth	16·5

The piano is specially favourable to Equal Temperament; in fact, this system of tuning was first applied to the piano and subsequently made its way to other key-board instruments. In the first place, the tones of the piano are loud only at the moment of striking, and die away before the beats due to the imperfect intervals have time to become very prominent: and further, music for the piano abounds in rapid passages, and usually, the chords are so frequently changed, that beats have very little time in which to make themselves heard. Indeed, Helmholtz makes the suggestion, that it is the unequal temperament, which has forced on the rapid rate of modern music, not only for the piano, but for the organ also.

On the Harmonium and Organ, the effects of Temperament become very apparent in sustained chords. On the latter it is especially so in Mixture Stops, the tempered Fifths and Thirds of which, dissonating against the pure Fifths and Thirds in the overtones, producing what Helmholtz terms the "awful din" so often heard, when these stops are drawn.

Many attempts have been made from time to time to construct Harmoniums, Organs, &c., in such a way, and with such a number of tones to the Octave, that the intervals they yield shall be more or less close approximations to True Intonation. The best known of these instruments are:

1. *Helmholtz's Harmonium.* This instrument has two Manuals, the tones of each being such as would be generated by a

succession of True Fifths, but those of the one manual are tuned a comma sharper than those of the other. A full description will be found in Helmholtz's "Sensations of Tone."

2. *Bosanquet's Harmonium*, which possesses eighty-four tones to the Octave and a specially constructed key-board. A full account may be found in "Proceedings of the Royal Society," vol. 23.

3. *General Thompson's Enharmonic Organ*, which possesses 3 manuals and seventy-two tones to the Octave. For particulars of its construction the reader is referred to General Perronet Thompson's work "On the Principles and Practice of Just Intonation."

4. *Colin Brown's Voice Harmonium*, the finger-board of which differs entirely from the ordinary one. The principles of its construction are given in "Music in Common Things," parts I & II.

SUMMARY.

To perform music *with true Intonation* in one key only, without using Chromatics and in the Major Mode, *eight* tones to the Octave are required.

In general, every transition of one remove, either way, from the original key and still keeping to the Major Mode only, requires *two* new tones. In changes of three or more removes, the number of new tones required is not quite so large as if the changes were made through the intervening keys.

To modulate therefore in *all* keys and in both the *Major* and *Minor Mode in true intonation* requires a very large number of tones to the Octave—between 70 and 80, in fact.

This presents no difficulty in the case of the voice and stringed instruments of the Violin Class, for such instruments can produce tones of any required gradation of pitch; the difficulty is only felt in instruments with a limited number of fixed tones; and for such instruments some system of Temperament is necessary.

A *Temperament* is any *system* of tuning other than true intonation; Intervals tuned on any such system are termed *tempered intervals.* The *object* of temperament is so to tune a certain limited number of fixed tones, as to produce, *on the whole*, the least possible departure from true intonation.

The limited number of fixed tones just referred to is almost always *twelve* to the Octave.

The systems of Temperament, which have been most extensively used in Modern Music are *Equal Temperament* and *Mean Tone Temperament.*

Mean Tone Temperament.

Chief features:

(1) The Major Thirds are true.

(2) The Fifths are $\frac{1}{4}$ comma flat.

(3) There is no distinction between the Greater and Smaller Tone.

When there are but 12 tones to the Octave, however, (1) and (2) are true in *only half-a-dozen keys.*

The great disadvantage of this temperament is, that only music in a limited number of keys can be performed on instruments tuned according to this system.

Equal Temperament.

Chief features:

(1) The Octave is divided into 12 equal intervals, the vibration ratio of each of which is $\sqrt[12]{2} = 1{\cdot}0595$ or $1{\cdot}06$ nearly

(2) The Fifths are $\frac{1}{11}$ comma *flat.*

(3) The Major Thirds are $\frac{7}{11}$ comma *sharp.*

The above facts are true *in all keys.*

The chief advantage of this temperament is, that music in *all keys* can be performed on instruments tuned according to this system; that is to say, all keys are equally good or equally bad.

Though it is impossible to obtain true intonation from instruments with but twelve fixed tones to the Octave, yet in the case of the *Voice*, *Violin*, and other instruments which may be made to produce tones of any desired pitch, it seems self-evident that true intonation should be the thing aimed at; inasmuch as it is just as easy with these instruments to make the intervals true as to make them false, provided the ear of the performer has not been already vitiated by the tempered intervals.

QUESTIONS.

CHAPTER I.

1. What is meant by a periodic motion?

2. Describe three methods of obtaining a periodic movement.

3. What is the physical difference between musical sounds and noises?

4. How can it be demonstrated, that the air in a sounding flue-pipe is in periodic motion?

5. Under what circumstances does a periodic motion *not* give rise to a musical sound?

CHAPTER II.

6. How can it be proved, that some medium is necessary for the transmission of sound?

7. Draw a diagram of a water wave, showing clearly what is meant by its *length*, *amplitude*, and *form*.

8. Draw three water waves of same length and amplitude, but of different forms; three of same amplitude and form, but of different lengths; and three of same length and form but of different amplitudes.

9. Describe the way in which a vibrating tuning-fork originates a sound wave.

10. In what direction do the air particles in a sound wave vibrate?

11. What is meant by the *length*, *amplitude*, and *form* of a sound wave?

12. State the connection, in a sound wave, between (1) length of wave and duration of particle vibration, (2) amplitude of wave and extent of particle vibration, (3) form of wave and manner of particle vibration.

13. Describe the way in which a sound wave transmits itself through the air.

14. What is an *associated* wave?

15. Describe any method, by which the velocity of sound in air can be determined.

16. How is the velocity of sound in air affected by temperature?

17. What is the velocity of sound in air at 0° C.? What is it at 20° C.? What at 60° Fah.?

18. What is the velocity of sound in water? How has it been determined?

19. State what you know of the velocity of sound in solids.

20. Describe an experiment, which illustrates the fact, that solids are, as a rule, good conductors of sound.

21. A person observes that ten seconds elapse between a flash of lightning and the succeeding thunder clap. What is the approximate distance of the thunder cloud from the observer?

22. A vessel at sea is seen to fire one of its guns. Thirty-five seconds afterwards, the report is heard. How far off is the vessel? (Temperature 25° C.)

CHAPTER III.

23. What is the use of the External ear?

24. Describe the relative positions in the ear of the (1) Tympanum, (2) Fenestra Ovalis, (3) Fenestra Rotunda.

25. How is the vibratory motion of the Tympanum transmitted to the Fenestra Ovalis?

26. In what part of the internal ear are the Fibres of Corti situated? What is supposed to be their function?

27. What is the Eustachian Tube? What would be the result of this tube becoming stopped up?

28. The cavity of the Middle Ear is in most persons, completely separated from the external air by the Tympanum; but occasionally there is an aperture in this latter. Does this necessarily affect the hearing? Give reasons for your answer.

29. What is the special function of the labyrinth?

CHAPTER IV.

30. What are the three *elements* which define a musical sound?

31. What is the physical cause of variation in pitch? Describe a simple experiment in support of your answer.

32. What is meant by the vibration number of a musical sound?

33. Mention three of the most accurate methods of experimentally determining the vibration number of a given musical sound.

34. Describe the Wheel Syren.

35. What are the disadvantages of Cargnard de la Tour's Syren?

36. Describe the construction of Savart's Toothed Wheel.

37. Describe the Sonometer or Monochord.

38. Describe Helmholtz's or Dove's Double Syren.

39. Describe the principle of the Graphic method of ascertaining the vibration number of a tuning-fork.

40. Given the vibration number of a musical sound, how can its wave length be determined? What are the lengths of the sound waves emitted by 4 forks, which vibrate 128, 256, 512, and 1024 times per second, respectively? (Take velocity of sound as 1100).

41. The vibration number of a tuning-fork is 532. What will be the length of the sound wave it originates (1) in air at 32° Fah., (2) in air at 60° Fah.?

42. If the length of a sound wave is 3 feet 6 inches when the velocity of sound is 1100 feet per second, what is the vibration number of the sound?

43. Calculate the length of the sound wave emitted by an organ pipe, which produces $C_3 = 32$

44. Calculate the length of the sound wave produced by a piccolo flute, which is sounding $C^4 = 4096$.

45. What are the approximate vibration numbers of the highest and lowest sounds used in music?

46. Give the vibration numbers of (1) the C^1 in Handel's time, (2) the French Diapason normal, (3) a Concert Piano, and organ (approximately).

47. When a locomotive sounding its whistle is passing rapidly through a station, to a person on the platform, the pitch of the whistle appears sharper while the engine is approaching, than it does after it has passed him. Explain this.

CHAPTER V.

48. What is meant by the *vibration ratio* of an interval? If the vibration numbers of two sounds are 496 and 465 respectively, what is the vibration ratio of the interval between them? What is this interval called?

49. What are the vibration ratios of an Octave, a Fifth, and a Major Third?

50. What are the vibration ratios of a Major and Minor Sixth, and a Minor Third?

51. What is the best way of experimentally proving that the vibration ratios of an Octave, Fifth and Major Third are exactly $\frac{2}{1}$, $\frac{3}{2}$, and $\frac{5}{4}$ respectively?

52. Given that the vibration numbers of **s**, **m**, **d**, are as 6 : 5 : 4, and that **d** = 300; calculate *from these data*, the vibration numbers of the other tones of the Diatonic Scale.

53. Given **d** = 320, and that the vibration numbers of the tones of a Major Triad, in its normal position, are as 6 : 5 : 4; calculate *from these data* the vibration numbers of the other tones of the diatonic scale.

54. Given **d** = 256, and vibration ratios of a Fifth and Major Third are $\frac{3}{2}$ and $\frac{5}{4}$ respectively; calculate *from these data*, the vibration numbers of the other tones of the diatonic scale.

55. Given $\mathbf{d} = 240$, $\mathbf{r} = 270$, $\mathbf{m} = 300$, $\mathbf{f} = 320$, $\mathbf{s} = 360$, $\mathbf{l} = 400$, $\mathbf{t} = 450$, $\mathbf{d}^{\prime} = 480$; calculate *from these data* the vibration ratios of a Diminished Fifth, the Greater Step, the Smaller Step, and the Diatonic Semitone.

56. With the data of question 55, calculate the vibration numbers of **fe** and **se**, and the vibration ratios of the Greater Chromatic $\left\{\begin{matrix}\mathbf{fe}\\\mathbf{f}\end{matrix}\right.$, and the Lesser Chromatic $\left\{\begin{matrix}\mathbf{se}\\\mathbf{s}\end{matrix}\right.$.

57. From the data of question 55, show that the interval $\left\{\begin{matrix}\mathbf{l}\\\mathbf{r}\end{matrix}\right.$ is not a perfect Fifth; and calculate the vibration number of the note $\grave{\mathbf{r}}$, which would form a perfect Fifth to **l**.

58. By means of the results of question 57, ascertain the vibration ratio of the interval from ray to rah. What is this interval called?

59. Show from the numbers given in question 55, that $\left\{\begin{matrix}\mathbf{f}\\\mathbf{r}\end{matrix}\right.$ is not a perfect Minor Third, but that $\left\{\begin{matrix}\mathbf{f}\\\grave{\mathbf{r}}\end{matrix}\right.$ is so.

60. What musical peculiarity does the Octave possess, which is shared by no smaller interval?

61. Given: $\left\{\begin{matrix}\mathbf{s}\\\mathbf{f}\end{matrix}\right. = \frac{9}{8}$ and $\left\{\begin{matrix}\mathbf{se}\\\mathbf{s}\end{matrix}\right. = \frac{25}{24}$; calculate ratio of $\left\{\begin{matrix}\mathbf{se}\\\mathbf{f}\end{matrix}\right.$.

62. Given: $\left\{\begin{matrix}\mathbf{s}\\\mathbf{r}\end{matrix}\right. = \frac{4}{3}$ and $\left\{\begin{matrix}\mathbf{se}\\\mathbf{s}\end{matrix}\right. = \frac{25}{24}$; calculate vibration ratio of $\left\{\begin{matrix}\mathbf{se}\\\mathbf{r}\end{matrix}\right.$.

63. Given: $\left\{\begin{matrix}\mathbf{t}\\\mathbf{ta}\end{matrix}\right. = \frac{135}{128}$ and $\left\{\begin{matrix}\mathbf{t}\\\mathbf{le}\end{matrix}\right. = \frac{16}{15}$; calculate vibration ratio of the interval between **ta** and **le**.

64. Given: $\left\{\begin{matrix}\mathbf{fe}\\\mathbf{m}\end{matrix}\right. = \frac{9}{8}$ and $\left\{\begin{matrix}\mathbf{ba}\\\mathbf{m}\end{matrix}\right. = \frac{10}{9}$; calculate vibration ratio of the interval between **fe** and **ba**.

65. Given: $\left\{\begin{matrix}\mathbf{r}\\\mathbf{d}\end{matrix}\right. = \frac{9}{8}$, $\left\{\begin{matrix}\mathbf{m}\\\mathbf{r}\end{matrix}\right. = \frac{10}{9}$, $\left\{\begin{matrix}\mathbf{f}\\\mathbf{m}\end{matrix}\right. = \frac{16}{15}$, $\left\{\begin{matrix}\mathbf{s}\\\mathbf{f}\end{matrix}\right. = \frac{9}{8}$, and $\left\{\begin{matrix}\mathbf{se}\\\mathbf{s}\end{matrix}\right. = \frac{25}{24}$; calculate vibration ratio of the interval $\left\{\begin{matrix}\mathbf{se}\\\mathbf{d}\end{matrix}\right.$.

66. Given: $\left\{\begin{matrix}\mathbf{d}^{\prime}\\\mathbf{t}\end{matrix}\right. = \frac{16}{15}$, $\left\{\begin{matrix}\mathbf{t}\\\mathbf{l}\end{matrix}\right. = \frac{9}{8}$, $\left\{\begin{matrix}\mathbf{l}\\\mathbf{se}\end{matrix}\right. = \frac{16}{15}$; calculate vibration ratio of the interval $\left\{\begin{matrix}\mathbf{d}^{\prime}\\\mathbf{se}\end{matrix}\right.$.

67. Given: $\left\{\begin{matrix}\mathbf{d}^{\prime}\\\mathbf{s}\end{matrix}\right. = \frac{4}{3}$ and $\left\{\begin{matrix}\mathbf{la}\\\mathbf{s}\end{matrix}\right. = \frac{16}{15}$; calculate vibration ratio of $\left\{\begin{matrix}\mathbf{d}^{\prime}\\\mathbf{la}\end{matrix}\right.$.

68. From the data of No. 55, calculate the vibration ratios of a Major Tenth, a Minor Tenth, and a Twelfth.

69. How is the vibration ratio of the sum of two intervals calculated, when the vibration ratios of these latter are known? Give an example.

70. Given the vibration ratios of two intervals, show how the vibration ratio of the difference between these intervals can be ascertained. Give an example.

CHAPTER VI.

71. What is meant by the intensity of a musical sound? How can it be shown experimentally, that the intensity of a sound depends upon the amplitude of the vibrations that give rise to it?

72. State the law of Inverse squares. Why does it not appear to be correct, under ordinary circumstances?

73. What is the principle of the speaking tube?

74. Explain the phenomenon of echoes.

75. If two seconds elapse between a sound and its echo, what is the distance of the reflecting surface?

76. Explain one of the causes of the bad acoustical properties of some buildings, and state any remedy you know of.

77. Why does one's voice appear louder in an empty unfurnished room, than in the same room furnished?

78. Give a theoretical proof of the law of Inverse Squares.

CHAPTER VII.

79. How can the phenomenon of resonance or co-vibration be illustrated with two tuning-forks? What conditions are necessary to the success of the experiment?

80. Describe some experiments with stretched strings to illustrate the phenomenon of resonance.

81. Explain in detail the cause of resonance or co-vibration in the case of tuning-forks or stretched strings.

82. Explain in detail the cause of resonance in open tubes, showing clearly why the tube must be of a certain definite length, if it is to resound to a note of given pitch.

83. Explain in detail the cause of resonance in *stopped* tubes, showing clearly why the tube must be of a certain definite length, if it is to resound to a note of given pitch.

84. How would you construct a resonator to resound to G'? Calculate approximate dimensions.

85. To illustrate the phenomenon of resonance with two tuning-forks, they must be in the most perfect unison; whereas an approximate unison is sufficient in the case of two strings stretched on the same sound-board. Explain this.

86. I have a tube 1 inch in diameter, open at both ends, which resounds powerfully to G♯. What length is it? (C' = 512).

87. Calculate the length of a stopped tube about 1 inch in diameter, resounding to E♭.

88. A tube open at both ends, and about an inch in diameter, is 10½ inches long. Calculate approximately the note it resounds to.

89. A tube closed at one end is 14 inches long, and about $1\frac{1}{2}$ inches diameter. Calculate approximately the note to which it resounds.

90. What are resonance boxes? What are they used for? What are resonators? What are they used for? Explain the best method of using them.

91. Describe an experiment to show how the resonance of air-chambers can be optically demonstrated.

92. Why does the sound of a vibrating tuning-fork die away more quickly when attached to a resonance box, than when held in the hand?

93. While singing the other day, I happened to sound D loudly. Immediately a gas globe in the room was heard to give out a tone of the same pitch. I found that this occurred whenever D was sounded in its vicinity, but a tone of any other pitch produced no effect. Explain why the globe emitted this particular note and no other.

94. Why does a vibrating tuning fork give forth a louder sound, when its handle is applied to a table, than when merely held in the hand?

CHAPTER VIII.

95. Define the terms: Simple Tone, Clang or Compound Tone, Partial, Overtone, Fundamental Tone.

96. Write down in vertical columns the Partials, that may be heard when any low note is loudly sounded on the pianoforte or harmonium, or by a bass voice, calling the fundamental, $\mathbf{d}_{|}$, $\mathbf{r}_{|}$, $\mathbf{m}_{|}$, $\mathbf{f}_{|}$, $\mathbf{s}_{|}$, $\mathbf{l}_{|}$, $\mathbf{t}_{|}$, successively.

97. Write down the vibration numbers of the partials which may be heard on a harmonium, calling the fundamental 100.

98. Write down in vertical columns, the partials that may be heard, when the following tones are struck on a piano:—$C\sharp_2$, D_2, E_2, F_2, $G\sharp_2$, $B\flat_2$.

99. The chord $\left\{\begin{array}{l} C^{|} \\ A \\ C \\ F_{|} \end{array}\right.$ is sounded on a harmonium, or smartly struck on a a piano. Write down in 4 columns the various partial tones that may be heard, keeping sounds of the same pitch on the same horizontal line.

100. From what instruments can simple tones be obtained?

101. Write down in a column the relative vibration numbers of the first 20 partial tones, naming all those which are constituent tones of the musical scale.

102. Many persons find great difficulty in hearing overtones. Explain any methods you know of, which will assist such persons in hearing them.

103. State what you know concerning the relative intensities of partials on various instruments.

104. Upon what does the quality of a Compound Tone depend? Explain fully the meaning of your answer.

105. Explain how a Tuning Fork can be kept in a state of continued vibration by an electro-magnet.

106. Describe the apparatus, which is used for the purpose of keeping a number of forks in continued vibration, by means of electro-magnets and a single current. What relation must exist between the vibration numbers of these forks?

107. How may the relative intensities of the sounds of the forks in the above, be modified?

108. Describe the apparatus used by Helmholtz, in his experiments on the synthesis of Compound Tones.

109. What is a pendular vibration? Describe a method of obtaining a graphic representation of one.

110. Show by a diagram how to compound two simple associated waves.

111. Given the quality of a musical tone: is it possible to determine the corresponding wave form? If not, why not?

CHAPTER IX.

112. Describe an experiment illustrating the use of the sound-board in stringed instruments.

113. What acts as the sound-board in the harp, and in the violin?

114. State the laws of stretched strings, relating to Pitch; and illustrate them by reference to musical instruments.

115. Give the experimental proofs of the above.

116. Take a stretched string and set it vibrating as a whole. Stop it at half its length and set one of the halves vibrating. Do the same with $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$ and $\frac{1}{8}$, of its length. What notes will be heard in each case, calling the first one $\mathbf{d}_1$? What will be the relative rates of vibration?

117. Define the terms *node*, and *ventral segment*.

118. Describe an experiment to show that a stretched string can vibrate in several segments: say, four.

119. Explain the cause of the occurrence of partials in the tones which are given by stretched strings.

120. How can it be proved that any particular partial—say, the 5th—is produced by the string vibrating in 5 segments with 4 nodes?

121. How could you ensure the absence of the 7th partial in the tone produced by a violin string?

122. How is the quality of tone from a stringed instrument affected by the weight and flexibility of the string?

123. How could you ensure the absence of any given partial in the tone from a stretched string?

124. What is the best method of proceeding, if we wish to ensure the presence of a particular partial in the tone from a stretched string?

125. Explain clearly how nodes are formed in a stretched string. Give an experiment in illustration.

126. What are the principal circumstances, which determine the presence and relative intensities of partials, in the tones of stringed instruments?

127. Explain fully the effect of pressing down the loud pedal in pianoforte playing.

128. I press down the loud pedal of a pianoforte and strike E♭ in the Bass Clef sharply; name the strings that will be set in vibration.

129. How can a Tuning Fork be used, to produce vibrations in a stretched string?

130. Explain how the material of the hammer and the kind of blow, affect the quality of tone in the pianoforte?

CHAPTER X.

131. Describe the phenomenon of the reflection of a sound wave at the end of an *open* pipe.

132. Describe the phenomenon of the reflection of sound at the end of a *stopped* pipe.

133. Define the terms *node* and *ventral segment* as applied to organ pipes.

134. Explain clearly how a node is formed in an open organ pipe.

135. Explain how nodes are formed in a *stopped* organ pipe.

136. Show by diagrams the positions that the nodes may take in an *open* organ pipe, and state the pitch of the tone produced in each case relatively to the fundamental tone of the pipe.

137. Do the same with regard to a *stopped* organ pipe,

138. Explain why only the *odd* series of partial tones occurs in the tones from stopped organ pipes, while open pipes give the complete series.

139. What is the difference between Harmonics on the one hand, and Overtones or Partials on the other?

140. Explain the principle of mixture stops on the organ—the "Sesquialtera" for example.

141. Describe an experimental method of proving the existence of nodes in an organ pipe.

142. I have two organ pipes each 4ft. 4in. long, one stopped and the other open. Calculate the approximate pitch of the fundamental tone in each.

143. What will be the pitch (approximate) of an open organ pipe 6ft. 6in. long? What would be the approximate pitch if it were stopped?

144. To what length (approximate) must an open organ pipe be cut to produce the F^1 on the top of the Treble staff? To what length, if stopped?

145. What is the general effect of rise of temperature upon organ pipes, and why has it this effect?

146. I have two organ pipes, one of metal and the other of wood. They are exactly in unison on a cold day in winter, while on a hot summer's day they are out of tune with one another. Explain the reason of this. Which will be the sharper of the two in warm weather?

147. How is the pitch of a pipe affected by the wind pressure?

148. Detail the various circumstances that affect the pitch of an organ pipe.

149. Why cannot the intensity of the sound from an organ pipe be increased by augmenting the wind pressure?

150. Describe the two kinds of reed used in musical instruments, and explain their action.

151. Explain why the tones of a harmonium are so rich in partials.

152. What is the function of the pipe in an organ reed-pipe?

153. How are reeds sharpened and flattened?

154. What is the effect of a rise of temperature on the pitch of a reed?

155. What is the cause of the peculiar quality of the tones of the Clarionet?

156. Describe the principle of the French Horn, showing the origin of its Tones, and how variation in pitch is effected.

157. To what class of instruments would you assign the Human Voice, and why?

158. What circumstances affect the production of partials, and therefore the quality of the tones of the Human Voice?

159. I sing successively the vocal sounds "a" and "oo;" state briefly the changes that take place in the shape and size of the mouth, in passing from the former to the latter; and show how these changes cause the difference in the above sounds.

160. By increasing the wind pressure in blowing the Flute, the tone rises an octave; in the Clarionet, it rises a Twelfth. Explain this discrepancy.

CHAPTER XI.

161. State the laws connecting the vibration number with the length of a rod, vibrating longitudinally—

1st. When the rod is fixed at both ends.

2nd. When it is fixed at one end only.

3rd. When it is free at both ends.

161*b*. What partials may occur in each of the above cases?

162. State the laws connecting the vibration number with the length of a rod, vibrating transversely—

1st. When the rod is fixed at both ends.

2nd. When it is fixed at one end only.

3rd. When it is free at both ends.

163. What partials may occur in each of the above cases?

164. State what you know of the partials which may be produced by a tuning fork.

165. Give sketches showing the various ways in which a tuning fork may vibrate.

166. When the handle of a tuning fork is applied to a table, the fork's vibrations are communicated to the latter. Explain how this is effected. Account also for the louder sound thus produced.

167. How does change of temperature affect the pitch of a tuning fork?

168. How can the velocity of sound in air be approximately ascertained, with no other apparatus than an open organ pipe giving C=518?

169. Given the velocity of sound in air, how can the velocity of sound in other gases be ascertained?

170. Explain the methods by which the velocity of sound in solid bodies is ascertained.

171. A silver wire is stretched between two fixed points, and caused to vibrate longitudinally. Its length is varied till the tone it produces is in unison with a C^1 fork (C^1=512). It then measures 8ft. 4in. Calculate the velocity of sound in silver.

172. A copper rod 2ft. 10in. long is fixed in a vice at one end and rubbed longitudinally with a resined leather. The tone it emits is in unison with a C^2 tuning fork (=1024). Find the velocity of sound in copper.

173. If a lath of metal 4ft. long, fixed at one end, vibrates laterally once a second; how many vibrations per second will it perform, when its length is reduced to 4in.?

174. How can the nodal lines of a vibrating plate or membrane be discovered? Explain the principle of the method.

175. Describe by means of sketches two or three ways in which a square and a round plate may vibrate.

CHAPTER XII.

176. What are Combination Tones? How many kinds of Combination Tones are there? What are they respectively termed?

177. How is the vibration number of a differential tone related to the vibration numbers of its generators?

178. Describe some method of producing Differential Tones

179. Calculate the Differentials produced by an Octave, Fifth, and Major and Minor Thirds.

180. Calculate the Differentials produced by a Fourth, Major and Minor Sixth, a Tone, and a Semitone.

181. When the following is played on a Harmonium, what Differential Tones may occur?

:s | d' :— | t :l | s :— | f :m | r :— | r :— | d :— | — ||
:m | m :— | s :f | m :— | r :d | d :— | t_1 :— | d :— | — ||

182. Upon what instruments are Differential Tones very prominent?

183. When is the Differential intermediate in pitch between its two generators?

184. What is meant by a Differential Tone of the 1st order? Show by an example how Differential Tones of the 2nd and 3rd order are produced.

185. How is the vibration number of a Summation Tone related to the vibration numbers of its generators?

CHAPTER XIII.

186. Explain by means of diagrams or otherwise, what is meant by Interference of sound waves.

187. What is meant when it is said that two waves are in the same or opposite *phase?*

188. Describe and sketch any apparatus that may be used to demonstrate the fact, that two sounds may be so combined as to produce silence.

189. Describe by help of a diagram the sound waves that emanate from a vibrating fork, clearly showing their alternate phases.

190. If a tuning-fork be revolved before the ear, alternations of intensity are observed; clearly explain the whole phenomenon and its cause.

191. How may Chladni's plates be used to illustrate the phenomenon of interference?

192. How are the intensities of sounds related to the amplitudes of their corresponding sound-waves?

193. What is a *beat*? What is the law connecting the vibration numbers of two tones and the number of beats they generate per second?

194. Carefully explain how a *beat* is produced.

195. How would you proceed to prove experimentally that the rapidity of the beats increases as the interval between the two generating tones increases?

196. Explain the principle of the Tonometer.

197. A Tonometer consists of 50 forks, each fork is 4 vibrations per second sharper than the preceding, and the extreme forks form an exact Octave. What is the pitch of the lowest fork?

198. Two harmonium reeds when sounded together produce 4 beats per second. The pitch of one of them is then slightly lowered, and on again sounding it with the other, two beats per second are heard. It is then further flattened and again two beats per second are heard. Was it originally sharper or flatter than the unaltered one? Answer fully.

199. What is the use of the Tonometer? Describe the method of using it.

200. Why is the Reed Tonometer inferior in accuracy to the tuning-fork Tonometer?

CHAPTER XIV.

201. What use is made of slow beats in music?

202. What is the physical cause of dissonance? How would you experimentally prove your answer to be correct?

203. What is meant by the *Beating Distance?* State generally how it varies in different parts of the scale.

204. How can the phenomenon of beats be imitated by the use of one sound only?

205. Why are beats unpleasant to the ear?

206. Explain a method of detecting very faint musical sounds by means of beats.

207. Why do beats cease to be unpleasant when they are sufficiently rapid?

208. Upon what does the harshness of a dissonance depend.? Illustrate by examples.

209. Show by examples that the harshness of a dissonance does not depend entirely on the rapidity of beats.

210. What are the Beating Distances, in the regions of C_2, C_1, C, C^1, and C^2?

211. "The sensation of a musical tone in the region of $C_2 = 64$ persists for $\frac{1}{16}$ of a second after the vibrations that give rise to it have ceased." What evidence is there for this statement?

212. What is the cause of Dissonance between Simple Tones that are beyond Beating Distance?

213. Two forks, the vibrational numbers of which are 100 and 210, dissonate when sounded together. Explain why. How many beats per second may be counted?

214. Two forks, the vibration numbers of which are 200 and 296, generate slow beats when sounded together. Explain the cause of this. How many beats per second will be heard?

215. Show that a compound tone may contain dissonant elements in itself.

216. Show clearly, how it is that $\left\{\begin{matrix}\mathrm{f}\\ \mathrm{m}\end{matrix}\right.$ is a harsher discord than $\left\{\begin{matrix}\mathrm{f}^1\\ \mathrm{m}\end{matrix}\right.$ and $\left\{\begin{matrix}\mathrm{l}\\ \mathrm{t}_1\end{matrix}\right.$ than $\left\{\begin{matrix}\mathrm{l}\\ \mathrm{t}_2\end{matrix}\right.$.

217. The interval $\left\{\begin{matrix}\mathrm{f}\\ \mathrm{s}_1\end{matrix}\right.$ sounds less harsh when played by two clarionets, than when played on a harmonium. Explain the reason.

218. Show by sketches the beating elements present in a Major and Minor Third when played on a piano.

219. Two harmonium reeds, the vibration numbers of which are 199 and 251, produce slow beats when sounded together. Explain the reason. How many beats per second will be heard?

220. Two forks, the vibration numbers of which are 256 and 168, produce faint beats when sounded together. How many beats per second will be heard?

221. How many *audible* beats per second will two harmonium reeds generate, which vibrate 149 and 301 times per second respectively?

CHAPTER XV.

222. How is the interval of an Octave between *Simple Tones* defined?

223. How would you proceed to tune two *Simple Tones* to the interval of a perfect Fifth; 1st, if you had no other tones to assist you, and 2nd, if you already possessed the Octave of one of the tones?

224. How are the intervals of a Fifth and Fourth between *Simple Tones* defined?

225. How are the intervals of an Octave and a Fifth between Compound Tones defined?

226. Explain the principle involved in tuning two violin strings at the interval of a perfect Fifth.

227. In the interval of a Fourth, why is it necessary that the vibration numbers of the two tones should be in the exact ratio of 4 : 3?

228. If two harmonium reeds vibrate 501 and 399 times per second respectively, how many beats per second will be heard when they are sounded together?

229. The vibration numbers of the C and E♭ in an equal-tempered harmonium are 264 and 314 respectively; how many beats per second will be heard when they are sounded together?

230. On an equal-tempered harmonium, C = 264 and A = 444. When they are sounded together, how many beats per second will be heard?

231. Two harmonium vibrators an exact Octave apart are sounded together. Explain fully the result of again sounding them together after one of them has been flattened by one vibration per second.

232. How is the Octave defined, in the case of stopped organ pipes, the tones of which consist of the 1st and 3rd partials only?

233. Which is the easier interval to tune, and why:—the Fifth or Major third?

234 Two tones are sounded on a harmonium, the vibration numbers of which are 300 and 401 respectively. How many beats per second may be counted?

CHAPTER XVI.

235. What musical intervals are perfectly harmonious under all circumstances? Why are they so?

236. Why does a Fifth sound rough when very low in pitch?

237. How is it that singing in Octaves is usually styled singing in *unison?*

238. Compare theoretically the following Fifths as played 1st on a harmonium, and 2nd on a stopped diapason:— 

239. Compare the relative smoothness of a Fourth and Major Sixth under similar conditions of quality and pitch.

240. Compare the relative smoothness of Major and Minor Thirds between tones of ordinary quality.

241. Why can the Major Third be used at a lower pitch than the Minor Third?

242. Account for the inferiority of the Minor Sixth to its inversion, the Major Third.

243. Mention any facts that serve to explain why Thirds were not admitted to the rank of Consonances until comparatively recent times.

244. Why do such intervals as Thirds, Sixths, &c., sound smoother on the Stopped than on Open pipes?

245. In a duet for Oboe and Clarinet, the Fifth sounds smoother when the latter instrument takes the lower tone, but the Fourth is smoother when the Oboe takes the lower tone. Explain this.

246. Compare in smoothness the intervals of a Fifth and a Twelfth.

247. Compare the Thirds and Tenths in smoothness.

248. Show which are the better, Thirteenths or Sixths.

249. Compare the Fourth with the Eleventh.

250. Give general rules referring to the relative smoothness of an interval, and its increase by an Octave.

CHAPTER XVII.

251. Name, and give the vibration ratios of the Consonant intervals smaller than the Octave.

252. Combine the above, two at a time and calculate the vibration ratios of the intervals thus formed, which are less than an Octave.

253. Show from the above, that there are only six consonant triads within the Octave.

254. Name the six consonant triads, and show how they may be considered as derived from two.

255. Show on physical grounds, that the 1st inversion of the Major triad is inferior to its other two positions.

256. Prove from acoustical considerations, that the 2nd inversion of a Minor Triad is inferior to its other two positions.

257. Account for the fact that  is more harmonious than 

258. Why are Major Triads as a rule more harmonious than Minor Triads?

259. In selecting the most harmonious distributions of the Major Triad, what intervals should be avoided, and why?

260. Give examples of the more perfect and less perfect distributions of the Major Triads.

261. What considerations guide us in selecting the more harmonious distributions of the Minor Triad?

262. What three distributions of the Minor Triad have only one disturbing differential?

263. How are consonant Tetrads formed from consonant Triads?

264. Compare the relative harmoniousness of the following two Tetrads after the manner of Fig. 87:— KEY F. $\left\{\begin{matrix} d^{\prime} \\ m \\ s \\ d \end{matrix}\right.$ and KEY D♭. $\left\{\begin{matrix} s \\ d \\ s \\ m \end{matrix}\right.$

265. What considerations serve as a guide in selecting the best positions and distributions of a Major Tetrad? Give Helmholtz's rule.

266. Why may neither the 3rd nor 5th of a Major Triad be duplicated by the double Octave?

267. Give examples of good positions and distributions of the Major Tetrad.

268. State any rule you know of, concerning the distribution of the 1st inversion of the Major Tetrad.

269. Within what limits should the 2nd inversion of a Major Tetrad lie?

270. Write down a Minor Tetrad which has only one disturbing differential.

271. State any rule you know of, concerning the distribution of the 2nd inversion of the Minor Tetrad.

272. Within what limits should the 1st inversion of a Minor Tetrad lie?

CHAPTER XVIII.

273. Given that the vibration ratios of the tones of a Major Triad are as 6 : 5 : 4, show that $\left\{\begin{matrix} l \\ r \end{matrix}\right.$ is not a perfect Fifth.

274. With the data of the last question show that $\left\{\begin{matrix} f \\ r \end{matrix}\right.$ is not a true Minor Third.

275. Calculate, from the data of 273, the vibration ratio of $\left\{\begin{matrix} \grave{r} \\ r \end{matrix}\right.$.

276. When is **rah** required, (1) in harmony (2) in melody?

277. What are the vibration ratios of $\left\{\begin{matrix} \grave{m} \\ r \end{matrix}\right.$ and $\left\{\begin{matrix} \grave{r} \\ d \end{matrix}\right.$?

278. How many new tones are required to form the Major Diatonic Scale of the new Key, when a transition of one remove to the right occurs? What are they? Show that your answer is correct.

279. When a transition of one remove to the left occurs in a piece of music, show that two new tones will be required to form the Major Diatonic Scale of the new key.

280. Show what new tones will be required to form the Major Diatonic Scale, when a piece of music changes suddenly from C to E.

281. Show what new tones will be required, when a piece of music passes suddenly from the Key of C Major into that of A♭ Major.

282. If music in C Major passes through the Keys of F and B♭ to E♭, show that the C of this last Key is not of the same pitch as the original C.

283. What is meant by temperament?

284. Starting with **d**, if four true fifths be tuned upwards, and then two octaves downwards, show that the note thus obtained is one comma sharper than the true **m**.

285. Show how the twelve notes of the Octave, in mean-tone temperament, are determined.

286. Why is mean-tone temperament so called?

287. Compare the Consonant intervals in mean-tone temperament with the same in true intonation.

288 What is the principle of Equal Temperament?

289. What is the Comma of Pythagoras? How is it obtained?

290. Compare the consonant intervals in equal temperament with the same in true intonation.

291. Compare the consonant intervals in equal, with the same in mean-tone temperament.

292. What are the advantages and disadvantages of (1) mean-tone temperament, (2) equal temperament?

293. In equal temperament, if C=522, what should be the vibration number of C♯?

294. Taking A=435, calculate the vibration numbers of the other eleven tones of the scale in equal temperament.

295. Taking C=522·, calculate the vibration numbers of the notes of the diatonic scale in true intonation.

296. Taking C=522, calculate the vibration numbers of the notes of the diatonic scale in equal temperament.

297. Taking C=522, calculate the vibration numbers of the notes of the diatonic scale in mean-tone temperament.

298. Ascertain the vibration numbers of C♯, E♭, G♯, F♯, and B♭ in question 295.

299. Ascertain the vibration numbers of C♯, E♭, G♯, F♯, and B♭ in question 296.

300. Ascertain the vibration numbers of C♯, E♭, G♯, F♯, and B♭, in question 297.

THE ROYAL COLLEGE OF ORGANISTS.

1896 and 1897.

1. Write out the harmonic series up to the twelfth harmonic commencing with the following note as the generator.—

Answer.

* Approximately.

2. Explain the difference between equal temperament and just temperament.

Ans.—This question is wrongly worded; there is no such thing as just temperament, just or true intonation is meant. In true intonation, the relations betwen the tones of the scale are those given on p. 231, middle column. In equal temperament the octave is divided into twelve equal parts, each part forming an equal tempered semitone, two of these semitones forming a tone. Consequently none of the equal tempered intervals, except the octaves, are the same as in true intonation. The amount by which these intervals differ from the true intervals is given on pp. 241 and 242.

3. What is the vibration ratio of a minor tone? State between which degrees of the major diatonic scale a minor tone occurs when tuned according to just temperament.

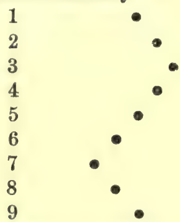
Ans.—Temperament should be intonation. Vibration ratio of a minor tone is $\frac{10}{9}$. Between 2nd and 3rd, and 5th and 6th—that is, between r m, and s l.

MUS.BAC. (Cantab.).

MAY 21, 1895. 9 till 12.

1. **Explain with help of a diagram how a given particle of the air alters its position as a train of sound-waves passes over it. Show how we can determine graphically what happens when two trains of waves pass over the particle simultaneously.**

Ans.



Let the dot on line 1 represent the particle at rest. When the condensed part of the wave reaches it from left, it began to travel to the right, as seen in line 2. In 3, it has reached its extreme position and begins to return as at 4. In 5, it has reached its orignal position, and is still travelling towards left as at 6. In line 7, it has reached its extreme left hand position, and then returns, as at 8, to its original position at line 9, the wave having now just passed over it.

The second part of the question is fully answered at p. 82.

2. Describe experiments that show the intensity of sound to be connected with the amplitude of vibration of the air, and the pitch with the period of vibration.

Ans.—For first part, see p. 52, last eight lines, and p. 53, first fourteen. For second part, either Wheel Syren, pp. 29 and 30, or the Syren of Cagniard de Latour, pp. 31 and 32, or Savart's Toothed Wheel, p. 6.

3. (*a*) **Explain how or why a rise of temperature affects the pitch of the wind instruments in an orchestra.**

(*b*) If the velocity of sound is 1,120 ft. per sec. at 60° and 1,140 ft. per sec. at 77°, how much would a trumpet player have to alter the length of the tube of his instrument in order to keep to his original pitch, if the temperature of the concert room rose from 60° to 77°? (Assume the length of tube in a trumpet to be 5ft.).

Ans. (*a*) See p. 100.

(*b*) Length of trumpet is 5ft., therefore length of sound-wave at 60° is 10ft., and vibration number of fundamental is $\frac{1120}{10} = 112$.

Hence length of sound-wave at 77° must be $\frac{1140}{112} = \frac{285}{28} = 10\frac{5}{28}$: and length of trumpet must be $5\frac{5}{56}$ ft. The player therefore would have to lengthen his instrument by $\frac{5}{56}$ ft., that is $\frac{5}{56} \times \frac{12}{1} = \frac{60}{56} = \frac{15}{14} = 1\frac{1}{14}$ in.

4. (*a*) Describe the construction of the principal classes of pipes used in an organ, explaning how each is tuned.

(*b*) What is the scientific explanation of the effect of mixture stops?

Ans. (*a*) See pp. 99 and 112, for construction; and pp. 100 and 112, for tuning.

(*b*) See p. 109.

5. (*a*) State how the pitch of a vibrating string depends on the density, tension, and length of the string.

(*b*) Explain the application of these laws to the construction, method of tuning, and use of a violin.

Ans. (*a*) See p. 87, (4) (3) and (1).

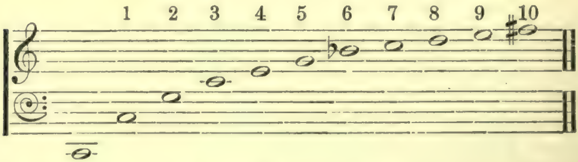
(*b*) See pp. 88, 89, 90.

6. (*a*) What are partials or overtones?

(*b*) Give the first ten overtones of an open pipe whose fundamental is—

(*c*) State which of these would differ most markedly from the corresponding notes on an equally tempered pianoforte, and which would agree most nearly.

Ans. (*a*) See p. 84.



6 and 10 only represent the pitch approximately.

1, 3, and 7 would agree perfectly.

2 and 5 would agree very nearly.

8 would agree fairly well.

4 and 9 would be $\frac{9}{11}$ comma sharp.

7. A vibrating tuning-fork is held over a tall cylinder, into which water is gradually poured.

(*a*) Describe and explain the variation that takes place in the sound of the fork.

(*b*) How could you employ this apparatus to find the velocity of sound, the period of vibration of the fork being given.

Ans. (*a*) Probably at first the sound of the fork would not be much altered, but as the water is gradually poured in, the sound will gradually increase in intensity up to a maximum and then fall off again. If the cylinder is long enough and the water is still gradually poured in, this effect may be repeated. For explanation, see pp. 61 and 62. For explanation of repetition, it will be seen from pp. 61 and 62 that resonance takes place when length of vibrating column is $\frac{1}{4}$, $\frac{3}{4}$, or $\frac{5}{4}$, &c., the length of sound-wave, but the maximum intensity is obtained in first case.

(*b*) First ascertain by repeated trials, the length of air column that gives maximum resonance; measure this length; add to it $\frac{4}{5}$ radius of cylinder; multiply four times this result by the vibration number of the fork.

8. (*a*) Discuss the relative consonance of an octave, a fourth, and a minor third.

(*b*) Why is it that in any system of temperament, the octaves must be true, whilst the minor thirds may be considerably different from true minor thirds?

Ans. (*a*) See pp. 187, 188, 189, 191.

(*b*) This follows from the perfect definition of the octave and the vague definition of the minor third. See pp. 177 and 181.

9. Two tuning-forks very nearly an octave apart and free from overtones give beats when sounded together. What is the cause of the beats?

Ans.—Say vibration numbers of forks are 202 and 400. These generate a third tone the vibration number of which = 198; and this with the lower fork produces 202—198 = four beats per sec.

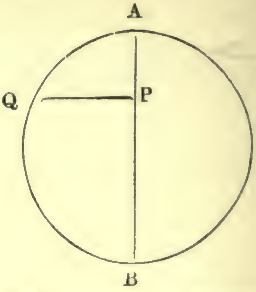
MUS. BAC. (Cantab.).

MAY 26, 1896. 9 till 12.

1. (*a*) Explain the meaning of the terms—simple harmonic vibration, wave front, difference of phase, wave length.

(*b*) Give a diagram illustrating the motion of a series of particles as a transverse wave passes over them.

(*c*) Show that the wave length is equal to the velocity of the wave multiplied by the time of vibration of one of the particles.



Ans. (*a*) Suppose a point Q revolves round the circle A Q B with uniform speed, and also let a point P be vibrating to and fro, along the line A D, then if the imaginary straight line connecting P and Q is always parallel to P Q, then point P is said to be executing simple harmonic or pendular vibrations. See p. 81. In figure 30, p. 54, in which the vibrating body is supposed to be at A; the spherical surface at D is a wave front, that is, a continuous surface, at every point of which the air is in the same stage of vibration at the same moment. B and C are also wave fronts.

Suppose two series of equal waves pass over the same particles of air, if one series is half a wave length before the other, they are said to be in opposite phase; if one is a quarter wave length behind the other, the difference of phase is one quarter; and so on. See pp. 136, 137, and 144. For wave length see p. 22.

(*b*) See either p. 12 or 14.

(*c*) It is shown on p. 13 that each particle makes one complete vibration in same time as wave takes to travel its own length. Now if n be vibration number of the sound, $\frac{1}{n}$ is the time it takes to travel a wave length—that is, the time of vibration of each particle. Now it is shown on p. 39 that wave length = velocity of sound (v) divided by the vibation number (n) or

wave length $= \frac{v}{n}$

(*i.e.*) ,, $= v \times \frac{1}{n}$

,, = velocity of wave × time of vibration of one of the particles.

2. (*a*) Describe some accurate method of finding the velocity of sound in air.

(*b*) How would the velocity be affected if the height of the barometer changed, or

(*c*) If the temperature changed?

Ans. (*a*) See pp. 19 and 20. The air between the two stations should be at rest.

(*b*) Not at all.

(*c*) Velocity is increased by a rise of temperature. See pp. 20 and 100.

3. (*a*) A string attached to two rigid supports gives out very little sound when made to vibrate. If attached to a sound board the sound is much louder. Explain this and give instances of similar effects.

(*b*) Does the presence or absence of the sound-board have any effect on the length of time the string will continue to vibrate?

Ans. (*a*) See pp. 86 and 87.

(*b*) Yes; the string if connected with a sound-board does not vibrate so long as it would if it were not so connected.

4. (*a*) When a closed organ pipe is blown too strongly the note goes up a twelfth. In the case of an open pipe, it goes up an octave. Give the reason for this.

(*b*) Calculate the approximate vibration number of a closed organ pipe 2 ft. long, assuming the velocity of sound to be 1,100 ft. per sec.

Ans.—See pp. 107, 108, and 102.

Approximate vibration number $= \frac{1100}{2 \times 4} = 137{\cdot}5$

5. (*a*) Why is the interval beween two notes estimated by the ratio of the vibration numbers and not by their difference?

(*b*) What are the vibration ratios of a fifth, a major third, and an equal temperament semitone.

Ans. (*a*) Because the ratio of the vibration numbers for any particular interval is constant; while the vibration numbers themselves vary with the pitch, and therefore their difference would vary.

(*b*) $\frac{3}{2}$; $\frac{5}{4}$; $\sqrt[12]{\frac{2}{1}} = \frac{53}{50}$ nearly.

6. Explain the production of the scale on the slide trombone.

Ans.—See p. 114.

7. Why are the various notes of a flute or clarinet put out of tune with each other when the joints of the instrument are pulled out so as flatten the pitch as much as possible.

Ans.—The fundamental note of such an instrument is that due to the whole length from mouth-piece to the other end. The other notes are due to the distance between the mouth-piece and the apertures corresponding to these notes. Now these distances are at certain *ratios* to the whole length. If the joints be lengthened *all* these ratios are changed, and therefore the intervals are not the same as before.

8. (*a*) Investigate by Helmholtz's method the relative consonance of a fourth and a major third.

(*b*) Why does the ear recognize want of correct intonation in the case of the octave more easily than in the case of the major third?

Ans. (*a*) See p. 191.

(*b*) In the octave, powerful beats are found between loud first and second partials. In the major third much fainter beats between weaker fourth and fifth partials, see pp. 176 and 180.

ROYAL UNIVERSITY OF IRELAND.

Mus. Bac. First Examination, 1895.

1. Male and female voices differ in pitch and quality. Acount for these differences.

Ans.—The larynges of women are smaller than those of men, the vocal cords are smaller and vibrate more quickly, hence the difference in pitch. When a man and a woman are singing a tone of same pitch and with about the same intensity there is no specific difference of quality. A man's low tones are richer in partials than a woman's high tones, hence the corresponding difference in quality.

2. (*a*) What is meant by the wave-length of a note?

(*b*) When sound passes from one medium to another (as from one gas to another of different density), what change, if any, takes place in the wave-length?

Ans. (*a*) See p 22.

(*b*) The velocities of sound in different gases vary inversely as the square roots of their densities. (See p 20.) Therefore when a sound passes from one gas into another of greater density its velocity is diminished. Now the wave length equals the velocity divided by the vibration number (see p. 39); therefore, in this case, the wave-length is diminished. Similarly, if a sound passes into a gas of less density its wave-length is increased.

3. Describe a method of accurately determining the velocity of sound through air.

Ans.—See p. 268.

The result will be the more accurate, the farther the experimenters are apart. It will also be more exact, if *each* observer make a signal, say fires a cannon and *both* note the time between the flash and the report, the mean of the two observations being taken.

4. (*a*) The temperature of air through which sound-waves are propagated is supposed to be subject to changes of an alternating character.

Describe the nature of these changes, and give an explanation of them.

(*b*) Is there any change of temperature of a continuous character, and if so, to what would you attribute it?

Ans. (*a*) When a gas is compressed, its temperature rises, heat being evolved; when a gas expands freely, its temperature falls. Hence the temperature of the air through which a sound-wave passes rises very slightly at a point through which a condensation is passing, and falls very slightly when the rarefaction follows.

(*b*) The temperature of the air through which a series of sound-waves is passing is very slightly raised in temperature, in consequence of the kinetic energy of the moving air particles being slowly transformed into the kinetic energy of heat, just as the energy of a train in motion is changed into heat when the brakes are applied and it is brought to rest.

5. Two organ pipes, one open and the other closed, are sounding the same fundamental note.

(*a*) What is the ratio of the lengths of the pipes?

(*b*) Are the notes exactly alike?

(*c*) And if not, account for their difference.

Ans. (*a*) Length of the open pipe : length of the closed pipe as 2 : 1.

(*b*) No.

(*c*) The notes are both more or less compound, that from the open pipe will consist of the fundamental, its octave, octave fifth, and so on; while that from the closed pipe will consist of the partials of the odd series only, viz., the fundamental, octave fifth, &c.

6. (*a*) What is meant by resonance, and how are resonant effects explained?

(*b*) A column of air resounding to a note 256 complete vibrations per second is obtained by filling a tall narrow jar with water, and then allowing the water to escape gradually through a stop-cock at the bottom, till maximum resonance is obtained. Find at least two other notes to which the same column of air would resound.

Ans. (*a*) See pp. 67 and 58.

(*b*) $256 \times 3 = 768$ and $256 \times 5 = 1280$.

ROYAL UNIVERSITY OF IRELAND.

Honour Examination in Music, 1895.

1. Give an expression for the velocity of propagation of transverse vibrations along a stretched string. Deduce Bernoulli's Laws for the vibrations of stretched strings.

Ans.—See p. 87.

2. Explain how it is that in a column of air in stationary vibration, the length of a ventral segment is a half-wave length.

Ans.—See pp. 102, 105, 106.

3. (*a*) What is meant by a gamut of equal temperament?

(*b*) Find the arithmetical value of the interval of a tone on this gamut.

Ans. (*a*) See p. 238.

(*b*) Nearly $\frac{1{\cdot}1225}{1} = \frac{449}{400}$

4. Describe the changes in form observed in the Lissajous' figure obtained by using two tuning-forks, the pitch of one being nearly the octave of the other.

Ans.

5. (*a*) How would you produce a flame sensitive to sound.

(*b*) Give a method of measuring the wave-length of a note of very high pitch by means of a sensitive flame.

Ans. (*a*) Burn coal gas from a pin-hole burner at a pressure of ten inches of water. Adjust the supply of gas by the tap, so that the flame is just on the point of flaring. The flame, which is now from 15 to 20 inches long, is very sensitive to any movement of the air just above the burner.

(*b*) Place the source of sound at some distance from a wall. The direct and reflected waves will then interfere, producing stationary nodes along a line drawn from the source of sound at right angles to the wall; and if the jet of the sensitive flame is moved along this line it will flare everywhere except at the nodes, the positions of which are thus discovered. The wave-length is obviously twice the distance between two consecutive nodes.

6. Describe the construction and uses of Helmholtz' Double Syren.

Ans.—For construction, see pp. 33 and 34.

Uses: 1st, To ascertain the vibration number of any note, by means of a counting apparatus not shown in fig. 22. 2nd, To demonstrate the exact vibration ratios of the most important musical intervals. 3rd, To illustrate and prove the fundamental facts concerning the theory of beats. Beats are produced in this instrument by rotating the handle of the wheel shown in fig. 22 at the top right hand side. When this wheel is rotated in one way the upper sound chest revolves in same direction as the discs, thus diminishing the vibration number of the upper syren; when rotated the other way, the upper sound chamber turns in the contrary direction to the discs, so increasing the vibration number of this same syren. Beats are produced in either case.

Honour Examination in Music.

1 and 2. Not acoustical questions.

3. (*a*) Distinguish between Difference and Summation Tones.

(*b*) And give in approximate notation, those which are attached to the octave, fifth, fourth, and major third.

Ans. (*a*) See p. 135.

(*b*) See pp. 130 and p. 134.

4. In equal temperament, to what extent does each interval of the major scale depart from accuracy?

Ans.—See pp. 241, 242, 244

5 and 6. Not acoustical questions.

7. Write out the fractions representing the vibration ratios which each interval of the major scale bears to that next below it.

Ans.—See p. 227.

8 (*a*) What was the Mean Tone system of Temperament?

(*b*) About what period was it in general use?

Ans. (*a*) See pp. 234 to 238.

(*b*) It prevailed all over the Continent and England for centuries. It disappeared from pianofortes in England about 1840, but at a later date from organs. All the organs at the Exhibition of 1851 were tuned in this temperament.

9. Describe any one of the modern contrivances for obtaining "just intonation."

Ans.—See pp. 245, 246.

10. Not an acoustical question.

ROYAL VICTORIA UNIVERSITY.

FIRST MUS.B, 1896. THREE HOURS.

1. (*a*) How has it been ascertained that the velocity of sound in air is about 1,120 ft. per second?

(*b*) How does a rise in the thermometer affect the velocity?

Ans. (*a*) See p. 270

(*b*) See p. 100.

2. Define the terms *amplitude*, *wave-length*, *phase*, and *form* of a vibration.

Ans.—See p. 22 and p. 268.

3. From the major and minor harmonic triads 1, $1\frac{1}{4}$, $1\frac{1}{2}$; and 1, $1\frac{1}{5}$, $1\frac{1}{2}$, deduce the intervals of (*a*) the diatonic scale; (*b*) comma, (*c*) Pythagorean comma, and (*d*) diesis.

Ans. (*a*) Multiply the ratios $1 : 1\frac{1}{4} : 1\frac{1}{2}$ by 4, and we get $4 : 5 : 6$. Then proceed as at pp. 45, 46, and 47.

(*b*) See p. 50.

(*c*) See p. 239.

(*d*) The Greater Diesis is the difference between an octave and three major thirds.

$$= \frac{2}{1} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{128}{125}$$

The Smaller Diesis is difference between five major thirds and a twelfth.

$$= \tfrac{5}{4} \times \tfrac{5}{4} \times \tfrac{5}{4} \times \tfrac{5}{4} \times \tfrac{5}{4} \times \tfrac{1}{3} = \tfrac{3125}{3072}$$

4. If a copper wire, density 9, 75 cm. in length, and ·6 m.m. in thickness, gives 200 vibrations per second when stretched by a weight of 10 kilogrammes, how many vibrations per second will a steel wire, density 7·5, one metre in length, and 1 m.m. in thickness, give when stretched by 27 kilogrammes?

Ans.—By the laws given on pp. 87 or 96 number of vibrations per sec.

$$= 200 \times \frac{\sqrt{9}}{\sqrt{7 \cdot 5}} \times \frac{75}{100} \times \frac{\cdot 6}{1} \times \frac{\sqrt{27}}{\sqrt{10}} =$$

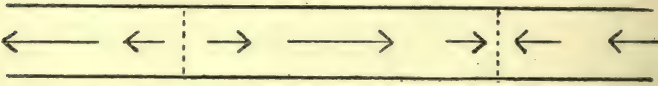
$$\frac{200 \times 3 \times 6 \times 3 \times \sqrt{27}}{4 \times 10 \times \sqrt{7 \cdot 5} \times \sqrt{10}} = 5 \times 3 \times 6 \times 3 \times \frac{\sqrt{27}}{\sqrt{75}} =$$

$$270 \times \frac{\sqrt{9}}{\sqrt{25}} = 270 \times \frac{3}{5} = 162$$

5. (*a*) Find all the overtones that are possible in an organ pipe closed at one end. (*b*) Draw a figure to show the directions in which the air particles are moving at a given instant when the pipe is emitting its first overtone.

Ans. (*a*) See pp. 106 and 107.

(*b*)



6. Give a brief account of what causes difference in timbre or quality in sounds.

Ans.—See pp. 74 and 75.

7. (*a*) Describe how reeds vibrate.

(*b*) Explain why the tones they give are generally harsh.

Ans. (*a*) See p. 110.

(*b*) See p. 111.

8. (*a*) Explain what is meant by the interference of two sounds.

(*b*) Describe experiments illustrating it.

Ans. (*a*) See pp. 136, 137.

(*b*) See p. 139 or 140 or 141.

9. Draw diagrams to show the result of compounding a note with its octave in two different phases.

Ans.—See p. 83.

UNIVERSITY OF LONDON.

INTERMEDIATE MUS.B. EXAMINATION, 1895.

Morning, 10 to 1.

1. (*a*) How would you prove that the pitch of a note depends solely on the number of vibrations received per second by the hearer.

(*b*) And that the same number per second always gives the same note.

Ans. (*a*) In Savart's Toothed Wheel and in the Syren, as the speed increases or decreases, the pitch rises or falls, and nothing is altered except the number of vibrations received per second by the hearer.

(*b*) Tuning-forks vibrating the same number of times per second are found to give the same note. The same with other instruments.

2. (*a*) Describe and explain the mode of using a tonometer, consisting of a series of forks for the determination of frequency of vibration.

(*b*) For what reason are forks better than reeds in such a tonometer?

Ans. (*a*) See pp. 150 to 152.

(*b*) See p. 151.

3. (*a*) How would you produce (1) transverse and (2) longtitudinal vibrations in a string; and (*b*) how would you in each case show that the vibrations were of the kind stated?

(*c*) How would you obtain the various harmonics?

Ans. (*a*) (1) See p. 86. (2) See p. 118.

(*b*) By placing a rider on the string; or if the vibrations were very small, by viewing an illuminated point on the vibrating string with a low power miscroscope.

(*c*) (1) See p. 92. (2) See p. 118.

4. A telegraph wire is 50 metres long, and is stretched with such a force that a transverse wave travels along it with velocity 125 metres per second, while a longitudinal wave travels with velocity 3,700 metres per second. (*a*) Find the frequency of the fundamental mode of vibration for each kind of vibration. (*b*) To what kind of vibration do you think it most likely that the sound heard at a telegraph pole belongs? (*c*) Give a reason for your opinion. (*d*) How would you explain the beating often heard near the pole?

Ans. (*a*) Transverse wave.

$$N = \frac{V}{2L} \quad \text{(See p. 87.)}$$

$$\text{Therefore } N = \frac{125}{1} \times \frac{1}{100} = 1\tfrac{1}{4}$$

Longitudinal wave.

$$N = \frac{V}{26} \quad \text{(See p. 119.)}$$

$$= \frac{3700}{100} = 37$$

(*b*) Transverse vibrations; harmonics.

(*c*) Exciting agent: the wind which cannot excite longitudinal vibrations.

(*d*) The beating is due to high partials.

5. (*a*) What is meant by a combination tone?

(*b*) How may one be produced so as to be directly audible?

(*c*) Give a general explanation of the production of such a tone.

Ans. (*a*) See p. 135.

(*b*) See p. 131.

(*c*) See pp. 133 and 134.

AFTERNOON. 2 to 5.

1 (*a*) What effect will be produced by a rise in temperature on the pitch of the notes given out (1) by stretched strings, (2) by organ pipes? (*b*) An organ pipe sounds at 0° C. a note with 256 vibrations per second. What will be the frequency of the note given out by the same pipe at 20° C.?

Ans. (*a*) 1. See p. 90. 2. See p. 100.

(*b*) $\text{Frequency} = 256 \sqrt{1 + \frac{20}{273}}$ (See p. 100.)

$= 265{\cdot}2$

2. The velocity of sound through air at 0° C. is 1,100 ft. per second. What will be the velocity of sound through hydrogen at the same temperature? 1 litre of hydrogen weighs ·0896 grms.: 1 litre of air, 1·293 grm. at 0° C. and atmospheric pressure.)

Ans.—Velocity in hydrogen

$$= 1100 \times \sqrt{\frac{1.293}{{\cdot}0896}} \quad \text{(See p. 20.)}$$

$$= 4{,}179 \text{ ft. per second.}$$

3. (*a*) Explain, by the aid of carefully drawn diagrams, how beats are produced, and

(*b*) Show how to find the number of beats per second when the frequencies of the component beats are given.

Ans.—See p. 145.

(*b*) See p. 153.

4 (*a*) Define the terms node and loop.

(*b*) How would you demonstrate their existence (1) in vibrating strings, (2) in organ pipes?

Ans. (*a*) See p. 96 (loop = ventral segment) and p. 102.

(*b*) (1) See p. 92.

(2) See pp. 103 and 104.

5. If the stem of a tuning-fork is pressed against a table, the sound is much louder than before.

(*a*) Explain this and (*b*) give as many illustrations as you can of the same principle.

(*c*) Will the tuning-fork vibrate for a longer or shorter time when pressed against the table than when held in the hand.

Ans. (*a*) See p. 123 and p. 186.

(*b*) String and sound-board. See p. 186.

(*c*) See p. 64.

1896.

MORNING. 10 to 1.

1. How has the velocity of sound been determined (1) in air, (2) in water?

Ans. (1) See page p. 270.

(2) In a similar manner. The sound was made by a hammer striking a bell under water, the same lever which moved the hammer igniting some gunpowder above. The sound through the water was perceived by lowering into the water the wide end of an ear trumpet covered with membrane and turned towards the direction of the sound.

2. Describe fully the apparatus you would use, and the mode in which you would use it, to prove that the frequencies of C, E, and G are in the ratios of 4 : 5 : 6.

Ans.—See Helmholtz' Syren, pp. 33, 34, and 45.

3. A steel wire, density 7·8 is stretched by a weight of 39 lbs. and gives a note of frequency n. A string, density 1, has double the length and double the diameter of the steel wire. With what weight must it be stretched to give frequency $\frac{n}{2}$

Ans.—Let l and d denote length and density of steel wire, and let x denote weight required then, from p. 87

$$n = \frac{\sqrt{39}}{l \times d \times \sqrt{7{\cdot}8}} = \frac{\sqrt{5}}{l \times d}$$

$$\text{and } \frac{n}{2} = \frac{\sqrt{x}}{2\,l \times 2\,d \times \sqrt{1}} = \frac{\sqrt{x}}{4\,l \times d}$$

$$\text{therefore } n \times \frac{2}{n} = \frac{\sqrt{5}}{l\,d} \times \frac{4\,l\,d}{\sqrt{x}}$$

$$i.e. \qquad 2 = \frac{4\sqrt{5}}{\sqrt{x}}$$

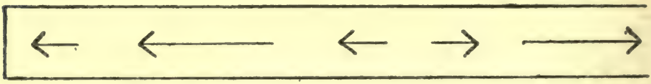
$$i.e. \qquad \sqrt{x} = 2\sqrt{5}$$

$$\text{or} \qquad x = 20$$

4. An organ pipe, open at both ends, is blown so hard that it sounds only the octave of the fundamental tone. (*a*) Describe the mode of vibration. (*b*) Draw a figure indicating the directions of motion in the different parts of the pipe at a given instant. (*c*) How would you experimentally show the positions of the nodes?

Ans. (*a*) See p. 103.

(*b*)



(*c*) See pp. 103 and 104.

5. Give a brief account of Helmholtz' Theory of Dissonance.

Ans.—See Summary, p. 172.

AFTERNOON. 2 to 5.

1. How could you show that the velocity of sound through air at a constant temperature is independent of the pressure?

Ans.—The pitch of an organ pipe depends upon the velocity. Therefore, by showing that the same pipe, at same temperature gave a note of precisely the same pitch upon two different days when the pressure of the air was distinctly different, the above point would be proved.

2. Explain by the use of diagrams, how the quality of the note emitted by a plucked string depends upon the manner of plucking.

Ans.—See pp. 94 and 95.

3. Explain why sound travels badly against the wind.

Ans.—The velocity of wind is less near the surface of the earth than above because of friction. Consider in the first place, a wave surface travelling with the wind. For simplicity of explanation, suppose this surface to be plane and perpendicular to the earth's surface; then, as the velocity of the wind is greater above, this plane will not remain vertical, but will be tilted over more and more towards the earth; that is, the sound will take a more or less downward direction and keep near the surface. Now consider a wave surface travelling against the wind. The upper part of the wave surface will now lag behind the lower so that the wave surface will be tilted upwards, that is, the sound soon leaves the earth altogether and passing over the observer's head is lost above.

4. (*a*) Describe an experiment proving the interference of two sounds, and (*b*) give some familiar examples of the effects of interference.

Ans. (*a*) See p. 139 or 140.

(*b*) See pp. 141, 147.

5. (*a*) What is meant by "combination tones?"

(*b*) What is their origin?

(*c*) What are the frequencies of the combination tones for two notes of frequencies 256 and 384?

Ans. (*a*) See p. 135.

(*b*) See p. 133.

(*c*) $384-256=128$.

$384+256=640$.

LONDON UNIVERSITY.

INTERMEDIATE DOC. MUS. EXAMINATION.

1896. Morning 10 to 1.

1. What evidence can you give to show that periodic compressions and rarefactions are occurring in the air in the neighbourhood of a sounding body?

Ans.—Best by means of a manometric jet (for constuction of which see fig. 55 left hand top corner and accompanying description) and a revolving mirror used as explained on p. 5.

2. How does the pitch of the note given out by a stretched string depend (1) on the length of the string, (2) on the weight of unit length of the string, and (3) on the tension of the string?

Ans.—See p. 87.

3. How could you analyse a complex sound so as to determine whether a note of any particular pitch was present?

Ans.—See p. 65.

4. Explain the method of production of beats, illustrating your answer by carefully-drawn figures. What effect is produced when two sounds of nearly equal periods, but of somewhat different amplitudes, are sounded together?

Ans.—See pp. 144, 145, and 146.

5. Give a brief summary of the various theories which have been put forward to explain a consonance and a dissonance.

Ans.—See p. 172.

AFTERNOON. 2 to 5.

1. (*a*) Using a graphic representation of sound-waves in air, point out what, in your representation, determine respectively the pitch, loudness, and the quality of the sound?

(*b*) State very briefly the evidence for your statement.

Ans. (*a*) See pp. 19 and 22.

(*b*) Pitch varies with vibration rate. Proved by Syren, &c.

Wave-length=Velocity divided by vibration rate (see p. 39);

Therefore pitch depends upon length of sound-wave.

Loudness depends upon amplitude. See p. 52.

Quality depends upon form. See p. 80.

2. If you stand near equally spaced palisading, with vertical stakes a few inches apart, and sharply clap your hands, there is an echo with a musical tone. Explain this, and show how the tone may be determined from the velocity of sound and the distance between the stakes.

Ans.—The sound will be reflected back by each of the stakes. Suppose, for simplicity, you are close to one of the stakes and that the clap caused one sound-wave only; part of this sound-wave striking the next stake is reflected back to your ear; another part travels to next stake, is reflected and reaches your ear in its turn, and so on. The length of this sound-wave therefore is twice the distance between the stakes, and thus the vibration number of the tone produced is found by dividing the velocity of sound by twice this distance.

3. (*a*) How may the velocity of sound in a pipe, closed at one end by a movable piston, be determined by resonance to a fork of known frequency?

(*b*) Give the theory of the experiment, and a general explanation of the need for the end correction?

(*c*) How may the experiment be conducted so as to eliminate the end correction?

Ans. (*a*) See p. 119.

(*b*) See p. 61 and 62.

The reflection from the open end does not take place exactly at the end, but at a distance from it of about $\frac{4}{5}$ of the radius of the tube; this is called the end correction.

(*c*) Ascertain the position of the upper node in B fig. 56 (see p. 103). Measure distance of this node from closed end and multiply this by $\frac{3}{4} \times 4$ to obtain the wave-length.

4. Describe a stroboscopic method of determining the frequency of a fork.

Ans.—See pp. 37 and 38.

5. What are Combination Tones? Explain how they are probably produced in the ear when they have no external existence. In what case have they been shown to exist externally, and how?

Ans.—p. 134.

The rest of this question is still under discussion. For a full account of the matter, the student is referred to Helmholtz' "Sensations of Tone," 2nd English Edition, pp. 152, and 156 to 159.

Also Appendix XX, Section L.

TONIC SOL-FA COLLEGE.

FIRST STAGE. 1896.

1. (*a*) Explain clearly what you understand by a sound-wave?

(*b*) In what respects may sound-waves differ?

(*c*) What difference in the sound results from these differences in the wave?

(*d*) What is the length of the sound-waves given off from an ordinary tuning-fork?

(*e*) Why are the sound-waves from the same fork not always of the same length?

Ans. (*a*) See pp. 17 and 18.

(*b*) Length, amplitude, and form.

(*c*) Pitch, intensity, and quality.

(*d*) $1100 \div 517 = 2$ ft. 2 in. nearly.

(*e*) Because of temperature.

2. (*a*) If a stretched string sounds the note $C_{|}$, what note will $\frac{1}{5}$ of the same string give?

(*b*) If a certain string stretched by a weight of 25 lbs. gives C, what note will the same string give when stretched by a weight of 9 lbs.

Ans. (*a*) $E^{|}$.

(*b*) $E\flat_{|}$.

3. (*a*) Explain and illustrate the terms "node," and "ventral segment" as applied both to strings and pipes.

(*b*) A closed organ pipe, 2ft. 2ins. long is sounding G in treble staff. Give a sketch showing position of its nodes.

Ans. (*a*) See pp. 96 and 102. Also, nodes in pipes are situated where the changes in density are greatest.

(*b*) See p. 106, fig. 56 B.

4. On keyboard instruments, four minor thirds are taken as equal to an octave. (*a*) Is this correct? (*b*) If not, state as a vibration ratio the interval by which one exceeds the other, and show clearly which is the greater.

Ans. (*a*) No.

$$(b)\ \tfrac{6}{5} \times \tfrac{6}{5} \times \tfrac{6}{5} \times \tfrac{6}{5} = \tfrac{1296}{625}$$

$$\tfrac{2}{1} \quad = \quad \tfrac{1250}{625}$$

therefore the former is the greater by the interval $\frac{1296}{1250} = \frac{648}{625}$

5. (*a*) State precisely what you understand by "Simple Tone," "Compound Tone," "Partial," "Overtone," and "Fundamental," and

(*b*) Explain how you would demonstrate to anyone the existence of each.

Ans. (*a*) See Summary, pp. 83 and 84.

(*b*) Simple Tone. See p. 84.

Compound Tones are obtained from any ordinary musical instrument. Partials, overtones, and fundamentals may easily be discriminated by resonators (see pp. 69 and 70.)

6. (*a*) Explain clearly and illustrate what is meant by resonance.

(*b*) What are the relations between the length of an air column and the pitch of its note of maximum resonance?

Ans. (*a*) See pp. 67 and 62, or 63 or 64.

(*b*) The length of air column must be $\frac{1}{2}$ or $\frac{1}{4}$ the wave-length (that is 1100 ÷ vibration number) according as the air column is in an open or stopped tube.

Second Stage.

1. Wherabouts are the hammers made to strike the middle strings of a pianoforte? Why is this?

Ans—See p. 95.

2. Show clearly why $\left\{\begin{matrix} d^{\prime} \\ se \end{matrix}\right.$ is a dissonant interval, and $\left\{\begin{matrix} d^{\prime} \\ la \end{matrix}\right.$ a consonant interval.

Ans.—See p. 168 for former and pp. 188 and 191 for latter, which is a major third.

3. Show clearly how it comes about that the major third is too sharp in equal temperament.

Ans.—See p. 240.

4. Explain as fully as you can why a primary dissonance is worse than a secondary, and a secondary worse than a tertiary.

Ans.—See pp. 165 and 166 and figures.

5. (*a*) Explain clearly what is meant by interference of sound-waves. What is meant by "phase?"

(*b*) Describe in detail how it is that the sound-waves that issue from a tuning-fork are alternately in opposite phase.

Ans. (*a*) See pp. 136, 137, with figs. 67 and 68.

(*b*) See pp. 141 and 142, with fig. 74.

6. Compare by neat diagrams and explanations the intervals of a minor third and minor tenth in respect to their harmoniousness.

Ans.—See pp. 188, 200, and 201.

CAMBRIDGE.

Examination for the Degree of Mus Bac., Part I, and special examination in Music for the Ordinary B.A. Degree.

Tuesday, May 23, 1899. 9 to 12.

ACOUSTICS.

1. A tuning-fork is set into vibration. Describe fully the motion of a particle on one of the prongs, and the way in which sound from the fork reaches the ear of an observer.

Ans.—See p. 18, with fig. 14.

2. Upon what physical characteristics of the vibrations of a body do the loudness and pitch of the note emitted depend? What experiments can you adduce in support of your statements?

Ans.—Loudness depends upon amplitude, and pitch upon vibration number (see pp. 29, 30, et seq. for pitch and p. 32 for loudness).

3. (*a*) Describe some form of resonator, and explain how resonators can be applied to the analysis of compound sounds.

(*b*) A note on a pianoforte is struck *staccato*—(1) when the octave above is held down; (2) when the octave below is held down. Describe and explain what is heard in each case.

Ans.—(*a*) See pp. 65, 66, and 67 with figs. 37 and 38. See also p. 70.

(*b*) The strings of the octave above sound by resonance with the two vibrating halves of the string struck; (*b* 2) the two halves of the strings an octave below sound by resonance with the original strings struck (see also p. 70).

4. (*a*) Describe the motion of the air in an open organ-pipe sounding its fundamental note.

(*b*) Taking the velocity of sound in air to be 1100 ft. per second, find the approximate length of an open pipe giving 256 vibrations per second. In what respects is this calculation incomplete?

Ans.—(*a*) See pp. 102, 103, 104 with fig. 54 (A).

(*b*) Approximate length $= \frac{1100}{2 \times 256} = \frac{550}{256} =$ 2 ft. 2 ins. nearly.

In this calculation it is assumed that the reflection of the sound-wave at the open end of a pipe takes place exactly at the end. This is not the case, and in that respect the calculation is incomplete.

5. (*a*) Describe and explain the phenomena of beats.

(*b*) If two notes a semitone apart give six beats per second, when sounded together, what are their vibration-numbers?

Ans.—(*a*) See pp. 144, 145, and 156 with fig. 76.

(*b*) Let x denote vibration number of the lower tone, then $x + 6$ denotes vibration number of the upper tone,

$$\text{and} \quad \frac{x+6}{x} = \frac{16}{15}$$

$$\text{therefore} \quad 1 + \frac{6}{x} = 1 + \frac{1}{15}$$

$$\text{therefore} \quad \frac{6}{x} = \frac{1}{15}$$

$$\text{and} \quad x = 90$$

Thus the vibration numbers are 96 and 90.

6. (*a*) What are combination tones, and under what conditions are they audible?

(*b*) Explain the observation that with mounted tuning-forks excited very gently there is but slight dissonance in an impure octave, but that if the forks are excited vigorously the dissonance becomes marked.

Ans.—(*a*) See pp. 128 and 131.

(*b*) When the forks are vigorously excited, the differential combination tone beats with the lower fork, but when very gently excited, the probability is that the differential is not produced.

7. Explain Helmholtz's theory of consonance and dissonance, and employ it to prove that a fifth is a more consonant interval than a fourth.

Ans.—See Chap. XV and Chap. XVI, p. 191, with fig. 80.

8. Explain how the need for tempered intonation in the case of keyed instruments arises, and describe the system of equal temperament.

Ans.—See pp. 226 to 233 and pp. 238 to 241.

Tuesday, May 22, 1900. 9 to 12.

1. (*a*) Explain the mode in which sound is transmitted through the air.

(*b*) How do the motions of the particles of air within a sounding organ-pipe differ from the motions of the particles of air outside the pipe by which the sound is transmitted to a distance?

Ans.—(*a*) See pp. 16, 17, and 18.

(*b*) See pp. 102 and 104 with fig. 54 (A).

2. In what respect does a musical sound differ from a noise? What is the evidence in support of your statements?

Ans.—See p. 2.

3. (*a*) What is meant by resonance? (*b*) Explain why a resonator responds most strongly to a note of a certain definite pitch.

Ans.—(*a*) See p. 57.

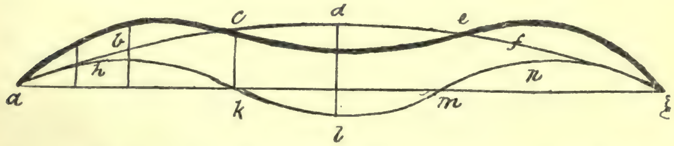
(*b*) See pp. 60 and 61.

4. (*a*) What are the relations between the upper partial tones of a sounding string and the fundamental tone?

(*b*) Draw diagrams illustrating the motion of a string which is sounding its fundamental and second upper partial simultaneously.

Ans.—(*a*) See p. 93.

(*b*) Let *a b c d e f g* and *a h k l m n g* represent two positions of the string at some given instant, the former as if the fundamental alone were being produced and the latter the second upper partial alone.



Then the thicker curve will represent the resultant motion, that is to say one position of the string when the two are produced simultaneously. For details as to construction see p. 82.

5. Describe the mechanism by which the human voice produces musical sounds, and explain how the pitch and quality of these sounds are controlled.

Ans.—See pp. 114 and 115.

6. (*a*) Describe and explain what is heard when two notes very nearly in unison are sounding together.

(*b*) If two organ-pipes in exact unison be sounded at opposite ends of a large room, what will be heard by a man who walks from one of the pipes to the other.

Ans.—(*a*) See pp. 144, 145, and 146.

(*b*) As the man walks *to* one of the organ-pipes, more sound-waves per sec. from that pipe will enter his ear than if he were at rest, because he advances to meet them. On the other hand, fewer sound-waves per sec. will reach his ear from the other pipe. One pipe will consequently *appear* sharper than the other, and therefore slow beats will be heard.

7. (*a*) Explain the formation of combination tones.

(*b*) Why is it that in the case of some instruments these tones can be reinforced by the use of resonators, while in other cases resonators are of no assistance?

Ans.—(*a*) See pp. 133 and 134.

(*b*) When the same mass of air is agitated by both generators as in the case of an harmonium bellows, the combination tone has an objective existence and can be reinforced by resonators. In other cases the tones are subjective, formed in the ear, and cannot be thus reinforced.

8. (*a*) Distinguish between exact and tempered intonation, and describe the system of equal temperament.

(*b*) If the pitch number of C is 264, what will be that of A—(1) in exact; (2) in tempered intonation.

Ans.—(*a*) See pp. 238 to 241.

(*b*) (1) $$\frac{264}{1} \times \frac{5}{6} = 220$$

(2) $$\frac{264}{1} \div \left(\sqrt[12]{2}\right)^3 = \frac{264}{1} \div \sqrt[4]{2} = 222$$

UNIVERSITY OF LONDON.

Intermediate Examination for Degree of Bachelor of Music.

Tuesday, December 14, 1898. 10 to 1.

1 (*a*) Explain why the interior length of a stopped organ-pipe is approximately equal to one quarter the length of the wave of sound which it limits.

(*b*) Why is it not precisely equal thereto?

Ans.—(*a*) See pp. 61 and 62.

(*b*) See answer to 4*b* Mus.Bac. (Cambridge) Exam., 1899.

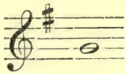
2. In what way is the pitch of an organ-pipe affected by changes in the temperature and in the humidity of the air.

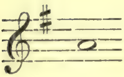
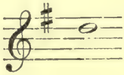
Ans.—See pp. 100 and 101. The greater the humidity the sharper the pipe.

3. (*a*) State the ratios between the frequencies (*i.e.* vibration number) of any fundamental note and the frequencies of the seven successive notes of its major scale (untempered) up to its octave.

(*b*) Given that  is tuned to have a frequency of 263,

find the frequency of  of  and of 

(*c*) From frequency of  so found, calculate those of

 and of 

Ans.—(*a*) See p. 51.

(*b*) $$D = 263 \times \frac{9}{8} = \frac{2367}{8} = 295\tfrac{7}{8}$$

$$G = 263 \times \frac{3}{2} = \frac{789}{2} = 394\tfrac{1}{2}$$

$$A = 263 \times \frac{5}{3} = \frac{1315}{3} = 438\tfrac{1}{3}$$

(*c*) $$A = 394\tfrac{1}{2} \times \frac{9}{8} = \frac{7101}{2 \times 8} = \frac{7101}{16} = 443\tfrac{13}{16}$$

$$D^{\prime} = 394\tfrac{1}{2} \times \frac{3}{2} = \frac{789 \times 3}{4} = \frac{2367}{4} = 591\tfrac{3}{4}$$

4. **(*a*) Explain the production of beats. (*b*) Suppose a set of forks to be tuned to philosophic pitch in which $C = 256$; and that a fork mistuned to 260 vibrations per second is procured. How many beats per second will the mistuned fork make (1) when sounded only with $C = 256$; (2) when sounded with $\sharp C = 264$; and (3) when sounded with $C^{\prime} = 512$?**

Ans.—(*a*) See pp. 144, 145, 146.

(*b*) (1) $260 - 256 =$ **4 beats per second.**

(2) $264 - 260 =$ **4 ,, ,,**

(3) $512 - 260 =$ **252 Differential.**

$260 - 252 =$ **8 beats per second.**

5. (*a*) Enunciate the laws which govern the frequency of vibration of a stretched string; (*b*) and point out how these laws come in, in the construction, tuning, and playing a violin.

***Ans.*—(*a*) See p. 87.**

(*b*) See pp. 88, 89, and 90.

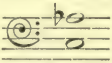
AFTERNOON. 2 to 5.

1. What explanation can you offer of the following?

(*a*) The throbbing sound so often heard when the tone of a large bell is dying away.

(*b*) The circumstance that the upper partial tones of church bells do not even approximately follow the harmonic series 1 : 2 : 3 : 4 &c.

Ans.—(*a*) and (*b*) See pp. 125 and 126.

2. (*a*) Briefly state Helmholtz's Theory of Consonance. (*b*) It is found that if two notes  are played simultaneously upon an open and a stopped pipe, the interval sounds harsher, if the lower note is taken by a stopped pipe while the upper note is played upon an open pipe, than is the case if the upper is played upon a stopped pipe and the lower upon an open pipe. Explain this.

Ans.—(*a*) See Chapter XV.

(*b*) See p. 198 with fig. 82, last interval.

3. Sketch the wave forms for a note, for its octave, and for the note that is a fifth above its octave, assuming that each is of equal loudness; also show how to find the complex wave form of the sound produced by all these three notes sounding together as a chord.

Ans.—See pp. 82 with fig. 45.

4. What is the effect on the apparent pitch of a note under the following various circumstances:—(*a*) when the instrument which sounds the note is moving rapidly (as when the performer is carried on board a railway train) towards the observer; (*b*) when a wind is blowing towards the observer from the place where the instrument is situated; (*c*) when the observer is moving rapidly (say, on board a train) towards the place where the instrument is situated.

Ans.—(*a*) The note is apparently sharpened in all three cases.

Tuesday, December 13th, 1899. Morning, 10 to 1.

1. (*a*) What change does a rise of temperature produce upon the velocity of sound in air? (*b*) Suppose an organ-pipe to be in tune when blown with air at the freezing point, how much will its pitch rise when blown with air at 80°F? (*c*) Why do not stringed instruments change in pitch to the same extent.

Ans.—(*a*) It increases the velocity: See p. 20 and p. 100.

$$(b)\ 80^\circ\mathrm{F} = \left(80 - 32\right)\frac{5}{9}\,\mathrm{C}^\circ = 48 \times \frac{5}{9} = \frac{80}{3} = 26\tfrac{2}{3}{}^\circ\mathrm{C}$$

Therefore if v denotes velocity of sound at 0°C

$v\sqrt{1 + \frac{80}{3} \times \frac{1}{273}}$ will be the velocity at 80°F (see p. 100).

Therefore $\dfrac{\text{vib. No. at } 80^\circ \text{ F}}{\text{vib. No. at } 32^\circ \text{ F}} = \dfrac{\sqrt{1 + \frac{80}{819}}}{1} = \sqrt{\dfrac{899}{819}}$

$$= \frac{2998}{2862} = \frac{150}{143} \text{ very nearly.}$$

That is, pitch will rise about 7 vibrations in 143, or about 1 in 20. More simply, 80° F is $80^\circ - 32 = 48^\circ$ above freezing point. Therefore if velocity of sound at 0° is 1090 above velocity at 48° F will be nearly $1090 + 48 = 1138$.

Therefore $\dfrac{\text{vib. No. at } 80^\circ \text{ F}}{\text{vib. No. at } 32^\circ \text{ F}} = \dfrac{1138}{1090} = \dfrac{114}{109}$ nearly.

That is, a rise of about 5 in 109, or about 1 in 20 as before.

(*c*) Because strings whether of catgut or metal do not expand anything like so much as air for the same increase of temperature, but see p. 90.

2. Explain how it is that a sound can be reflected back from the open end of a tube. Explain the difference between this and the reflection taking place at a closed end of a tube.

Ans.—See pp. 60 and 61.

3. (*a*) What are Lissajous' figures? (*b*) How are they produced? (*c*) What do they demonstrate?

Ans.—(*a*) Lissajous' figures are the figures described by a particle which is simultaneously vibrating in two directions, say at right angles to one another; or we may say, they are the figures obtained by optically combining the motions of two particles vibrating in directions at right angles to one another.

(*b*) There are various methods of producing them: one of the simplest is by means of such a compound pendulum as is figured in fig. 42. Fasten the lower end of this at one corner of the board with a piece of thread, load the funnel with sand and then release by burning the thread. The pendulum will then vibrate as a whole across the board in a direction parallel to *a b*—but the lower part will also vibrate in a direction at right angles to this parallel to A' A, and the result will be a Lissajous' figure in sand on the board below, which remains at rest.

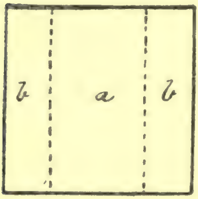
(*c*) They demonstrate the exact proportion or otherwise between the two vibration periods.

4. If mixed sand and lycopodium powder are scattered over a square brass plate clamped at its centre, and the plate is so bowed at its edge as to emit a shrill note, the sand is observed to settle down in lines, and the lycopodium in small patches. Explain the significance of these facts.

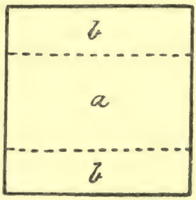
Ans.—See pp. 124 and 125. The lycopodium powder being so light is entangled by the little whirlwinds of air produced by the vibrations of the plate; it cannot escape from these tiny whirlwinds, but the heavier sand particles are readily driven through them.

5. Find the form of the sand figures produced on a square plate so bowed that its vibrations, so far as they are parallel to one edge are taking place with two nodal lines, and also with two nodal lines, so far as they are taking place parallel to the other edge. Sketch and explain the sand figures when (*a*) the corners are in the same and (*b*) when they are in opposite phases of vibration due to these two components.

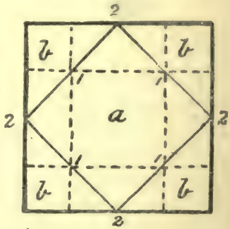
Ans.—Let fig. A represent the plate as if vibrating parallel to one edge only, and at the moment when the middle rectangle is above the horizontal (denoted by *a*) and the lateral rectangles below (denoted by *b*), the two lines representing the nodal lines. Let fig. B represent the



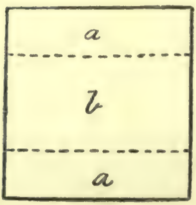
A.



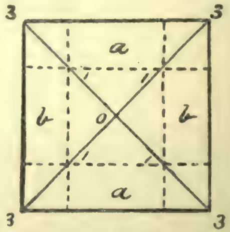
B



C



D.



E.

same plate, as if vibrating parallel to the other edge only, and at the moment when the inner rectangle is above and the outer rectangles below the horizontal lines (denoted by *a* and *b* as before). Now let C represent A and B superposed. On examination it is seen that the centre square marked *a* will have a double amplitude above and the corners *b* a double amplitude below the horizontal. Also the points marked 1 will be points of rest, since they are on both sets of nodal lines. The points 2 also are nodal points, for they have equal amplitudes in opposite directions. A little consideration will show also that the lines 2, 1, 2 connecting these points of rest will be nodal lines. Hence the sand figure will be the square 2, 2, 2, 2. It can be obtained by clamping the plate at 2 and bowing close to one of the corners. Now (*b*) let fig. D represent the plate B, but vibrating in opposite phase as shown. Let E represent D superposed on A. Then the rectangles *a a b b* will have twice the amplitude due to each set of vibrations singly. The points 1, 1, 1, 1 are of course nodal points: the four corners 3, 3, 3, 3 and the centre 0 are also nodal points, since they have equal and opposite amplitudes. Consequently the sand figure will be the cross along the diagonals 3, 0, 3. It can be obtained by clamping the plate at the centre and bowing near the middle of one of the sides.

6. (*a*) What is a good form for a resonator? Explain (*b*) why a well-made resonator is not set into active vibration, even by a loud tone, unless the tone is accurately of the pitch to which the resonator is tuned; (*c*) in what way resonators provide a means for the analysis of compound sounds.

Ans.—(*a*) See p. 65. (*b*) See pp. 60, 61, and 62. (*c*) See pp. 66 and 69.

AFTERNOON. 2 to 5.

1. Describe the structures of the inner ear, and explain how they aid in the perception of pitch, the quality, and the direction of sounds.

Ans.—See pp. 25, 26, 27, 28.

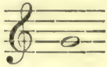
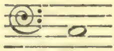
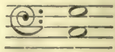
2. What is the reason why the ear is more sensitive to slight variations of pitch in moderately high than in low notes.

Ans.—In the Cochlea (see pp. 27 and 28) the radial fibres are probably tuned with fair regularity, like the forks of a Tonometer (see p. 151). Suppose for the sake of explanation they are tuned at say about 2 vibrations difference between successive fibres. In that case the ear would be able to detect difference of vibration number with equal keenness in all parts of the scale. But two vibrations at low pitch means a much larger interval than at high pitch. For example, 2 vibrations difference at d = 30 would mean a semitone, while 2 vibrations above d = 300 would mean only a small fraction of a semitone. Thus the ear would be much more sensitive to difference of pitch at the latter pitch than at the former.

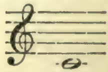
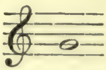
3. In arithmetic, a fourth and a fifth added together do not make an eighth; yet in music a "fourth" and a "fifth" added together make an "octave." Also in music a major "third" and a "fourth" added together make a sixth; yet in arithmetic neither do three and four make six, nor do a third and a fourth added together make a

sixth. Explain the peculiarity of musical nomenclature which has given rise to these anomalies.

Ans.—A fourth in music does not mean a fourth part of anything as in arithmetic, but merely the fourth note from that with which we start in the diatonic scale.

4. (*a*) If the note  on a pianoforte be silently depressed and held down, and the note  be struck vigorously and allowed to rise again, what note will then be heard? (*b*) If the notes  are similarly held down silently, while the notes  are vigorously struck, which of the upper notes will be heard sounding after the two lower notes have been released.

Ans.—(*a*)  (*b*) 

5. (*a*) Explain the nature of the "beats" heard in a mistuned interval such as a mistuned octave or twelfth. It has been stated that it is easier to tune a major third (by altering the pitch of the upper of the two notes) if there is sounding at the same time the fifth. For example, to tune  by comparison with  is easier if  is also present. Can you give a physical reason for this?

Ans.—(*a*) See p. 176.

(*b*) The C and G would produce a Summation Tone E' and the E must be tuned so that its first over-tone is in unison with this E'; also the Differentials produced by the E and G and by the E and C would be identical.

6. Give some account of the rise of pitch since the time of Handel. What are the causes tending to force up the pitch?

Ans.—See p. 42.

Tuesday, December 19th, 1899. 10 to 1.

1. Why is the tuning-fork rather than any other simple musical instrument selected as a standard of pitch? Does the pitch of a tuning-fork alter with temperature, time, or climate?

Ans.—See p 123.

2. A steel rod (such as the prong of a long tuning-fork) is free to vibrate at one end and is fixed at the other. Where will the nodes be situated in the first and in the second of its higher partial nodes of vibration, and what will be approximately the relative pitches of those higher tones compared with the fundamental?

Ans.—For 1st overtone the node will be about a fourth or a fifth of its length from the free end. For 2nd overtone the one node will be nearer than this to the free end, while the other will bisect the length between this node and the fixed end. The relative pitches of the fundamental, and 1st and 2nd overtones will be approximately as 4 : 25 : 70.

3. Many organists will tell you that when the room where the organ is gets hot, the reeds in the trumpet stop and other reed-pipes "get flat" in tone. Is this allegation true? Explain the facts that have originated this idea.

Ans.—Reeds do slightly flatten with rise of temperature (see p. 111), but the other pipes of the organ, the flue-pipes, sharpen much more rapidly with increase of temperature, so that the reeds seem to flatten by contrast.

4. Describe a method of analysing a complex tone and of discovering its constituent tones.

Ans.—See pp. 69, 70, 71.

5. Violinists in order to produce the effect of tremolo upon an open string resort to the following method: The open string alone is bowed whilst a tremolo fingering is performed upon the next higher string at the point corresponding to the octave above the note of the open string. Give physical explanations to account for the effect being the same as though the tremolo had been executed on the bowed string.

Ans.—The open string when bowed vibrates not only as a whole but also in halves, producing the upper octave; this sets the string above vibrating by resonace and as this is subjected to the tremolo fingering, a tremolo effect is produced, and this in its turn reacts upon the open string.

6. (*a*) Why are dampers always provided in a pianoforte? (*b*) What is the evidence for the existence of damping in the mechanism of the ear.

Ans.—(*a*) Dampers are provided: (1) to silence a string after it has been struck, and (2) to prevent other strings sounding by resonance.

(*b*) The shake in music (say 10 tones to a second) can be clearly heard throughout the greater part of the musical scale, but as we get very low in pitch, they sound bad and rough, and their tones begin to mix. This is not due to the instrument producing the tones, for they are so on all instruments. It is consequently due to the ear, that is the damping in the ear is not so rapid and complete for the lower tones as for those above.

Mus. Bac., Edinburgh, 1898.

TWO HOURS.

1. Write the first 16 Partial Tones, taking as the fundamental $E\flat$, and indicate those notes which do not agree with the actual sounds, saying whether these latter are sharper or flatter.

Ans.—$E\flat$, $E\flat^1$, $B\flat^1$, $E\flat^2$, G^2, $B\flat^2$, $\overset{*}{D\flat^3}$, $E\flat^3$, F^3, G^3, $\overset{\dagger}{A\flat}$, $B\flat^3$, $\overset{\dagger}{B}$, $\overset{*}{D\flat^4}$, D^4, $E\flat^4$. See p. 72.

[The notes marked * are flatter than indicated, those marked † sharper.]

2. Define the expressions Partials, Upper Partials, Overtones, and Harmonics.

Ans.—See pp. 84, 71. Upper Partials are all the partials above the Fundamental. See also p. 267, No. 4.

3. If the tone c'' is produced by 528 vibrations, by how many vibrations are produced the tones C g'' and d''.

Ans.—$C = c'' \div 8 = 528 \div 8 = 66.$

$$g'' = c'' \times \frac{3}{2} = 528 \times \frac{3}{2} = 264 \times 3 = 792.$$

$$d'' = c'' \times \frac{9}{8} = 528 \times \frac{9}{8} = 66 \times 9 = 594.$$

See Chapter V.

4. What are the ratios of a major third, a minor third, and a major sixth?

Ans.—See p. 51.

5. What causes the difference in the quality of tone of different instruments; for instance, the flute, the clarinet, and the violin.

Ans.—See pp. 96, 109, 113.

6. How do you explain the fact that the quality of tone of a pianoforte is changed by changing the place where the strings are struck by the hammers?

Ans.—See pp. 94 and 95.

7. Define the terms Difference Tones and Summation Tones.

Ans.—See p. 135.

8. What Difference Tones are produced by the following couples of tones:—$d' - a'$; $c'\sharp - a'$; $e' - g'\sharp$?

Ans.—1st, d, nearly.
2nd, e.
3rd, E.

See p. 130.

9. What Summation Tones are produced by the same couples of tones?

Ans.—1st, $f''\sharp$
2nd, $f''\sharp$. Approximately.
3rd, $f''\sharp$

See p. 134.

10. How arise Beats?

Ans.—See pp. 144, 145, 146.

11. (*a*) What is the nature and object of Equal Temperament? (*b*) Are there other Temperaments? (*c*) And what is tempered?

Ans.—(*a*) See p. 238, &c.
(*b*) See p. 233.
(*c*) See p. 232.

Mus.Bac., Edinburgh, 1899.

1. State what you know about the velocity of sound in air and other media.

Ans.—See pp. 20 and 21.

2. What are the respective causes of Noise and Tone?

Ans.—See p. 2.

3. Explain what is meant by the words Length, Amplitude, and Form in connection with sound-waves.

Ans.—See p. 22.

4. Name the 6th, 8th, and 10th Upper Partials of the tone A$\flat$.

Ans.—(6th) Approximately G$\flat$, a little flatter.
(8th) B$\flat$.
(10th) Approximately D$\flat$, somewhat sharper.

See p. 72.

5. (*a*) Which is the interval above or below a tone that blends with it most readily; (*b*) and which is the interval that comes next in this respect?

Ans.—(*a*) Octave; (*b*) twelfth.

6. If the tone C^{II} is produced by 528 vibrations, by how many vibrations are produced the tones of C^{IIII}, G, a^{I} and b flat?

Ans.—$C^{IIII} = 528 \times 2 \times 2 = 2112.$

$$G = 528 \times \frac{3}{4} \times \frac{1}{2} = 66 \times 3 = 198.$$

$$a^{I} = 528 \times \frac{5}{6} = 88 \times 5 = 440.$$

$$\text{b flat} = 528 \times \frac{8}{9} \times \frac{1}{2} = \frac{2112}{9} = 234\tfrac{2}{3}$$

See Chapter V.

7. What is Helmholtz's Theory of Beats?

Ans.—See p. 144, 145, 146.

8. How many kinds of Combinational Tones are there? What are their names? Give some examples of them.

Ans.—See p. 135, and pp. 130 and 134.

9. What are Tartini's Tones?

Ans.—See p. 129.

10. How has difference in the quality of the tone (tone colour) been accounted for?

Ans.—See p. 74.

11. Explain the cause of the effect produced by the use of what is popularly called the "loud pedal" of the pianoforte.

Ans.—See p. 95.

Mus.B., Victoria University.

THREE HOURS.

1. Describe two different methods by which the velocity of sound has been determined.

Ans.—For one method see p. 19. For another method: Stand opposite a wall which gives back an echo. Clap the hands at such a rate that the sounds and their echoes occur at equal intervals of time. Count the number of claps and echoes in (say) 2 minutes. Measure the distance of the observer from the wall. This distance multiplied by the number of claps and divided by 60 will give the velocity of sound.

2. Taking the co-efficient of expansion of air to be $\frac{1}{273}$, and the velocity of sound in air at 0° C to be 33200 cm. per second find its velocity at 20° C and 100° C.

Ans.—Velocity at 20° C $= 33200\sqrt{1 + \frac{20}{273}}$

$$= 33200\sqrt{\left(\frac{293}{273}\right)}$$

$$= 34395$$

Velocity at 100° $= 33200\sqrt{\frac{373}{273}}$

$$= 38807$$

3. Describe the construction of the siren, and show how it is used to find the vibration number of a given note.

Ans.—See pp. 31 and 32.

4. Show that the addition of a fifth to a fourth gives an octave.

Ans.— $\frac{3}{2} \times \frac{4}{3} = \frac{2}{1}$

5. What ought the stretching weight of a wire to be to give a note one-fifth above that of another wire of the same thickness and material, one metre long, and stretched by a weight of 10 kilogrammes?

Ans.—Let x represent the weight required.

$$\text{Then } \sqrt{\frac{x}{10}} = \frac{3}{2} \quad \text{or} \quad \frac{x}{10} = \frac{9}{4} \quad \text{therefore } x = 22\tfrac{1}{2} \text{ kilos.}$$

6. An open organ pipe being 20 ft. long, find the wave-lengths, and the vibration numbers of its fundamental note and its first two overtones ($v = 1100$ ft. per second).

Ans.—Wave-lengths are 40, 20, $13\frac{1}{3}$ ft.

Vibration numbers are $27\frac{1}{2}$, 55, $82\frac{1}{2}$ ft.

See pp. 39 and 99.

7. What are the Diatonic and the Chromatic Semitones, and their vibration numbers?

Ans.—See pp. 48 and 49.

8. A note of 226 vibrations per second and another of 340 are sounded together, each being accompanied by its first two over-tones. Show that two of the over-tones will give two beats per second.

Ans.—Second over-tone of the first $= 226 \times 3 = 678$.

First over-tone of the second $= 340 \times 2 = 680$.

Difference $= 2$.

Royal University of Ireland, 1898.

1. Describe a sound wave in air.

Ans.—See pp. 16, 17, and 18.

2. How may the pitch of a musical note be found by experiment.

Ans.—See pp. 31, 32; 35, 36, 37, or 151 and 152.

3. A stretched string gives the note *doh*. How must its tension be altered so that the note emitted may be *sol*?

Ans.—The tension must be increased $2\frac{1}{4}$ times. See p. 87.

4. Describe the condition of the air in an open organ-pipe when sounding its first overtone.

Ans.—See p. 105 with fig. 54 (B).

5. Why do sounds travel faster in water than in air?

Ans.—Because the elasticity of water in proportion to its density is greater than the elasticity of air in proportion to the density of air. See p. 20.

6. Give some account of the physical basis of harmony. See Chapter XVI.

7. What interval added to a Major third will make an Octave?

Ans.—A Minor Sixth: for $\frac{5}{4} \times \frac{8}{5} = \frac{2}{1}$

See Chapter V.

8. Describe Helmholtz's Double Siren.

Ans.—See p. 34 with fig. 22.

Royal University of Ireland, 1899.

1. Explain the terms *Pitch* and *Timbre*, giving illustrations. See p. 29. For Timbre or Quality see p. 74.

2. Find the vibration frequency of a note whose wave-length in air at 20° C is 2 ft.

Ans.—Velocity of sound in air at 20° C is $1090 + 20 = 1110$, therefore vib. frequency $= \frac{1110}{2} = 555$

3. How may the interference of sound-waves be illustrated by means of a tuning-fork?

Ans.—See pp. 141 and 142.

4. Describe the construction and use of a siren.

Ans.—See pp. 31 and 32.

5 What difference exists, as a rule, in notes of the same pitch when sounded on closed and open pipes, respectively?

Ans.—See p. 116.

6. Describe an experiment illustrative of resonance.

Ans.—Any of those described on pp. 59, 60, 62 or 63 will do.

7. What difference is observable in the pitch of the note emitted by the whistle of a railway locomotive when it is approaching the listener as compared with that when it is going away from the listener? Explain.

Ans.—When the locomotive is approaching, the note is sharper than when going away. See No. 6, p. 285.

Royal University of Ireland, 1900.

1. What are the ratios of the vibration-numbers of three notes which form a major common chord? Show how from these ratios the frequencies of the notes forming a diatonic scale may be built up.

Ans.—See p. 45.

2. (*a*) Explain the necessity for a "tempered scale"; (*b*) What ratio corresponds to the interval of a fifth on this scale?

Ans.—See first 8 pp., Chapter XVIII; (*b*) *Equal* temperament presumed: $\sqrt[12]{2}^{\,7} = 1{\cdot}4982$

3. Define "overtone" and "harmonic." Name some instruments in which the overtones are harmonies, and others in which this is not the case.

Ans.—For overtone, see p. 84.

Harmonic is evidently used here to denote overtones which are harmonic with the fundamental. For the former stringed instruments, organs, &c. For the latter anv of the instruments mentioned in Chapter XI.

4. (*a*) Describe the possible modes of vibration of a stretched string; (*b*) Draw diagrams showing the forms assumed by the string when the first two of these modes are superposed.

Ans.—(*a*) See p. 90 with fig. 46. (*b*) See No. 4 p.

5. Describe the most accurate method known to you for determining the vibration frequency of a tuning-fork.

Ans.—Either the Graphic method, p. 36 and 37, or the Tonometric, pp. 150 and 151.

6. (*a*) Explain the production of beats. (*b*) How many beats will be heard per second when two open organ pipes are sounding together, their frequencies being—

(1) 200 and 203.
(2) 200 and 403.

Ans.—(*a*) See pp. 144, 145, 146.

(*b*) (1) 3 beats per second. (2) 403 — 400 = 3 beats per second.

7. Give the formula for the rate at which sound travels through a gas, and point out how this rate will be affected by alterations in the temperature and pressure of the gas.

Ans.— $$V = \sqrt{1{\cdot}41\ \frac{P}{D}\left(1 + \frac{t}{273}\right)}$$

where P denote pressure of gas, D its density, and t denotes temperature. See also p. 20.

ROYAL COLLEGE OF ORGANISTS.

QUESTIONS.

1. What intervals do the following vibration fractions represent?

$$\frac{15}{16} \quad \frac{8}{5} \quad \frac{5}{3}$$

Ans.—See p. 51.

2. Give an acoustical reason why one interval is more discordant than another.

Ans.—See Chapter XIV.

3. What is the length of the C C pipe of the Twelfth stop?

Ans.—Take $C^{\prime} = 540$, $C\,C = 67\frac{1}{2}$, then length of $C_{\prime}$ pipe $= \frac{1120}{4 \times 67\frac{1}{2}} =$ 4 ft. 2 in. approximately.

4. Why is the sound produced by a stopped Flue pipe an octave lower than that of an open pipe of the same length?

Ans.—See pp. 60 and 61.

5. Describe the constituent parts of (*a*) a Flue pipe, (*b*) a Reed Pipe.

Ans.—(*a*) See pp. 98 and 99; (*b*) pp. 111 and 112.

6. Why does a stopped pipe give a hollow sound compared with the sound of an open pipe?

Ans.—See pp. 106, 107.

7. Why does a major seventh sound more dissonant than a major second.

Ans.—Because in the first case the higher tone beats at a *semitone* distance with the second partial of the lower, while in the second case the beating distance is about twice that interval.

8 Why should students of music study the elements of Acoustics?

Ans.—Briefly, because the laws of Acoustics are the foundations of music; for example, a knowledge of acoustics is necessary in order to understand the construction of the scale; the estimation of intervals; the fundamental facts of harmony; the nature and origin of dissonance, temperaments, &c

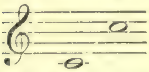
Miscellaneous Questions.

1. Find the approximate length of the C C♯ pipe on the open diapason 8 ft. What is the pitch of note on open diapason 8 ft. length, the pipe of which is 1 ft. 6 ins.?

Ans.—To find approximate length of C C♯ pipe, C C being 8 ft., it is first necessary to assess the interval between the two. This is a chromatic semitone. The vibration fraction for this interval being $\frac{25}{24}$ (see p. 51) the 8 ft. length of the C C pipe must be divided by 25 and multiplied by 24—

$$8 \times 24 \div 25 = 7\tfrac{17}{25} = 7 \text{ ft. } 8 \text{ ins. approximately.}$$

To find out the pitch of a pipe of 1 ft. 6 ins. in length. In the first place it must be some note between the 2 ft. and 1 ft. C, viz., between



Now 2 ft. $\times$ 3 ft. $=$ 6 ft., and 6 ft. $\div$ 4 ft. $=$ 1 ft. 6 in., and as $\frac{4}{3}$ is the vibration ratio of a Fourth, the required pitch is F.

2. (*a*) How does a string vibrate as a whole, and at the same time in segments, to form harmonics? (*b*) How is the tone of the (one segmental) fundamental kept up, although the harmonics are sounding? (*c*) Does the fundamental vibrate first, then the segments for the 8ves, 5ths, 4ths, &c., one after the other respectively, or do they all vibrate together? (*d*) What causes the harmonics; *i.e.*, what causes the string to assume the different segments, seeing it only gets plucked to vibrate its own (one segmental) fundamental?

Ans.—(*a*) Students often have a difficulty in realising that a body can have several independent motions at the same time, yet the phenomenon is a very common one. Take the case of two persons in the opposite corners of a carriage of an express train, tossing a ball from one to the other. The ball has several independent motions. While it is being tossed across the carriage, it is also travelling in the direction in which the train is moving, and at the same speed as the train. Moreover, the ball and the train are both moving from west to east at the rate of several hundred miles an hour, a motion due to the earth's rotation. Lastly, the ball, train, and earth are whirling round the sun at a still more rapid rate. Thus, the ball has four distinct motions at the same time. In the same manner, there is no real difficulty in conceiving a stretched string vibrating as a whole, and at the same time in two or more segments. (*b*) The string vibrating as a whole gives the fundamental, and vibrating in two, three, four, or more segments *at the same time*, produces also the first, second, third, fourth, and higher harmonics, or as it is better to term them, *overtones*. (*c*) All together. (*d*) Stretch a heavy cord ten or twelve feet long between two fixed points; the heavier it is the more slowly it will vibrate, and thus its vibration will be the more easily seen. An india-rubber tube filled with sand answers the purpose very well. Set it vibrating by a gentle movement near the centre; it will vibrate as a whole, *i.e.*, with one vibrating segment. Bring it to rest and now strike it somewhat sharply at about $\frac{1}{4}$ of its length from one end. It will then vibrate in two segments. By following up this line of experiment, it may be made to vibrate in three or more segments, the number being only limited by the flexibility of the cord. Moreover, when the tube is set vibrating, say at $\frac{1}{4}$th its length from the end, it is not difficult after a few trials to get it to vibrate in halves and as a whole at the same time. Now, a stretched string producing a musical tone vibrates in exactly the same way, but owing to its lightness and flexibility, it is found impossible to get it to vibrate as a whole, that is, in one segment alone; some other segmental vibration is always present under ordinary musical conditions.

3. A vibrating tuning-fork is held over a tall cylinder, into which water is gradually poured. Describe and explain the variation that takes place in the sound of the fork. How could you employ this apparatus to find the velocity of sound, the period of the vibration of the fork being given?

Ans.—For first part of question, see pp. 61 and 62. For second part, Hold vibrating fork over cylinder; gradually pour in water until the air column in cylinder gives out its maximum resonance. Then measure distance from surface of water to mouth of cylinder. Multiply four times this distance by the vibration number of the fork and the product will be the distance sound travels per second; *i.e.*, the velocity of sound.

4. What name is given to the difference between a diatonic and a chromatic semitone?

Ans.—The difference between a diatonic semitone and the greater chromatic semitone $= \frac{16}{15} \div \frac{135}{128} = \frac{16}{15} \times \frac{128}{135} = \frac{2048}{2025}$ it is called the Diaskhisma. The difference between a diatonic semitone and the lesser chromatic semitone $= \frac{16}{15} \div \frac{25}{24} = \frac{16}{15} \times \frac{24}{25} = \frac{128}{125}$ It is called the Enharmonic or Great Diesis.

5. Supposing this note  to give 520 vibrations per second find the vibration numbers of the minor 7th above it, produced (1) by a perfect 5th added to a minor 3rd, and (2) by the sum of two perfect 4ths.

Ans.— $\frac{520}{1} \times \frac{3}{2} \times \frac{6}{5} = 936$

$$\frac{520}{1} \times \frac{4}{3} \times \frac{4}{3} = 924{\cdot}4$$

6. How is the formula $\sqrt{\frac{E}{D}}$ modified so as to apply to the velocity of sound in air?

In the case of air E represents the elasticity of air, which is measured by the ratio of the pressure applied to the compression produced. Thus, if a certain volume of air under a pressure P is submitted to a small additional pressure p, then, by Boyle's law, the diminution in volume thus caused will be $\frac{p}{P}$; consequently E is the ratio of p to $\frac{p}{P}$ that is—

$$E = p \div \frac{p}{P} = \frac{p}{1} \times \frac{P}{p} = P$$

This, however, is only true if the compression produces no rise in temperature; but heat is always produced during compression. And in the case of sound waves the compression is so rapid that the heat has not time to escape, and thus a rise of temperature occurs during the condensation. The amount of heat produced is proportional to the compression, and this increases the elasticity of the air in the same proportion. The amount by which the elasticity of the air is thus increased has been found by experiment to be 1·41, so that the formula—

$$V = \sqrt{\frac{E}{D}} \text{ becomes } V = \sqrt{\frac{P \times 1{\cdot}41}{D}}$$

Expressing P and D in our English units: D, the density or weight of air per cubic foot, is 565 grains at the freezing point, and at a pressure of 14 692 lbs. per square inch. Now 14 692 lbs. $=$ 14 692 $\times$ 7,000 grains, and therefore 14·692 lbs. per square inch is equivalent to 14·692 $\times$ 7000 $\times$ 144 grains per square foot. Thus our formula—

$$V = \sqrt{\frac{P \times 1{\cdot}41}{D}}$$

becomes $V = \sqrt{\dfrac{14{\cdot}692 \times 7000 \times 144 \times g' \times 1{\cdot}41}{565}}$

and taking $g = 32{\cdot}186$ (introduced to express P in dynamical units) we get $V = \sqrt{\dfrac{14\ 692 \times 7000 \times 144 \times 32{\cdot}186 \times 1{\cdot}41}{565}}$

7. (*a*) Has the system of tuning keyboard instruments been always the same? (*b*) If not, what different systems exist now, or have existed in the past? (Edinburgh, 1903.)

Ans.—(*a*) No. (*b*) Tuning in Mean-tone temperament. See p. 234, *et seq.* Tuning in Equal temperament. See p. 238, *et seq.*

8. What is the difference between noise and tone and how are they produced? (Edinburgh, 1903.)

Ans.—Noise is produced by irregular or non-periodic vibrations, tone by regular or periodic vibrations.

9. Give the etymology and meaning of the word Acoustics, and say of what the Science it names treats. (University of Edinburgh, March, 1904.)

Ans.—The word Acoustics is derived from the Greek *akouo*, I hear. Acoustics is that branch of physics which treats of the phenomena of sound, sound waves, and the vibrations of elastic bodies generally.

10. What are the vibration numbers of the tones of a Perfect Fifth, a Perfect Fourth, and a Major Third above, and a Perfect Fourth below a tone of 290 vibrations? (Edinburgh, 1904.)

Ans.— Perfect Fifth above $\frac{290}{1} \times \frac{3}{2} = 435.$

Perfect Fourth above $\frac{290}{1} \times \frac{4}{3} = 386\frac{2}{3}.$

Major Third above $\frac{290}{1} \times \frac{5}{4} = 362\frac{1}{2}.$

Perfect Fourth below $\frac{290}{1} \times \frac{3}{4} = 217\frac{1}{2}.$

11. Distinguish between longitudinal and transverse wave motion, and give instances of each from instruments used in the orchestra. (University of Cambridge, Easter Term, 1901.)

Ans.—Longitudinal wave motion occurs in the flute, oboe, clarinet, trombone, trumpet, cornet, bassoon, and horns. Transverse wave motion takes place in the violin, viola, 'cello, double bass, and harp.

12. A string 2 ft. long and stretched with a weight of 10 lbs. has a vibration number 300. Give three distinct methods of reducing its vibration number to 200. (Cambridge, 1901.)

Ans.—1st. Reduce tension. Let T be tension required—

$$\sqrt{\frac{T}{10}} = \frac{200}{300} = \frac{2}{3}$$

$$\frac{T}{10} = \frac{4}{9}$$

$$T = 4\tfrac{4}{9} \text{ lbs.}$$

2nd. Lengthen string. Let L be the length required—

$$\frac{L}{2} = \frac{300}{200} = \frac{3}{2}$$

$$L = \frac{3}{2} \times 2 = 3 \text{ ft.}$$

3rd. Reduce tension and shorten string. Reduce tension say to $1\frac{1}{9}$ lbs. and let V be corresponding vibration number. Then—

$$\sqrt{\frac{1\frac{1}{9}}{10}} = \frac{V}{300}$$

$$\frac{1}{9} = \frac{V^2}{90000}$$

$$V^2 = 10000$$

$$V = 100$$

Now take half the string and the vibration number will be 200. An alternative method would be to wind a wire round string, so as to weight it till its vibration number is reduced to 200.

13. Describe the way in which air vibrates in an open organ pipe when it gives its first overtone. What would be the result if the pipe were blown with hydrogen instead of air? (Cambridge, 1901.)

Ans.—For the first part see pp. 102 and 103, and Question 4 (p. 278) second part. Velocity of sound in different gases varies inversely as the square roots of their densities (see p. 20). The densities of hydrogen and air are as—

$$1 \quad : \quad 14{\cdot}435$$

Therefore—

$$\frac{\text{Velocity in Hydrogen}}{\text{Velocity in Air}} = \sqrt{\frac{14{\cdot}435}{1}}$$

$$= 3{\cdot}8 \text{ nearly.}$$

That is :—The velocity of sound in hydrogen is 3 8 times the velocity in air. Now vibration number of a pipe is found by dividing velocity by length of pipe. Therefore it is evident that the vibration number of the pipe, when blown with hydrogen, will be 3·8 times as great as when blown with air.

14. What is the effect on the pitch of a sound if its source is moved rapidly towards or from the observer? Explain the reason of any change of pitch. (Cambridge, 1901.)

Ans.— A C B

Let A be the source of the sound at rest, and B the observer, also at rest. Suppose that A vibrates 100 times per second. Then as long as B is at rest, his ear receives 100 waves per second. Now let B move up towards A; say he gets to C in one second. B will now, not only receive the 100 waves per second that he would have done if at rest, but also all the waves that lie between B and C; thus the note he hears is sharper than before. Similarly, if B were moving away from A, the note would obviously be flattened. Analogous reasoning shows a similar result, if B is at rest and A moves towards or from B.

15. Give a brief description of the general conception of wave motion, and explain, in regard to it, the terms *amplitude*, *wave-length*, *period*, *form*, *frequency*, and *phase*. Distinguish between the motions of progressive waves and stationary waves, and illustrate with diagrams. (Cambridge, 1902.)

Ans.—For first part of question see pp. 12, 13, 22, and 137; *period* is time of a vibration or time a wave takes to travel its own length; *frequency* is vibration number. Waves such as those figured and described on pp. 12, 13, and 14 are progressive waves. The "rope" waves, described and figured on p. 90, are stationary waves.

16. In what way does the pitch of the note given by a string vibrating transversely depend on its length and tension? Show that an increase of two per cent. in the tension or a decrease of one per cent. in the length produce very nearly the same rise in pitch. (Cambridge, 1902.)

Ans.—For first part see p. 87. Second part: Let n be vibration number of string when tension is 100, and let length of string be 100 centimetres. Then an increase of 2 per cent. raises the tension to 102 Let x be now the vibration number. Then—

$$\frac{x}{n} = \sqrt{\frac{102}{100}} = \frac{\sqrt{102}}{10} = \frac{10\,1}{10} = \text{(very nearly) } 1{\cdot}01.$$

$$\therefore\ x = n \times 1{\cdot}01 \text{ very nearly.}$$

A decrease of one per cent. reduces length of string to 99. Let y be now the vibration number, then—

$$\frac{y}{n} = \frac{100}{99} = 1{\cdot}01 \text{ very nearly.}$$

$$y = n \times 1{\cdot}01 \text{ very nearly.}$$

Thus the rise in pitch is very nearly the same in each case.

17. (*a*) On what does the pitch of a note depend? (*b*) What exact information in relation to the pitch difference of the common musical intervals is afforded by the syren? (University of Cambridge, 1903.)

Ans.—(*a*) Vibration rate, or frequency. (*b*) Helmholtz' syren proves that the vibration ratios of notes at the intervals of an octave, a fifth, and a major third are exactly—

$$\frac{2}{1},\ \frac{3}{2},\ \frac{5}{4}.$$

18. (*a*) Describe the simple modes of transverse oscillation of a tense string, and state how the pitch of the corresponding notes depends on the length and tension of the string. (*b*) At what points of its length may the string be touched to bring out the harmonic which is three octaves above the note of the open string. (Cambridge, 1903.)

Ans.—(*a*) See p. 87, *et seq.* (*b*) At $\frac{1}{8}$ or $\frac{3}{8}$ of its length from one end.

19. (*a*) Give any mechanical illustrations of sympathetic oscillation and explain how the principle involved applies to air resonators. (*b*) How is the pitch of a resonator affected (1) by increasing the volume, (2) by enlarging the orifice? (Cambridge, 1903.)

Ans.—(*a*) See pp. 57, 60, *et seq.* (*b*) (1) pitch is lowered, (2) pitch is raised.

20. (*a*) Discuss and explain briefly the phenomenon known as beats. (*b*) On what does the rapidity of the beats depend? (*c*) Two forks give a slow beat, but have a pitch difference too small to be detected by ear. How may the sharper fork be selected by testing with a third fork? (Cambridge, 1903.)

Ans.—(*a*) and (*b*) See pp. 144 to 147. (*c*) Let third fork be sharper than other two. Count the number of beats this third fork makes with each of the others in the same time. That which makes the fewer beats is the sharper of the two. If the third fork were flatter than the other two, that fork which makes the larger number of beats would be the sharper.

21. What are the advantages of a tuning-fork as a standard of pitch? (Royal University of Ireland, 1901.)

Ans.—Tuning-forks maintain their pitch well, and are little affected by ill-usage; a slight rusting, for instance, changes the pitch but little. Moreover, the alteration in pitch for change of temperature is very slight (see p. 123).

22. (*a*) Describe an experimental method of finding the wave-length of the note given by a tuning-fork. (*b*) How would you deduce the frequency of the fork? (University of Ireland, 1901.)

Ans.—(*a*) Hold the vibrating tuning-fork over the open end of a sufficiently long and not too wide tube closed at the other extremity. Pour in water until the maximum resonance is obtained. Measure the distance from the surface of the water to the open end. Four times the distance will give the wave-length approximately. (*b*) Divide the velocity of sound by this wave-length and the result will be the frequency.

23. Why does the harshness of a discord vary with the nature of the instrument on which the discordant notes are sounded? (University of Ireland, 1901.)

Ans.—Best answered by an illustration. Take for example the secondary dissonance, No. 2, on p. 165. If this is sounded on the stopped diapason, or by clarinets, nearly all the elements of roughness will disappear, for the $s_{|}$, s, and f are absent in the tones of such instruments.

24. The strings of a violin are tuned to the notes G, D, A, E, as correct fifths. In which of these keys will the open strings have the correct pitches as diatonic notes of the major scale? (University of Ireland, 1901.)

Ans.—In key G the open E will be too sharp (see Chap. XVIII).

,, ,, A the G is not in the Diatonic Scale.

,, ,, E the G and D are not in the Diatonic Scale.

Therefore it is only in key D that all the open strings have the correct pitches as diatonic notes of the major scale.

25. What do you know of the use which has been made of flames in studying the properties of musical sounds? (University of Ireland, 1901.)

Ans.—See pp. 4, 104, 148. If the manometric flame figured on the left in fig. 55 be placed at the base of a resonator tuned to a note of a particular pitch, then the reflection of the flame, in a revolving mirror (see p. 4), will at once show whether a note of this particular pitch is present in any compound tone or assemblage of tones. Thus manometric flames may be employed to detect or prove the presence of particular partials.

26. Write a short note on the determination of consonance and dissonance in musical intervals, and also state what bearing on this subject has the phenomenon known as beats. (University of Ireland, 1901.)

Ans.—There is no hard and fast line between consonance and dissonance, the one merges into the other; moreover, there are intervals which at one time were termed dissonances, but are now included among the consonances. It is difficult to treat of this matter in a *short note.* The shortest answer may be gathered from the Summaries to Chapters XIV, XV, and XVI.

27. (*a*) What class of musical instruments are tuned to the scale of equal temperament and why? (*b*) What is the difference between the tempered fifth and the diatonic fifth? (University of Ireland, 1902.)

Ans.—(*a*) Keyboard instruments, and generally, instruments with fixed tones, because having only a very limited number of tones (generally only 12 to the octave). It is best to tune them in equal temperament, as, on the whole, this produces the least divergence from true intonation. (See p. 238, *et seq.*, and the Chapter on Temperament generally.) (*b*) Difference between the tempered fifth and true fifth is very slight, only $\frac{1}{11}$ of a komma, or—

$$\frac{1500}{1498} = \frac{750}{749} \quad \text{See p. 298, } 2b.$$

28. What are the conditions necessary for the production of a strong echo? (University of Ireland, 1902.)

Ans.—1st, For a strong echo, the original sound must itself be strong. 2nd, A good reflecting surface is necessary, wood, brick, rock, &c. 3rd, No impediment in the path of the direct sound or its reflection. 4th, Not too great, but yet a sufficient distance between A and C, and C and B (fig. 31, p. 55). 5th, A homogeneous atmosphere for the sound to travel through, and absence of wind.

29. How would you find the pitch of a tuning-fork by means of a sonometer? (University of Ireland, 1902.)

Ans.—First, tune the wire on the sonometer in exact accord with the tuning-fork, by altering the tension, or length of wire. The vibration number of the wire, and therefore of the fork, may then be found from the formula given on p. 35.

30. Describe fully the condition of the air at a node, and at a loop in an organ pipe. How is it that the pitch of the note is unaltered when a small hole is opened in the wall of the pipe at a loop. (Royal University of Ireland, 1903.)

Ans.—As described on p. 104, the air at a node is undergoing rapid alternations of density, but at a loop or antinode no such changes of density occur; in other words, the air at a loop is of the same density as the external air. Hence a small hole in the wall of the pipe at a loop will leave things unaltered.

31. A string yields the notes D and E as consecutive harmonics. What is the pitch of its fundamental note? (University of Ireland, 1903.)

Ans.—Vibration ratio of E and D $= \frac{10}{9}$. Therefore E and D are the 10th and 9th partials of C, three octaves below.

32. Why, with a given change of temperature, does the pitch of an organ not vary altogether in one direction? (Royal College of Organists, Associate, 1902.)

Ans.—See pp. 101 and 111.

33. What length of pipe is required to give this note  in each of the following organ stops:—Open Diapason, Stopped Diapason, and Harmonic Flute? (Royal College of Organists, Associate, 1903.)

Ans.—Taking middle C $= 256$ and 1120 as velocity of sound on the Open Diapason length is about—

$$\frac{1120}{256 \times 2} = 2 \text{ ft. } 2\tfrac{1}{4} \text{ in.}$$

On the Stopped Diapason—

$$\frac{1120}{256 \times 4} = 1 \text{ ft. } 1 \text{ in.}$$

On the Flute—

$$\frac{1120}{512 \times 4} = 6\tfrac{1}{2} \text{ in.}$$

34. Give the fundamental and the two succeeding harmonics of a stopped organ pipe 8 ft. long. (Royal College of Organists, Associate, 1903.)

Ans.—Vibration number of fundamental $= \frac{1120}{8 \times 4} = 35$.

Then the vibration numbers of the two succeeding harmonics must be—

$$3 \times 35 = 105, \text{ and } 5 \times 35 = 175.$$

These correspond to notes a little flatter than D_3 A_2 and $F\sharp_1$.

35. What happens when an organ pipe is overblown? (Royal College of Organists, Associate, 1903.)

Ans.—At first the pitch of the fundamental rises, and finally the fundamental disappears and the 1st overtone is heard.

36. How is the sound of flue-pipes produced? (Royal College of Organists, Associate, 1903.)

Ans.—See p. 99.

37. If the temperature of a concert room rises rapidly how would it affect the various groups of orchestral instruments? (Royal College of Organists, Fellowship, 1901.)

Ans.—The brass and wood-wind rise in pitch with increase of temperature. A rise in temperature alone would lengthen the strings, and this would decrease the tension and so cause a fall in pitch, but in the case of strings of cat-gut and the like, the matter is complicated by the increase of humidity which accompanies rise of temperature in a concert room, and this has an opposite effect, increasing the tension.

38. Taking the middle C as having 264 vibrations a second, how many vibrations would the following notes above the C have, respectively: E, G♯, A♭? (Royal College of Organists, Fellowship, 1901.)

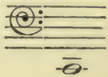
Ans.—

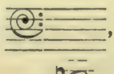
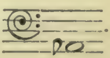
$$E = \frac{264}{1} \times \frac{5}{4} = 330.$$

$$G\sharp = \frac{330}{1} \times \frac{5}{4} = 412\tfrac{1}{2}.$$

$$A\flat = \frac{264}{1} \times \frac{2}{1} \times \frac{4}{5} = 422\tfrac{2}{5}.$$

39. Explain summational and differential tones. Give in notation the differential tones of the intervals whose ratios are $\frac{3}{2}$, $\frac{9}{8}$, $\frac{10}{9}$, $\frac{6}{5}$, taking  as the lower sound of each. (Royal College of Organists, Fellowship, 1902.)

Ans.— $\frac{3}{2}$ , $\frac{9}{8}$ ,

$\frac{10}{9}$ , $\frac{6}{5}$ .

40. How are (*a*) open pipes, (*b*) stopped pipes, (*c*) reeds, tuned. (Royal College of Organists, Fellowship, 1903.)

Ans.—(*a*) Roughly tuned by altering length and finally adjusted by the pressing in or out of the open end by the "tuning cone." (*b*) By pressing in or drawing out the stopper which closes the pipe. (*c*) See p. 111 and p. 112 with fig. 60

41. (*a*) A trumpeter sounds a note on a railway train. The note is heard by listeners at two stations, one towards which the train is travelling, the other from which the train is receding. Have the notes heard at the two stations the same pitch? Give reasons for your answer. (University of Wales, 1904.)

Ans.—No, as heard at the former station, the pitch is higher; and, as heard at the latter station lower than it really is, for reasons see p. 305, No. 14.

42. A wind is blowing from a bandstand towards a listener. State, with reasons, whether the pitch of the notes is affected by the wind. (University of Wales, 1904.)

Ans.—The pitch of the notes is not affected by the wind. It is true that the velocity of the sound would be increased by the wind, but the length of the sound waves would be increased in the same ratio: therefore, the number of waves per second which reach the listener's ear would be the same, whether the air be still or in motion.

43. The stretching force in a string is 8080 grammes weight, the vibrating length is 40 centimetres, and the mass of a centimetre of the string is ·0189 gramme. Taking gravity as 981 find the frequency. (University of Wales, 1904.)

Ans.—The formula required for working this is given on p. 87:—

$$N = \frac{1}{2L}\sqrt{\frac{T}{M}}$$

Here $L = 40$

$M = 0189$

$T = 8080 \times 981$ (expressed in dynamic units).

Therefore $N = \frac{1}{80}\sqrt{\frac{8080 \times 981}{0.89}}$

$= 256.$

44. (*a*) Explain the phenomenon of beats. (*b*) Illustrate by drawing curves to represent the varying displacement of the tympanic membrane for each of the two notes nearly but not quite in unison, and the resultant curve given by combining the displacements. (University of Wales, 1904.)

Ans.—(*a*) See pp. 144, 146, *et seq.* (*b*) The curves on p. 145 will answer this part of the question: A and B being the curves for the two notes not quite in unison and C the resultant curve obtained by combining their displacements.

45. In a determination made of the frequency of a fork, by means of a monochord, the notes were not got exactly in tune. Without putting them in tune how could you ascertain the exact frequency of the fork? (University of Wales, 1904.)

Ans.—First ascertain by ear which is the sharper, the fork or the monochord. Then count the number of beats the fork and monochord produce when sounded together for say 30 seconds. Divide number of beats by 30. This gives number of beats per second. If the fork is the sharper, add, if the flatter, subtract this number from the frequency number of the monochord.

46. State what you know regarding the mode of vibration of a violin string. How do you explain the maintaining action of the bow? (University of Wales, 1904.)

Ans.—The bow, aided by the resin with which it is rubbed, catches hold of the string at some point and carries it forward with it till the adhesion can no longer equal the elastic force of the string thus brought into play: the string therefore rebounds, only to be again caught. This action is repeated as long as the bowing is continued and thus the vibration is maintained. A violin string cannot vibrate in one segment only; usually it vibrates in 1, 2, 3, 4, 5, 6, 7, or even more segments, yielding a klang rich in partials. (See Chap. IX.)

47. The note emitted by a standard tuning-fork while it is being bowed is not precisely of the same pitch as the note it emits when left to itself. Explain why this is. How is it that the act of drawing a bow across the prongs is able to produce a smooth tone at all? (London, 1900.)

Ans—The bow, or rather the particles of resin on the bow, catch the prongs of the fork and move them from rest (see p. 86). Though the fork is thus slightly constrained by the movement of the bow and thus slightly altered in pitch, yet the energy of the moving prongs is sufficient to keep up an almost uniform vibration rate, and thus the tone produced is sufficiently smooth and uniform.

48. Draw separately the wave forms that will represent a tone and its higher octave of equal loudness. Then draw the resultant wave form for the compound sound produced by them jointly, under such phase relations that when the ordinate of the wave-form of the lower note is zero, that of the higher wave-form is either at a maximun or a minimum. (London, 1900.)

Ans.—Describe any circle, Fig. I, and divide it into any even number of equal parts (say twelve), numbered 1, 2, 3, &c. Join points

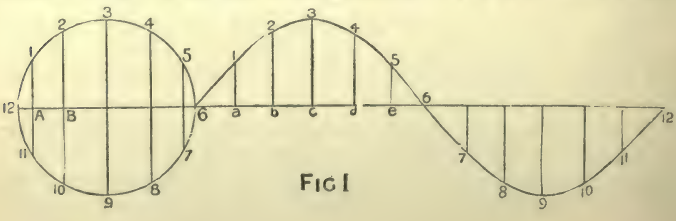


FIG I

as in figure. Prolong the horizontal diameter 12 6 to the right, and measure off twelve equal parts, *a*, *b*, *c*, and so on. Draw vertical lines from these points and make *a* 1 equal to *A* 1, *b* 2 equal to *B* 2, and so on. Join the tops of these verticals 1, 2, 3, &c., by a curve as shown; this curve will be the wave-form representing a tone. Do the same for Fig. II, but let the equal spaces *a*, *b*, *c*, &c., be exactly one half of what

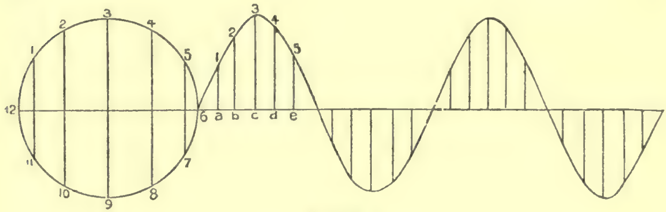


FIG II

they are in the Fig. I. The curve thus obtained will be the wave-form of the higher octave of equal loudness. Now draw the upper curve, Fig. I, again, as *X Y* in Fig. III. Then draw the curve in Fig. II, as *M N P* in Fig. III, but so that point 3 in Fig. II is exactly over

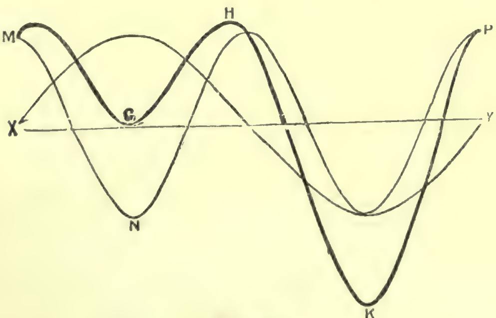


FIG III

X in Fig. III. Now combine these two curves in Fig. III, as explained on p. 82, and we get the thick curve *M G H K P*, which is the resultant wave-form required.

49. (*a*) Write down the vibration ratios for the ordinary musical intervals. (*b*) Show that three successive major thirds are not equal to one octave, and (*c*) that twelve successive fifths are not equal to seven octaves. (London, 1900.)

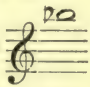
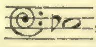
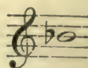
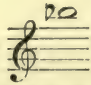
Ans.—(*a*) See p. 51. (*b*) Three major thirds $=$

$$\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} = \frac{125}{64} \quad \text{An octave} = \frac{2}{1} = \frac{128}{64}$$

$$\text{(c) Twelve fifths} = \left(\frac{3}{2}\right)^{12} = \frac{130}{1} \text{ very nearly.}$$

$$\text{Seven octaves} = \left(\frac{2}{1}\right)^{7} = \frac{128}{1} \text{ See p. 239.}$$

50. What do you know about the vowel sounds? How has their composition been investigated experimentally, and with what result? (London, 1900)

Ans.—The origin of vocal tone is the vibration of the vocal cords. The tone thus produced is highly compound, consisting of many partials belonging to the harmonic series. The different vowel sounds are produced by the resonance imparted to certain of these partials, by the varying shape and capacity of the buccal cavity (see pp. 114 and 115). Willis and Helmholtz have investigated their composition chiefly in the following way. The mouth was placed in the exact form required to produce a particular vowel sound, say the *a* in father. Vibrating tuning-forks of various pitches were then held close to the mouth, and the fork or forks selected (in this case B♭¹) which were most powerfully reinforced by it while in this position. By examining all the vowel sounds in this way it has been shown that "vowel qualities of tone are essentially distinguished from the tones of most other musical instruments by the fact that the loudness of their partial tones does not depend solely upon their order, but preponderantly upon the absolute pitch of those partials." For example, the characteristic tone of the vowel *a* (father) is  Now whether that vowel sound is produced by a bass upon , or by a soprano on , it is the  which is reinforced by resonance, though this is the sixth partial of the bass tone, and the second of the soprano.

51. (*a*) What are beats? (*b*) What is their cause? (*c*) What is the distinction between primary beats and secondary beats? (*d*) Why are beats between two mistuned tuning-forks, or those between two mistuned stopped organ pipes, more incisive than the beats between two equally mistuned strings in a piano? (London, 1900.)

Ans.—(*a*) and (*b*) See pp. 146, 147, and 152. (*c*) Primary beats: those between fundamentals. Secondary beats: those between overtones and differentials. (*d*) The tones of tuning-forks and stopped organ pipes are practically simple; thus the beats are primary and loud. The tones of the pianoforte are very complex and evanescent; thus the whole energy of the tone is broken up amongst a number of partials, and is quickly dissipated, and therefore the beats also are less incisive.

52. Assuming  to have 256 vibrations per second, calculate the vibration numbers of  and of . Then from  calculate , and from  calculate  (London, 1900.)

$$\textit{Ans.}\text{—A} = \frac{256}{1} \times \frac{2}{1} \times \frac{5}{6} = 426\tfrac{2}{3}$$

$$\text{F} = \frac{256}{1} \times \frac{2}{1} \times \frac{4}{3} = 682\tfrac{2}{3}$$

$$\text{C}\sharp = 426\tfrac{2}{3} \times \frac{5}{4} = 533\tfrac{1}{3}$$

$$\text{D}\flat = 682\tfrac{2}{3} \times \frac{4}{5} = 546\tfrac{2}{15}$$

53. In Joachim's edition of Courvoisier's book on the technique of the violin, it says that C♯ is the leading tone into D natural, and must be very close to D; while on the other hand D♭ must be very near to C, which is the third in the scale of A♭. This would make C♯ of higher pitch than D♭. Is this so? Is it always so? Discuss the theory of the point. (London, 1900.)

Ans.—As in the previous question, C♯, the leading tone to D, $= 533\frac{1}{3}$, and D♭, the fourth in scale of A♭, $= 546\frac{2}{15}$. Consequently, C♯ is not higher in pitch than D♭. As used in harmony this must always be the case, but solo violinists have a tendency to sharpen their leading tones for the sake of brilliancy, hence the remark in Courvoisier.

54. What is heard when the following pairs of pure tones are sounded together:—

(1) $C = 128$ with $e^{I} = 320$
(2) $C = 128$,, $B = 120$
(3) $C = 128$,, $f^{I} = 341{\cdot}3$
(4) $C = 128$,, $b = 240$
(5) $C^{II} = 512$,, $f^{II} = 682{\cdot}6$
(6) $C^{IV} = 2048$,, $b^{IV} = 3840$
(7) $C^{III} = 1024$,, $d^{IV} = 3840$
(8) $C^{IV} = 2048$,, a fork of frequency 2816

Ans.—(1) $320 - 128 = 192 = g$ (a differential)
(2) $128 - 120 = 8$ beats per second
(3) $341{\cdot}3 - 128 = 213{\cdot}3 = a$ (differential)
(4) $240 - 128 = 112$ (differential)
$128 - 112 = 16$ beats per second
(5) $682{\cdot}6 - 512 = 170{\cdot}6 = f$ (differential)
(6) $3840 - 2048 = 1792$ (differential, 14th harmonic to $c = 128$)
(7) $2804 - 1024 = 1280 = e^{III}$ (differential)
(8) $2816 - 2048 = 768 = g^{II}$ (differential)

55. What are Lessajous' figures? What points in accoustical science do they illustrate? To what practical service have they been put? (London, 1900.)

Ans—Lissajous' figures are obtained in the following manner:—A tiny mirror is attached to the end of one prong of a tuning-fork which vibrates in a vertical plane. A second fork with a similar mirror vibrates in a horizontal plane in such a way that when a spot of light in a darkened room is reflected from the mirror of the first fork it is again reflected from that of the second and from that to a screen. When only the first fork vibrates, a vertical line of light is seen on the screen; when only the second, a horizontal line. But when both vibrate a curve is obtained, due to the composition of the two. On p. 272, illustrations of such curves are shown, these particular ones being due to two forks tuned very nearly to an octave. Lissajous' figures enable us to tune two forks to an absolutely true interval. Thus, for example, as long as the forks just referred to are untrue, the curve produced continually changes from the first of the five figures shown through the second, third, and fourth, back to the first again, but as soon as the forks attain a perfectly exact octave, the figure ceases to change.

56. Write an account of the upper partial tones (sometimes called overtones), particularly dealing with the following points;—(*a*) Their presence or absence in different musical instruments; (*b*) the presence or absence of any particular members of them in particular instruments; (*c*) their dependence upon the mode of excitation; (*d*) their effect upon the character of a consonance or dissonance; (*e*) the use made of them in the theory of music by Rameau. (London, 1900.)

Ans.—(*a*), (*b*), (*c*), (*d*), fully answered in the text. (*e*) Rameau (1685-1764) puts these two points as the foundation of his theory; first, the resemblance between a tone and its first overtone, that is its octave; second, that musical tones possess the third and fifth partials (s and m). On this he founds the major chord $\left\{\begin{matrix} \mathrm{s} \\ \mathrm{m} \\ \mathrm{d} \end{matrix}\right.$, and insists that it is the most natural of chords and the foundation of harmony, and from this argues that harmony is a natural and not an artificial production.

57. Explain the fact that when two tuning-forks of respective frequencies, $c = 128$ and $e^{\prime} = 320$ are sounded together, $C = 64$ is heard, or that when forks of frequencies $C^{\prime\prime\prime\prime} = 2048$ and $b^{\prime\prime\prime\prime} = 3840$ are sounded together, $C^{\prime} = 256$ is heard. (London, 1901.)

Ans.—When a tuning-fork is struck and not applied to a resonance chamber in unison with its fundamental, the octave is almost always present. Thus $C = 128$ gives also $C = 256$, which with $e^{\prime} = 320$ gives the differential $320 - 256 = 64$. Similarly 2048 gives 4096, and $4096 - 3840 = 256$.

58. If a fork of high pitch is excited and is then held opposite a wall, and is moved towards or from the wall, beats are heard. Explain the circumstance and show how the number of beats per second depends on the pitch of the fork and the velocity of the movement (London, 1901.)

Ans.—When a fork of very high pitch is excited and held opposite a wall, the reflected waves, together with the direct waves, form a series of stationary undulations between the fork and the wall in such a manner that the point in contact with the wall is a node. Similar nodes occur between the wall and the fork at distances equal to the half wave-length of the fork. The case is very similar to the waves figured on p. 106, fig. 56 D. These stationary undulations may be looked upon as formed by the interference of two series of equal waves, one series from the fork and the other from the wall. If now the fork be moved rapidly towards or from the wall, the lengths of the waves of the first series will be diminished or increased before the second series is affected, and the effect is momentarily the same, as if the waves from two tuning-forks of slightly different pitch interfered. This interference, as shown on pp. 144, 5, and 6, produces beats, the number of beats per second depending on the difference of the vibration numbers of the two forks. Let V be the velocity of sound, v the velocity of the fork, n the vibration number of the fork. Then just as the fork is commencing a second vibration, the first one must have travelled $\frac{V}{n}$, but the fork has itself travelled $\frac{v}{n}$ in the same time;

consequently its wave length is—

$$\frac{V}{n} \pm \frac{v}{n} = \frac{V \mp v}{n}$$

and the frequency is $V \div \frac{V - v}{n} = \frac{Vn}{V \mp v}$

The number of beats per second is therefore—

$$n - \frac{Vn}{V + v} \quad \text{or} \quad \frac{Vn}{V - v} - n$$

59. (*a*) Why is equal temperament a necessity in keyboard instruments? (*b*) In what respects do the notes of the equally-tempered scale fail of just intonation? (*c*) Calculate the logarithmic interval between a true fifth and a tempered fifth, given the following:—

Log 2 = ·30103, log 3 = ·47712. (London, 1901.)

Ans.—(*a*) See chapter XVIII. (*b*) See pp. 241 and 242. (*c*) Equal fifth $= \sqrt[12]{2^7}$

$$\begin{array}{r|l} & \cdot 30103 \\ & \underline{\quad\quad 7} \\ 12 & 2 \cdot 10721 \\ & \overline{\cdot 17560} \end{array}$$

$$\text{True fifth} = \frac{3}{2}$$

$$\begin{array}{r} \cdot 47712 \\ \cdot 30103 \\ \hline \cdot 17609 \end{array}$$

Therefore logarithmic interval between them is—

$$\cdot 17609 - \cdot 17560 = \cdot 00049.$$

60. Criticise the following statements taken from Zahm's "Sound and Music:" (*a*) Since Mersenne's time, as is apparent, the rise in pitch has been very great indeed (p. 77). (*b*) Music written by Mozart, Handel, Beethoven, and Haydn must be sung more than a semitone higher than it was intended to be sung (p. 78). (*c*) Orchestras and military bands are, in the main, responsible for this undue elevation of pitch (p. 78). (London, 1901.)

Ans.—Mersenne (about 1650). His spinet was tuned to A = 402·9. Handel (1685-1759). Mozart (1756-1791). Haydn (1732-1809). Beethoven (1770-1827). Handel's fork, A = 422·5. Stein's fork, A = 421. Stein made pianos for Mozart and Beethoven. In 1810, Paris opera, A = 427. In 1820, London Philharmonic, A = 433. In 1830, Paris opera, A = 434 to 440. At present day, A = 455 to 457. See Ellis' "History of Musical Pitch in Europe." Hence (*a*) is abundantly justified. Also (*b*) music written by the composers mentioned is at the present day sung quite half a tone sharper than they intended, for $\frac{455}{422}$ is greater than $\frac{16}{15}$. (*c*) Is also doubtless true. Orchestras and military bands have continually raised the pitch in their efforts after supposed brilliancy.

61. How has the velocity of sound in air been exactly determined? How is it affected by temperature? (University of London, 1902.)

Ans.—Two stations, *A* and *B*, are chosen, a measured distance (several miles) apart. A canon is fired at *A*, and time between flash and report noted at *B*. As soon as possible after this a canon is fired at *B*, and time between flash and report noted at *A*. The average of these two times is taken. This eliminates any error due to the wind. There will still remain a slight error due to the unequal times of perception for sight and hearing. This can be got rid of by dispensing with the observer and substituting an electric current to fire the cannon, say at *A*, and at the same time mark the moment of its occurrence on a revolving cylinder at *B*. The moment of the arrival of the sound wave at *B* can be recorded on same cylinder by causing an accoustical pendulum, set in motion by this sound wave, to make or break contact. Knowing the rate of revolution of cylinder, the time is thus accurately ascertained. One difficulty still remains, viz., the ascertaining the average temperature of the air through which the sound wave has travelled.

62. A band is playing at the head of a procession 540 ft. long, and the men step 128 paces a minute, exactly in time with the music as they hear it. Those at rear are exactly in step with those in front. What is the velocity of sound? (London, 1902.)

Ans.—As the men in rear are in step with those in front, and as from front to rear is 540 ft., it is evident that the sound travels that distance in exactly same time as the men take to march one step, that is the sound travels—

$$540 \text{ ft. in } \frac{1}{128} \text{ minute, or } 540 \text{ ft. in } \frac{60}{128} = \frac{15}{32} \text{ seconds.}$$

$$\text{or } \frac{540 \times 32}{15} \text{ ft. in one second, or 1152 ft. per second.}$$

63. Show how the pitch of a note heard from a source of given frequency is affected by the motion of the source to or from the hearer. A policeman who believed that a steam car had passed him at least at 20 miles an hour, stated in cross examination that the note of the bell certainly did not change more than half a semitone as the car passed. If he was right in this, how many miles an hour might the car have been going? (London, 1902.)

Ans.—For first part, see Miscellaneous Questions, No. 14, p. 305. For second part, note of bell was not sharpened more than a quarter of a semitone by the advance of car, nor flattened more than the same amount by its retreat. Now vibration ratio of a quarter of a semitone is—

$$\sqrt[4]{\frac{16}{15}} = \cdot 01016.$$

Let n be vibration number of bell, and take 1120 as velocity of sound.

$$\text{Then } \frac{1120}{n} = \text{wave length.}$$

The policeman therefore had perceived at most $n + n \times \cdot 01016$ waves per second, and the $n \times \cdot 01016$ waves were due to the motion of the car. Therefore the velocity of the car very nearly equalled—

$$\frac{1120}{n} \cdot \frac{n \times \cdot 0106}{1} \text{ ft. per second} = 11\ 3792, \text{ say } 12 \text{ ft. per second.},$$

$$i.e., \frac{12 \times 3600}{5280} \text{ miles per hour} = 8\tfrac{1}{4} \text{ miles per hour.}$$

64. When a very gentle wind is blowing, sound is clearly heard a long way to leeward, but only a short distance to windward. Why is this? Why also can a speaker standing on the ground be heard only a short distance in the open air on a hot still day, while the distance is much increased if he mounts a pedestal? (London, 1902.)

Ans.—The velocity of the wind is less at the earth's surface than it is above, because of the friction of the current of air against the ground. Consequently, the sound waves, which are travelling in the same direction as the wind, near the ground lag behind those above. Now imagine a huge sheet of paper carried forward by the wind in such a way that it is vertical, and its surface at right angles to the wind's direction. If this is retarded below, what happens? It ceases to be vertical, the top inclining forward, and the direction of motion of the sheet of paper is now not onwards in a horizontal direction, but towards the ground. Precisely the same thing happens to the sound wave; its direction is towards the ground. When the sound wave is proceeding against the wind it is evident that precisely the opposite occurs, the wind delays the sound wave above more than it does below, and the waves are therefore tilted upwards, and are soon lost altogether as far as listeners on the ground are concerned. On a hot, still day the air near the ground, heated by the hot earth, is at a higher temperature than the air above. Sound travels faster in hot air than in cool, therefore on such a day it will travel faster near the ground than at a distance above. Now this is precisely the same condition as in the first part of this question when the sound waves are travelling against the wind. Consequently the result is the same, the sound waves are tilted upwards and are lost to listeners at a distance. If the speaker mount a pedestal, the sound-waves are emitted in the cooler air above, and will proceed in a horizontal direction, if emitted horizontally, or unaltered while in air at same temperature, that is at same level and spreading out into the warmer layers below, will be thus heard at a much greater distance.

65. How is the velocity of sound in air affected by temperature? (London, 1902.)

Ans.—If V_1 and V_2 are velocities of sound at temperatures t_1 and t_2,

$$\text{then } \frac{V_1}{V_2} = \sqrt{\frac{273 + t_1}{273 + t_2}}$$

$$\text{If } t_2 = o,\ V_2 = 1090; \text{ therefore } \frac{V_1}{1090} = \sqrt{\frac{273 + t_1}{273}},$$

or $V = 1090\sqrt{1 + \frac{t_1}{273}}$, which formula gives the velocity of sound V_1 in air at any temperature t_1.

66. What are combination tones? How have they been shown to exist outside the ear? (London, 1902.)

Ans.—For first part, see p. 135. Second part, it has been shown that their intensity can be increased by the use of appropriate resonators, and this can only be the case if they have an objective existence.

67. If a certain tone is produced by 260 vibrations, by how many vibrations are produced the tones a perfect fifth lower, and a major second, a minor third, and a major sixth higher? (University of Edinburgh, 1907.)

Ans.—$173\frac{1}{3}$, $292\frac{1}{2}$, 312, $433\frac{1}{3}$.

68. What change would require to be made in (*a*) the length (*b*) the tension of a string in order to raise its pitch by a major third? (Royal University of Ireland, 1906.)

Ans.—(*a*) It must be shortened to four fifths of its present length. (*b*) It must be increased to $\frac{25}{16}$ of its present tension.

69. Explain the dull and insipid character of the top notes of a pianoforte, compared to the bottom notes of that instrument. (University of London, 1906.)

Ans.—While the lower notes are extremely rich in overtones, the highest notes are nearly or quite simple tones.

70. The note sounded by the horn of a motor seems to fall a whole tone in pitch as it passes a stationary observer. Show that it must be travelling about 45 miles an hour. (University of Cambridge, 1904.)

Ans.—Let v, be velocity of motor in ft. per second.
V, the velocity of sound.
and n, the vibration number of horn.

Then length of sound wave as motor approaches observer is

$$\frac{V - v}{n}$$

and after it has passed,

$$\frac{V + v}{n}$$

Therefore $$\frac{V + v}{n} : \frac{V - v}{n} :: 9 \cdot 8$$

thus $$\frac{V + v}{V - v} = \frac{9}{8}$$

and $$v = \frac{V}{17}$$

Take $V = 1120$

then $$v = \frac{1120}{17} \text{ ft. per sec.}$$

thus $$v = \frac{1120}{17} \times \frac{60}{1} \times \frac{60}{1} \times \frac{1}{5280} \text{ miles per hour.}$$

$$= 45 \text{ nearly.}$$

71. The lower tone of a just scale of one octave has a frequency 96. Find the frequencies of the other seven notes. Find also exactly what alterations will be needed for a modulation into the dominant. (Camb. 1904.)

Ans —96 ($106\frac{2}{3}$), 108, 120, 128, 144, 160, 180, 192.
128 must be replaced by 135
and 162 will be required.

72. A C fork is struck a considerable blow. Explain (1) why the note C should sound at first too flat, (2) why the octave c should be audible, (3) why the note a' (or thereabout) should be audible also. (Camb., 1904.)

Ans.—In such a case, the vibrations of the fork no longer follow the pendulum law. Hence (1) at first the vibrations are somewhat slower, (2) the octave is audible, being formed in an analogous way to combination tones. It is as if the fundamental formed summation tones with itself: hence sometimes not only the octave but the Octave Fifth, Double Octave, &c, can be heard. (3) This is the first Overtone. (see p. 123.)

73. Why is it advantageous to have an orchestra or choir compactly arranged in as small a space as possible? (University of Cambridge, 1905.)

Ans.—So that all the simultaneous sounds from the orchestra or choir may strike the tympana of the hearer at the same instant.

74. What are the chief combination tones that can be heard accompanying the notes C, E, c, when sounded together. (Cambridge, 1905.)

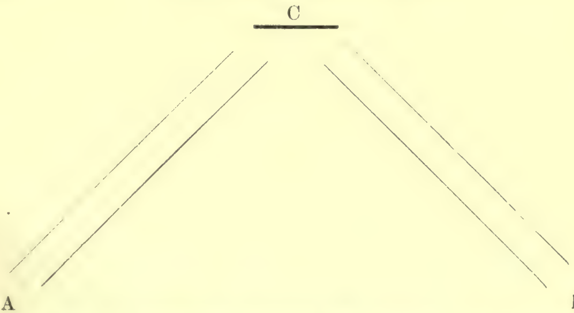
Ans.— Difference tone due to C and E :— C,,
Do. ,, ,, E and c :— G,

75. Explain how a sound can be heard at a much greater distance through a speaking tube than in the open, and why a sound can be heard at a much greater distance in a wind if the wind blows from the source of sound to the observer, than if the wind is blowing in the opposite direction. (Cambridge, 1905.)

Ans.— For first part see p. 55
For latter part see p. 320

76. Describe the phenomenon of reflection of sound. How can this phenomenon be shown in a room with the aid of a sensitive flame? Cambridge, 1905.)

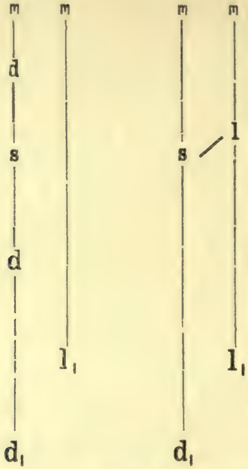
Ans.— For first part see pp. 54 and 55



For latter part :— Arrange two tubes, say 4 inches in diameter, and a few feet long, as in diagram. Let source of sound, say a watch, be placed at A, and the sensitive flame at B. Adjust the sensitiveness of the flame, till it just does not respond to the tick of the watch. If necessary have a screen between A and B. Now place a reflector C, a book will do, so that the angle of incidence is equal to the angle of reflection and the flame will at once respond.

77. A major sixth is played by an open and a stopped organ pipe. What difference in the effect will there be in the two cases when (1) the open pipe plays the lower note and (2) the stopped pipe plays the lower note? (University of Cambridge, 1906.)

Ans.—



As seen above, in (1) there is no element of roughness whatever, but in (2) there is the beating of first overtones at the interval of a tone.

78. A violoncello solo is reproduced on a phonograph, but the phonograph is run twice as fast as when the record was taken. Would the solo sound in tune? If so, would it be possible to distinguish the reproduction from the reproduction of the same solo played by a violin? (Camb., 1906.)

Ans.—The solo would sound in tune. It would be easy to distinguish the reproductions, because although the pitch might be the same, the quality of the tones of the 'cello differs from that of the violin.

79. A train is approaching a hill from which a well defined echo can be heard. The engine driver sounds his whistle; what will be the character of the echo heard by (1) the engine driver, (2) an observer who is stationary? (Camb., 1906.)

Ans.—(1) To the engine driver the echo will appear sharper than the whistle, owing to Doppler's Law (see p. 305, No. 14) and thus beats will be heard. (2) If the observer is between the engine and the hill, there will be no difference between the pitch of the whistle and the echo; but after the engine has passed him, the pitch of the whistle will apparently flatten (see p. 305, No. 14) and thus beats will be heard between this and the echo.

80. A horn sounds a note whose frequency is 250. What will be the effect, if any, on the pitch of the note heard by an observer of (1)

the observer approaching the sound rapidly, (2) the horn approaching the observer rapidly, (3) a strong wind blowing from the horn to the observer? (University of Camb., 1907.)

Ans.—(1) The pitch is raised above 250 (see p. 305, No. 14)

(2) The pitch is raised above 250 and a trifle more than in case (1)

(3) No change of pitch.

81. Two strings are tuned approximately a fourth apart. Their frequencies are 300 and 402·5. If the first six harmonics are present, what will be the character of the beats that can be heard? (Camb., 1907.)

Ans.—The frequencies of the first six harmonics are

600, 900, 1200, 1500, 1800, 2100, and
805, 1207·5, 1610, 2012·5, 2415, 2817·5,

of these only 1207·5 and 1200 are within beating distance, and these will produce 7·5 beats per sec.

82. A tuning fork when held above a column of air 13 inches long and closed at one end causes it to resound. Find the frequency of the tuning fork (velocity of sound = 1100). (Victoria University, 1907.

Ans.— $$\frac{1100 \times 12}{13 \times 4} = 254 \text{ nearly.}$$

83. If the vibration number of g is 192, what are the vibration numbers of c, e′, d′ and f′♯? University of Edinburgh, 1909.

Ans.—
$$c = \frac{192}{1} \times \frac{2}{3} = 128$$
$$e' = \frac{128}{1} \times \frac{2}{1} \times \frac{5}{4} = 320$$
$$d' = \frac{192}{1} \times \frac{3}{2} = 288$$
$$f'\sharp = \frac{288}{1} \times \frac{5}{4} = 360$$

84. If the vibration number of a′ is 440 what are the vibration numbers of d′, f′♯, c″♯ and e″? University of Edinburgh, 1910.

Ans.—
$$d' = \frac{440}{1} \times \frac{6}{5} \times \frac{1}{2} \times \frac{9}{8} = 297$$
$$f'\sharp = \frac{297}{1} \times \frac{5}{4} = 371\tfrac{1}{4}$$
$$c''\sharp = 371\tfrac{1}{4} \times \frac{3}{2} = 556\tfrac{7}{8}$$
$$e'' = \frac{440}{1} \times \frac{3}{2} = 660$$

85. How is the apparent pitch of a sound affected by motion of (1) the source of sound, (2) the observer?

Two horns on a moving motor car sound to the driver a perfect fifth. Will a stationary observer hear the same or a different interval? Mus. Bac. Cambridge, 1908.

Ans.—(1) For first part of question see page 305, No. 14.

(2) Let V, be velocity of sound,

and n, be vibration number of one horn when at rest,

then $\frac{V}{n}$ is the wave length at rest,

i.e., $\frac{V}{n}$ is distance the sound travels in $\frac{1}{n}$th of a second.

Now let a be velocity of motor car,

then $\frac{a}{n}$ is distance motor car travels in $\frac{1}{n}$th of a second.

Thus in this case the sound travels $\frac{V}{n} - \frac{a}{n}$ in $\frac{1}{n}$th of a second,

when motor is travelling with velocity a, that is $\frac{V-a}{n}$ is the wave length as regards an obsever in advance of the car.

And to such an observer $V \div \frac{V-a}{n} = n\left(\frac{V}{V-a}\right)$ is the vibration number.

Similarly to an observer behind the car the vibration number would be

$$n\left(\frac{V}{V+a}\right)$$

Let 300 and 200 be the vibration numbers as heard by driver, who is at rest relatively to the horn. Then when motor is moving with velocity a, the vibration numbers, as regards a stationary observer will be as

$$300\left(\frac{V}{V\mp a}\right) \text{ to } 200\left(\frac{V}{V\mp a}\right) \text{ that is 300 to 200}$$

so the observer hears the same interval, though the notes themselves are sharpened or flattened.

86. Two steel wires of lengths 8 and 12 inches, are found to give the same notes when stretched by weights of 40 and 30 lbs. respectively. Compare their thicknesses. Mus. Bac., Cambridge, 1909.

Ans.—Let vibration number of note be n.

Then if lengths were increased to 24 inches, other things remaining the same, the vibration numbers would be as

$$\frac{n}{3} : \frac{n}{2}$$

Now if stretching weights were increased to 120, the vibration numbers would be as

$$\frac{n}{3} \times \sqrt{3} : \frac{n}{2} \times \sqrt{4} \text{ or } \frac{n}{\sqrt{3}} : n$$

That is, if the two strings were similar in every respect except thickness, their vibration numbers would be as $1 : \sqrt{3}$
Therefore their thicknesses are as $\sqrt{3} : 1$

87. How would you ascertain the vibration number of any extremely high sound, one near the highest limit of ordinary perception? Please suggest some device (as simple as possible) by which to produce these high tones. What are "bird calls?"

Ans. The best way of producing extremely high notes is by means of the "bird call." In this instrument, a stream of air issuing from a circular hole in a tin plate impinges centrically upon a similar hole in a parallel plate held at a little distance. To make a "bird call," cut out from thin sheet metal (ordinary tin plate will do) a disc about the size of a shilling or less. Drill a hole at the centre of small diameter, say 1 millimetre, and solder a short supply tube to the disc, so that air blown through the supply tube will pass through the hole. Now cut out another plate of the shape of an equilateral triangle, just large enough to lie on the circular disc, or a trifle larger. Drill a hole in the centre of this triangle similar to the hole in the disc. Turn down the corners of the triangle at right angles and solder them to the disc in such a manner as to get the two holes exactly opposite and the distance between the plates small, say 2 or 3 millimetres.

To use "bird calls," simply blow down the supply pipe. The pitch is almost independent of the size and shape of the plates. It varies directly as the velocity of the jet and inversely as the distance between the plates, that is to say, as long as we keep to the same instrument, we raise the pitch by blowing harder down the supply pipe, and *vice versa*—but with different instruments the nearer the plates are, the higher the pitch of the note produced. For calls of medium pitch, the plates may be of tin plate as mentioned above, but for calls of very high pitch, thin brass or sheet silver is more suitable. In that case the hole may be as small as ½ millimetre in diameter and the distance between them as little as 1 millimetre. In any case the edges of the holes should be sharp and clean.

By means of these "bird calls," tones of higher and higher pitch may be obtained, until they exceed the limits of audition. It is quite impossible to ascertain the pitch of such high tones as are referred to in this question by any audible method. The only way with such exceedingly high tones and also with the still higher tones which are beyond the auditory limits is to actually measure the wave length and from this to deduce the vibration number. This is effected with the aid of the high pressure sensitive flame. The "bird call" blown at a constant pressure by means of an acoustical bellows is placed two or three feet from a plane reflecting surface. The direct and reflected waves will interfere so as to produce stationary waves with stationary nodes and ventral segments. The sensitive flame, carefully adjusted to the point of flaring, is then moved backwards or forwards between the call and the reflecting surface, till a point is reached where it is least affected. This point is the middle of a ventral segment. It is then moved cautiously till a successive point of least movement is reached. This is the middle of the succeeding ventral segment. Twice the distance between these two points is the wave length. This divided into the velocity of sound at the temperature of the room gives the vibration number.

Wave lengths as small as 1 centimetre, or with great care, even ·6 centimetre may be obtained, the latter corresponding to upwards of 50,000 vibrations per second. Of course such tones are absolutely inaudible. In practice it is found more convenient and more accurate to move the reflecting surface and keep the flame stationary.

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